




Radioactivity and the limits of the Standard Model

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- 1995: PhD at IPN Louvain-la-Neuve (Belgium)

  Experiments with the first post-accelerated radioactive beam (^{13}N)

  Main topics: nuclear astrophysics & elastic scattering

- 1995-1997: Postdoctoral position at GANIL (Caen)

  Experiments with ORION (neutron calorimeter)

  Main topics: halo nuclei & nuclear waste

- 1997: Permanent position at UNICAEN, LPC Caen

  Experiments with TONNERRE (neutron detector) and LPCTrap (Paul trap)

  Main topics: nuclear shell structure & Standard Model tests

  2005: Responsible of LPCTrap, research focused on SM tests

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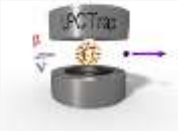


lpc caen Laboratoire de physique corpusculaire

Outline

I. Introduction (13 slides)

- Why and How (LE vs HE)?
- Current questions and goals of the lectures
- A quick reminder on beta decay (Prerequisites)

II. Nuclear beta decay: How testing the weak interaction? (61 slides)

- Some tracks on theory: from Golden rule to events distributions
- Which terms for which physics?
- A word on some approximations and consequences…
- A special case: the Fierz term
- The Standard Model (SM) and beyond (helicity, "ft" values,…)

III. From theoretical rates to correlation experiments (21 slides)

- Beta-neutrino correlations
- Correlations involving polarized decaying nuclei

IV. Last section: CVC, V_{ud} & CKM (20 slides)

- Pure Fermi decays
- Other sources: nuclear mirror decays
- Other sources: the neutron case

Introduction

Standard Model:

- 3 / 4 fundamental interactions:

  strong
  electromagnetic } electroweak
  weak
  gravitation

- Force mediating particles: bosons

  strong interaction: gluons
  electromagnetism: photon
  weak interaction: W^+, W^-, Z^0

- 3 generations of elementary particles: fermions

leptons	$\begin{pmatrix} \nu_e \\ e \end{pmatrix}$	$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}$	$\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$
quarks	$\begin{pmatrix} u \\ d \end{pmatrix}$	$\begin{pmatrix} c \\ s \end{pmatrix}$	$\begin{pmatrix} t \\ b \end{pmatrix}$

Precision measurement @ low energies in nuclear β decay = sensitive tool to test the electroweak Standard Model

- 3 / 4 fundamental interactions:

  strong
  electromagnetic } electroweak
  weak
  gravitation

- Force mediating particles: bosons

  strong interaction: gluons
  electromagnetism: photon
  weak interaction: W^+, W^-, Z^0 + ? ? ?

Search for
exotic couplings

CKM
unitarity

- 3 generations of elementary particles: fermions

leptons $\begin{pmatrix} \nu_e \\ e \end{pmatrix} \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix} \begin{pmatrix} ? \\ ? \end{pmatrix}$

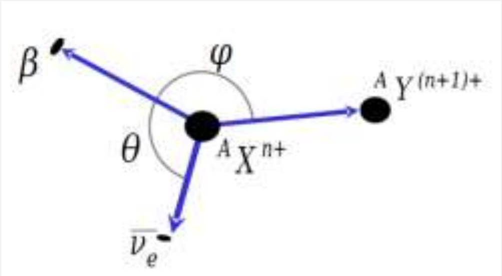
quarks $\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix} \begin{pmatrix} ? \\ ? \end{pmatrix}$

Precision measurement @ low energies in nuclear β decay
= complementary to high energies measurements

Hergé, "Tintin au Tibet", Ed. Casterman

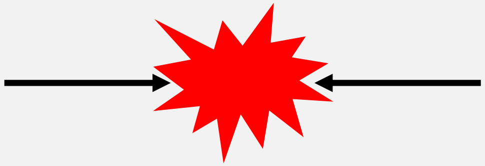
Search for "traces"

low energy (q < < M)



Meet the beast

high energy (E ~ M)



"Traces" = information hidden in a complex medium

→ Judicious selection of measured parameter & chosen transition

→ Be aware of the limits of the medium effects modelling

fundamental theory ⟷ ? experimental data

Hergé, "Tintin au Tibet", Ed. Casterman

Remark

Misinterpretation of particle physics data can also arise …

Role of nuclear physics experiments in the foundations of the Standard Model …

→ Discovery of a new « force »: weak interaction

→ Evidence of the smallness of neutrino mass: direct measurements of beta decay spectra

→ Determination of the nature of the weak interaction: "V-A" theory

→ Discovery of parity (P) violation → "helicity" structure of SM

→ Evidence of vectorial current conservation and quarks mixing matrix

→ …

… which are not the end of the story !

Some current key questions

→ Why do we observe matter and almost no antimatter in the universe ?

→ Why can't the SM predict a particle's mass ?

→ Are quarks and leptons actually fundamental ?

→ Are there exactly 3 generations of quarks and leptons ?

→ Are there other mediating particles ?

→ What are the properties and nature (Dirac or Majorana) of the neutrino ?

Some current key questions *with contributions from nuclear physics…*

→ Why do we observe matter and almost no antimatter in the universe ?

→ Why can't the SM predict a particle's mass ?

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→ Are there other mediating particles ?

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Goal of the lectures

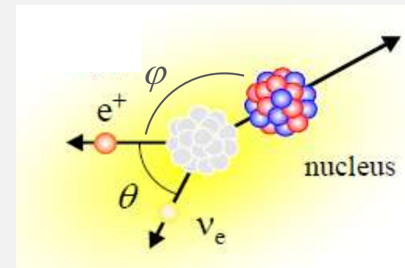
→ A first attempt to understand the link between fundamental equations (ie DIRAC) and events distributions in nuclear beta decays

→ Which parameters to which physics ?

→ Some illustrations …

A quick reminder on nuclear beta decay (prerequisites)

Nuclear beta decay = semi-leptonic process governed by **weak interaction**



I. Kinematics

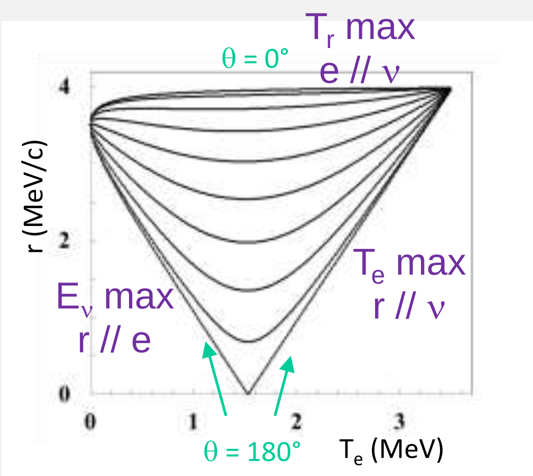
3 bodies → continuous spectra

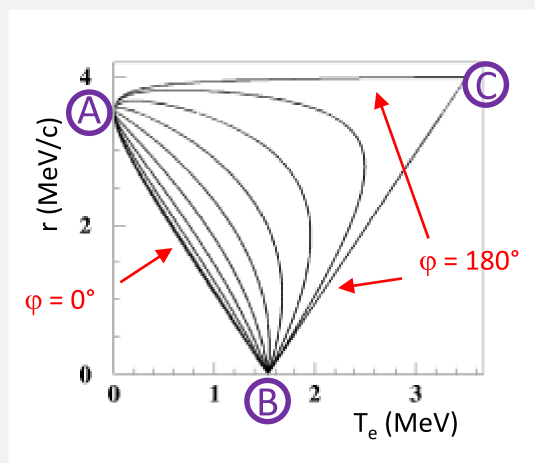
Energy conservation:	$Q_\beta = T_e + T_r + E_\nu$
Momentum conservation:	$\vec{0} = \vec{p_e} + \vec{r} + \vec{p_\nu}$

Electron axis = reference axis

θ : β–ν angle

φ : β–recoil angle





3 specific points :

- Ⓐ → $T_e = 0$
- Ⓑ → $T_r = 0$
- Ⓒ → $E_\nu = 0$

⇓

Description of the particles distribution in the border regions

At Ⓒ : $Q_\beta \simeq T_e^{max}$ and $p_e^{max} = r^{max} = \sqrt{Q_\beta^2 + 2m_e Q_\beta}$

A quick reminder on nuclear beta decay (prerequisites)

II. Fermi theory

Goal : reproduce energy distribution of β particles

Starting point : perturbation theory

$$\lambda = \frac{2\pi}{\hbar} \left| V_{fi} \right|^2 \frac{dN}{dE_0}$$

Fermi's Golden Rule

decay constant

transition probability

$(< f \mid H \mid i >)^2$

density of final states

Interaction is here !

→ dλ = N(chosen variables) → events distribution

→ λ = ∫dλ → (ft) values → classification & selection rules

A quick reminder on nuclear beta decay (prerequisites)

II. Fermi theory

Basic ingredients :

$$\boxed{(< f \mid H \mid i >)^2}$$

$$\boxed{\frac{dN}{dE_0}}$$

- $<f|H|i> = g \int(\varphi_e\, \varphi_\nu\, \Psi_f)^* O\, \Psi_i\, d\tau$
  where g: coupling constant O: operator
  $\Psi_{i,f}$: nuclear states $\varphi_{e,\nu}$: leptons states
- $\varphi_{e,\nu} \sim 1$: plane waves in *allowed approximation*
- $M_{fi} = \int(\Psi_f)^* O\, \Psi_i\, d\tau$: nuclear matrix element

⇒ $(< f \mid H \mid i >)^2 \sim g^2 |M_{fi}|^2$

- Nuclear states fixed, number of states given by leptons states
- What is the volume occupied by a quantum cell ?
- Value computed for an electron at a given p_e at ± dp_e → $N(p_e)$: events distribution

⇒ $d\left(\frac{dN}{dE_0}\right) \sim p_e^2(Q - T_e)^2\, dp_e$

$$\boxed{dN(p_e) = K\, F(\pm Z', p_e)\, p_e^2(Q - T_e)^2\, dp_e}$$

F : Fermi function → final state interaction (β vs nucleus) : Coulomb correction
K : constant for a given decay, containing g & M_{fi}

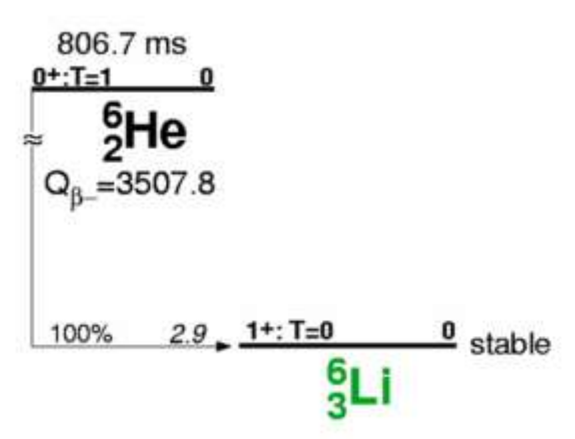
A quick reminder on nuclear beta decay (prerequisites)

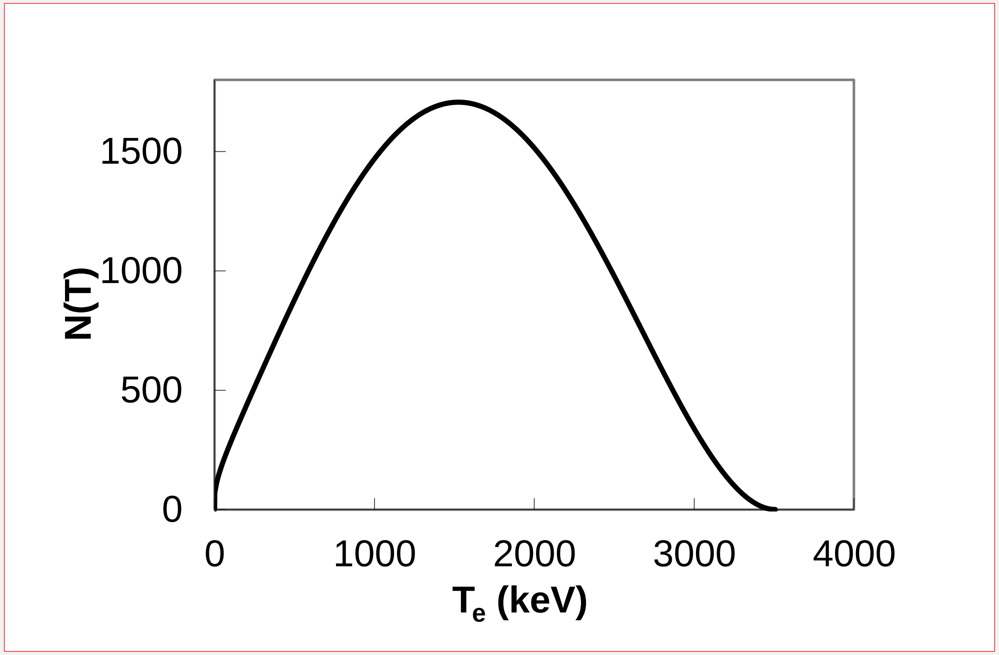
II. Fermi theory

$$\boxed{dN(p_e) = K\, F(p_e)\, p_e^2 (Q - T_e)^2\, dp_e}$$

$$\boxed{dN(T_e) = K'\, F(T_e)\, (T_e^2 + 2m_eT_e)^{1/2}\, (Q - T_e)^2\, (T_e + m_e)\, dT_e} \qquad (c = 1)$$

$$^{6}\mathrm{He} \rightarrow {}^{6}\mathrm{Li} + \beta^{-} + \overline{\nu}_e$$





A quick reminder on nuclear beta decay (prerequisites)

II. Fermi theory

$$\lambda = \int d\lambda = \int dN(p_e) = K \underbrace{\int_0^{p_e^{max}} F(pe)\, pe^2 (Q - Te)^2\, dpe}_{} = \ln(2)/t_{1/2}$$

$\longrightarrow$ $f(Z',E_0)$: statistical rate function (Fermi integral)

! *$t_{1/2}$ is a partial half-life:*
$t_{1/2} = T_{1/2}/BR$

ft value

$$\boxed{ft_{1/2} = \frac{4.79410^{-5}}{g^2 |M_{fi}|^2}\, s}$$

www.nndc.bnl.gov/logft/

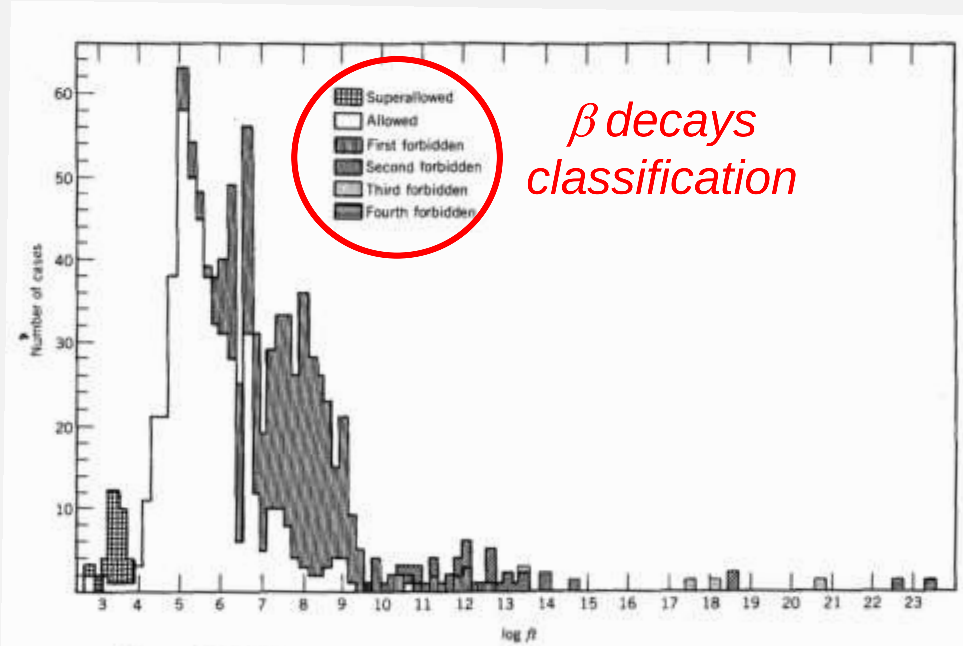


Figure 9.9 Systematics of experimental log *ft* values. From W. Meyerhof, *Elements of Nuclear Physics* (New York: McGraw-Hill, 1967).

(from K.S. Krane "Introductory nuclear physics")

A quick reminder on nuclear beta decay (prerequisites)

III. β decays classification and selection rules

- *Allowed approximation* → leptons do not carry orbital angular momentum : ℓ = 0

⇨ ΔJ linked to leptons spins alignement : anti // → ΔJ = 0 : Fermi (F) decays
// → ΔJ = 1 : Gamow-Teller (GT) decays
→ *"Allowed" transitions* : ΔJ = 0,1 without parity change (ℓ = 0)

⇨ Total momentum change has to be taken into account in GT transitions
→ $O_F = \tau$ isospin operator : n → p or p → n
→ $O_{GT} = \tau\sigma$ isospin-spin operator (includes Pauli matrices)

⇨
$$ft_{1/2} = \frac{4.794\,10^{-5}}{g_F^2 |M_F|^2} s \qquad ft_{1/2} = \frac{4.794\,10^{-5}}{g_{GT}^2 |M_{GT}|^2} s \qquad ft_{1/2} = \frac{4.794\,10^{-5}}{g_F^2 |M_F|^2 + g_{GT}^2 |M_{GT}|^2} s$$

"Pure" F — "Pure" GT — Mixed

$J_i = 0 \rightarrow J_f = 0$

"Mirror"

$$ft_{1/2} = \frac{4.794\,10^{-5}}{g_F^2 |M_F|^2 (1+\rho^2)} s$$

ρ is the mixing ratio

- *"Forbidden" transitions* → ℓ ≠ 0

beyond allowed approximation → f is modified

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Nuclear β decay: How testing the weak interaction ?

Some tracks on theory: from Golden rule to events distributions…

$$d^2\lambda = \mathrm{N(variables)} = \frac{2\pi}{\hbar}\left|V_{fi}\right|^2 \mathrm{d}\left(\frac{dn_e dn_\nu}{dE_0}\right)$$

transition probability

$\boxed{(< f \mid H \mid i >)^2}$

density of final states

To make experiments sensitive to fundamental interaction

II. Interaction is here !
It is necessary to go deeper in theory :
- How managing hadrons & leptons ?
- How involving *Dirac formalism*, the fundamental relativistic wave equation ?

I. Constrain the open space to reach correlations (angular correlations !)

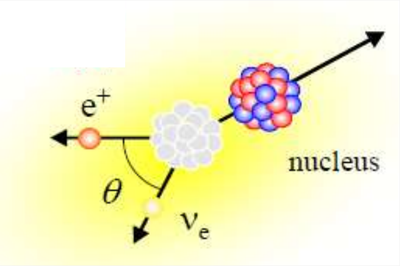
Some tracks on theory: from Golden rule to events distributions…

I. Constrain the open space to reach correlations (angular correlations !)

Fermi : $d\left(\frac{dn_e dn_\nu}{dE_0}\right) \Longrightarrow d\left(\frac{dN}{dE_0}\right) \sim p_e^2(Q - T_e)^2\, dp_e$

Here, the whole space is open: 4π for e^- & 4π for ν_e

To study β–ν correlations :



$d\Omega_e = 4\pi$
$d\Omega_\nu = 2\pi \sin(\theta)d\theta$

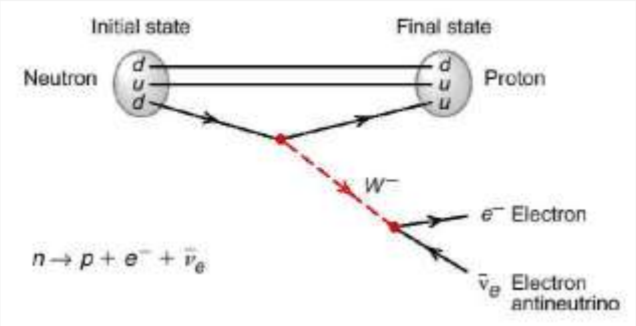
$$\boxed{d\left(\frac{dn_e dn_\nu}{dE_0}\right) \sim p_e^2(Q - T_e)^2 \sin(\theta)\, dp_e d\theta}$$

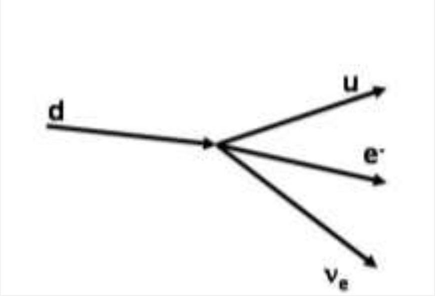
Some tracks on theory: from Golden rule to events distributions…

II. The transition probability (< f | H | i >)²

- How managing hadrons & leptons ? Fermi basic hypothesis :

1. Low energy ($q \ll M$) : point-like interaction with 4 fermions (no propagator)





2. Description // electromagnetism: interaction between a current and a radiation field

E-M interaction density: $H \sim e\, J.A$ *J: current, A: potential, e: interaction strength*

➡ β decay interaction density: $H_\beta \sim g\, J.L$

where *J: Hadronic "current", L: Leptonic "potential", g: interaction strength*

Q : What are expressions for J and L ?

Some tracks on theory: from Golden rule to events distributions…

II. The transition probability (< f | H | i >)²

H ~ g J.L

What are expressions for J and L ?

Responses are in Dirac fundamental formalism !

→ A quick reminder on quantum mechanics (*prerequisites*…)

1. Particles are waves described by specific equations of the form $H\Psi = E\Psi$ (1)

2. They have to comply with the equation of continuity

- Analogy with E-M : (charge conservation) $\frac{\partial \rho}{\partial t} + \vec{\nabla}.\vec{j} = 0$ (the charge variation in a volume = the current escaping the surface…)

- In quantum mechanics : a. equation deduced from Ψ^* x (1) – (1)* x Ψ

  b. ρ is interpreted as a density of probability

  c. $\vec{j}$ gives the expression of the "current" !

A quick reminder on quantum mechanics (*prerequisites*…)

1. Waves equations ?

- Non relativistic free particle → Schrödinger ! $\frac{\nabla^2}{2m}\Psi = -i\frac{\partial \Psi}{\partial t}$ ($\hbar$, c = 1)

- Relativistic free particle → Klein-Gordon ?

E² = p² + m² $\longrightarrow$ *correspondence principle* $(\nabla^2 - m^2)\Psi = \frac{\partial^2 \Psi}{\partial t^2}$

However $\rho \sim \Psi^* \frac{\partial \Psi}{\partial t} - \Psi \frac{\partial \Psi^*}{\partial t}$ can be < 0 ! Not satisfactory for a probability !!

- Relativistic free particle → Alternative approach of Dirac

Equation built

- with differentials at 1st order to avoid negative probability densities

- but respecting relativistic energy-momentum relation

$$(\vec{\alpha}.\vec{p} + \beta m)\psi = E\psi \longrightarrow \left(-i\vec{\alpha}.\vec{\nabla} + \beta m\right)\psi = i\frac{\partial \psi}{\partial t}$$

with α_i and β to be determined to retrieve E² = p² + m² !

A quick reminder on quantum mechanics (*prerequisites*…)

1. Dirac equation

$$\left[\left(-i\vec{\alpha}.\vec{\nabla}+\beta m\right)\psi = i\frac{\partial\psi}{\partial t}\right]^2 = Klein - Gordon\,!$$

$$\left(-i\vec{\alpha}.\vec{\nabla}+\beta m\right)\left(-i\vec{\alpha}.\vec{\nabla}+\beta m\right)\psi = \left(-i\vec{\alpha}.\vec{\nabla}+\beta m\right)i\frac{\partial\psi}{\partial t}$$

= K-N if $\alpha_i^2 = \beta^2 = 1$, $\alpha_i\alpha_j + \alpha_j\alpha_i = 0$ $(i \neq j)$, $\alpha_i\beta + \beta\alpha_i = 0$

α_i and β are at least matrices of dimension 4

Dirac-Pauli representation with
σ_i Pauli matrices for spin consideration

$$\alpha_i = \begin{pmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{pmatrix} \qquad \beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$$

Ψ have 4 components
and is called a Dirac spinor

$$\Psi = \begin{pmatrix} \Psi_1 \\ \Psi_2 \\ \Psi_3 \\ \Psi_4 \end{pmatrix}$$

A quick reminder on quantum mechanics (*prerequisites ?*)

1. Dirac equation in "covariant" form

"Covariant" form for a four-vector: $A_\mu = \left(A^0, -\vec{A}\right) \quad p_\mu = (E, -\vec{p})$

- *Equation multiplied on the left by β* $$\beta\left(-i\vec{\alpha}.\vec{\nabla} + \beta m\right)\psi = i\beta \frac{\partial \psi}{\partial t}$$
- *"m" is then isolated and products of quadrivectors can be rewritten* $$\left(i\beta\frac{\partial \psi}{\partial t} + i\beta\vec{\alpha}.\vec{\nabla} - m\right)\psi = 0$$

These are the so-called Dirac matrices or "γ" matrices

$$\boxed{\gamma^\mu = (\beta, \beta\vec{\alpha}) = (\gamma^0, \vec{\gamma})} \qquad \boxed{\partial_\mu = \left(\frac{\partial}{\partial t}, \vec{\nabla}\right)}$$

$$\Rightarrow \quad \boxed{\left(i\gamma^\mu.\partial_\mu - m\right)\psi = 0}$$

A quick reminder on quantum mechanics (*prerequisites ?*)

1. Dirac equation in "covariant" form

Dirac matrices $\boxed{\gamma^\mu = (\beta, \beta\vec{\alpha}) = (\gamma^0, \vec{\gamma})}$

in Dirac-Pauli representation $\gamma^k = \beta\alpha_k = \begin{pmatrix} 0 & \sigma_k \\ -\sigma_k & 0 \end{pmatrix}$ $\quad \gamma^0 = \beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$

k = 1,2,3

Very useful properties:

1) Anticommutation relations :

$\gamma^\mu \gamma^\lambda + \gamma^\lambda \gamma^\mu = 2\, g^{\mu\lambda}$ où $g^{\mu\lambda} =$ 1 si $\mu = \lambda = 0$
-1 si $\mu = \lambda \neq 0$
0 si $\mu \neq \lambda$

2) $(\gamma^0)^2 = I$, $(\gamma^k)^2 = -I$

3) $\gamma^{\mu+} = \gamma^0 \gamma^\mu \gamma^0$ *(hermitian)*

4) $\gamma^5 = i\gamma^0 \gamma^1 \gamma^2 \gamma^3$, $(\gamma^5)^2 = I$, $(\gamma^5)^+ = \gamma^5$, $\{\gamma^5, \gamma^\mu\} = 0$

A "quick" reminder on quantum mechanics ?

Dirac matrices

!

- Before ~ 70's another definition for the Dirac-Pauli representation:

$$\gamma^{k} = \begin{pmatrix} 0 & -i\sigma_{k} \\ i\sigma_{k} & 0 \end{pmatrix} \qquad \gamma^{4} = \gamma^{0} = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$$

and $\gamma^5 = \gamma^1 \gamma^2 \gamma^3 \gamma^4$, $(\gamma^5)^2 = I$, $(\gamma^5)^+ = \gamma^5$ ...

→ *Possible reverse sign in some expressions…*

It is used in papers published in 50's and 60's
while the "new" definition is often used in more recent papers

- Other representations exist, used for specific purpose

Example: Weyl representation → 2 components theory of neutrino (see later)

A "quick" reminder on quantum mechanics ?

2. Equation of continuity

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}.\vec{j} = 0$$

Ψ^{*} x (Equation) – (Equation)* x Ψ $\Rightarrow$ Ψ^{+} x (Equation) – (Equation)$^{+}$ x Ψ

because they are matrices

<u>Equation</u> : $\left(i\gamma^{\mu}.\partial_{\mu} - m\right)\psi = 0 \Rightarrow i\gamma^{0}\frac{\partial \psi}{\partial t} + i\sum_{k=1}^{3}\gamma^{k}\frac{\partial \psi}{\partial x_k} - m\psi = 0$

$\Rightarrow$ *hermitian*

$$-i\frac{\partial \Psi^{+}}{\partial t}\underbrace{(\gamma^{0})^{+}}_{=\gamma^{0}} - i\sum_{k=1}^{3}\frac{\partial \Psi^{+}}{\partial x_k}\underbrace{(\gamma^{k})^{+}}_{=-\gamma^{k}} - m\Psi^{+} = 0$$

$\Rightarrow \neq$ *signs: impossible to write a covariant form !*

Cunning: Multiplication on the right by γ^{0}...

$\Rightarrow$ *… exactly the reverse operation than performed at slide 24 !!*

A "quick" reminder on quantum mechanics ?

2. Equation of continuity

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}.\vec{j} = 0$$

Multiplication on the right by γ^0...

$$-i\frac{\partial \Psi^+}{\partial t}\gamma^0\gamma^0 - i\sum_{k=1}^{3}\frac{\partial \Psi^+}{\partial x_k}(-\gamma^k)\gamma^0 - m\Psi^+\gamma^0 = 0$$

$$= +\gamma^0\gamma^k$$

Thanks to anticommutation relation

If we define a new quantity called the *adjoint spinor* : $\overline{\Psi} = \Psi^+\gamma^0$

then we can write again a covariant form for the *adjoint equation*:

$$\boxed{i\partial_\mu\overline{\psi}\,\gamma^\mu + m\overline{\psi} = 0}$$

A "quick" reminder on quantum mechanics ?

2. Equation of continuity $\frac{\partial \rho}{\partial t} + \vec{\nabla}.\vec{j} = 0$

$\bar{\Psi}$ x (Equation) + (adjoint equation) x $\Psi = 0$

$$\bar{\Psi} \text{ x } \left(i\gamma^{\mu}.\partial_{\mu} - m\right)\psi \quad + \quad (i\partial_{\mu}\bar{\psi}\,\gamma^{\mu} + m\bar{\psi}) \text{ x } \Psi = 0$$

$$\boxed{\partial_{\mu}(\bar{\psi}\gamma^{\mu}\psi) = 0}$$

Form of a "current" : $\boxed{j^{\mu} = \bar{\psi}\gamma^{\mu}\psi}$

$\mu = 0$ correponds to the probability density $\rho = \bar{\psi}\gamma^{0}\psi = \psi^{+}\gamma^{0}\gamma^{0}\psi = \psi^{+}\psi$

Some tracks on theory: from Golden rule to events distributions…

II. The transition probability (< f | H | i >)^2

→ β decay interaction density: $H_\beta \sim g\, J.L$

where *J: Hadronic "current", L: Leptonic "potential", g: interaction strength*

Q : What are expressions for J and L ?

$J \propto \overline{\psi}\gamma^\mu\psi$ and $L \propto \overline{\psi}\gamma^\mu\psi$ too

to ensure H_β to be a Lorentz invariant !

→ $$\boxed{H_\beta \sim (\overline{\Psi}_p\gamma^\mu\Psi_n)\,(\overline{\Psi}_e\gamma^\mu\Psi_\nu)}$$

in a very "basic" version (Fermi theory in fact)…

Some tracks on theory: from Golden rule to events distributions…

II. The transition probability (< f | H | i >)²

A more general version can be built, involving all possible currents combinations

- as 16 independant matrices can be built from the Dirac matrices:

I	γ^μ	$\gamma^\mu \gamma^\nu \ (\mu < \nu)$	$\gamma^\lambda \gamma^\mu \gamma^\nu \ (\lambda < \mu < \nu)$	$\gamma^0 \gamma^1 \gamma^2 \gamma^3$	
[1]	[4]	[6]	[4]	[1]	*number of matrices*
	γ^0	$\gamma^0\gamma^1$	$\gamma^0\gamma^1\gamma^2$		
	γ^1	$\gamma^0\gamma^2$	$\gamma^0\gamma^1\gamma^3$		
	γ^2	$\gamma^0\gamma^3$	$\gamma^0\gamma^2\gamma^3$		
	γ^3	$\gamma^1\gamma^2$	$\gamma^1\gamma^2\gamma^3$		
		$\gamma^1\gamma^3$			
		$\gamma^2\gamma^3$			

- giving the following basic currents:

$$\overline{\psi}\psi \qquad \overline{\psi}\gamma^\mu\psi \qquad \overline{\psi}\gamma^\mu\gamma^\nu\psi \qquad \overline{\psi}\gamma^\lambda\gamma^\mu\gamma^\nu\psi \qquad \overline{\psi}\gamma^0\gamma^1\gamma^2\gamma^3\psi$$

Some tracks on theory: from Golden rule to events distributions…

II. The transition probability $(\langle f | H | i \rangle)^2$

Basic currents	*Behaviour under Lorentz transformation*		*Type of current*	
$\bar{\psi}\psi$	Invariant *Even for coordinates inversion (P)*	➡	Scalar	*S*
$\bar{\psi}\gamma^{\mu}\psi$	Like a vector *In particular sign change under P*	➡	Vector	*V*
$\bar{\psi}\gamma^{\mu}\gamma^{\nu}\psi$	Like a tensor of rank 2 *In particular invariant under P*	➡	Tensor	*T*
$\bar{\psi}\gamma^{\lambda}\gamma^{\mu}\gamma^{\nu}\psi$ replaced by $\bar{\psi}\gamma^{5}\gamma^{\mu}\psi$	Like a vector *But invariant under P*	➡	Axial-vector	*A*
$\bar{\psi}\gamma^{0}\gamma^{1}\gamma^{2}\gamma^{3}\psi$ replaced by $\bar{\psi}\gamma^{5}\psi$	Invariant *But sign change under P*	➡	Pseudoscalar	*P*

Some tracks on theory: from Golden rule to events distributions…

II. The transition probability (< f | H | i >)²

- General form of *β decay "hamiltonian"* (Lorentz invariant → scalar form)

$$H_\beta = \sum_{i=V,A,S,T,P} C_i\left(\overline{\psi}_p(x)\,\hat{O}_i\,\psi_n(x)\right)\left(\overline{\psi}_e(x)\hat{O}_i\psi_\nu(x)\right) + h.c.$$

with C_i *(~g_i)* : coupling constants, chosen complex in general case

h.c. : Hermitian conjugates written explicitely for symmetry

- After Wu's experiment (P violation), a component involving γ^5 was added "by hand"

$$H_\beta = \sum_{i=V,A,S,T,P} \left(\overline{\psi}_p(x)\,\hat{O}_i\,\psi_n(x)\right)\left(\overline{\psi}_e(x)\hat{O}_i(C_i + C_i'\gamma_5)\psi_\nu(x)\right) + h.c.$$

with *C_i' , C_i : 2 different coupling constants to control degree of P violation*

- Standard Model: only *V & A* → *"Standard" currents*

Not only…!

→ *The study of correlations in β decay enables to test existence of "exotic" currents*

Some tracks on theory: from Golden rule to events distributions…

II. The transition probability   (< f | H | i >)²

$$H_\beta = \sum_{i=V,A,S,T,P} \left(\overline{\psi}_p(x)\,\hat{O}_i\,\psi_n(x)\right)\left(\overline{\psi}_e(x)\,\hat{O}_i(C_i + C_i'\gamma_5)\,\psi_\nu(x)\right) + h.c.$$

→ *The study of correlations in β decay enables to test existence of "exotic" currents*

but also the degree of Parity (P) violation (weight of term containing γ^5)

and why not Time reversal (T) and Charge conjugation (C) ... ??

P *r → -r*
Space inversion

T *t → -t*
Time reversal

C *q → -q*
Charge conjugation

Mirror reflection + rotation by 180° (invariant)

Some tracks on theory: from Golden rule to events distributions…

Transformation of H_β under P, T or C ➡ Invariance conditions on C_i, C_i' ?

Technique:
1. Transformation of wave functions (using Dirac equation)
2. Transformation of Basic currents
3. Transformation of H_β

long & beyond the scope

Important effects:
1. Eventual change of signs (*P*, *T*, *C*)
2. Inversion of role of particles (*T*, *C*)
3. Change of sign for terms involving γ^5 (*P*, *T*)

requires to write explicitely h.c.

1. $C_i, C_i' \to C_i^*, C_i'^*$ (c.c.)
2. $(\overline{\Psi}_a O_i \Psi_b) \to (\overline{\Psi}_b O_i \Psi_a)$
3. $(\overline{\Psi}_a O_i \gamma^5 \Psi_b) \to -(\overline{\Psi}_b \gamma^5 O_i \Psi_a)$

$$h.c. = \sum_{i=V,A,S,T,P} \left(\overline{\psi}_n(x)\,\hat{O}_i\,\psi_p(x)\right)\left(\overline{\psi}_\nu(x)(C_i^* - C_i'^*\gamma_5)\hat{O}_i\psi_e(x)\right)$$

Some tracks on theory: from Golden rule to events distributions…

$\boxed{P}$

1. Eventual change of signs (*P*, *T*, *C*) → *2X (H & L)* → *no effect !!*
2. ~~Inversion of role of particles (*T*, *C*)~~
3. Change of sign for terms involving γ^5 (*P*, *T*)

$$H^P = \sum_i \left(\overline{\psi}_p(x')\,\hat{O}_i\,\psi_n(x')\right)\left(\overline{\psi}_e(x')\hat{O}_i(C_i - C_i'\gamma_5)\psi_\nu(x')\right) + h.c.$$

→

- invariance (means =) if $C_i' = 0$
- maximal parity violation: $|C_i| = |C_i'|$

Consistent with the addition of this term to account for P violation…

$$H_\beta = \sum_i \left(\overline{\psi}_p(x)\,\hat{O}_i\,\psi_n(x)\right)\left(\overline{\psi}_e(x)\hat{O}_i(C_i + C_i'\gamma_5)\psi_\nu(x)\right) + h.c.$$

Some tracks on theory: from Golden rule to events distributions…

$\boxed{T}$

1. Eventual change of signs (*P*, *T*, *C*) → *2X (H & L)* → *no effect !!*
2. Inversion of role of particles (*T*, *C*)
3. Change of sign for terms involving γ^5 (*P*, *T*)

$$H^T = \sum_i \left(\overline{\psi}_n(x'')\,\hat{O}_i\,\psi_p(x'')\right)\left(\overline{\psi}_\nu(x'')(C_i - C_i'\gamma_5)\hat{O}_i\psi_e(x'')\right) + $$
$$+ \left(\overline{\psi}_p(x'')\,\hat{O}_i\,\psi_n(x'')\right)\left(\overline{\psi}_e(x'')\hat{O}_i(C_i^* + C_i'^*\gamma_5)\psi_\nu(x'')\right)$$

- invariance (means =) if $C_i = C_i^*$ & $C_i' = C_i'^*$
  $\rightarrow$ C_i & C_i' real

$$H_\beta = \sum_i \left(\overline{\psi}_p(x)\,\hat{O}_i\,\psi_n(x)\right)\left(\overline{\psi}_e(x)\hat{O}_i(C_i + C_i'\gamma_5)\psi_\nu(x)\right) + $$
$$+ \left(\overline{\psi}_n(x)\,\hat{O}_i\,\psi_p(x)\right)\left(\overline{\psi}_\nu(x)(C_i^* - C_i'^*\gamma_5)\hat{O}_i\psi_e(x)\right)$$

Some tracks on theory: from Golden rule to events distributions…

$\boxed{C}$

1. Eventual change of signs (*P*, *T*, *C*) → *2X (H & L)* → *no effect !!*
2. Inversion of role of particles (*T*, *C*)
3. ~~Change of sign for terms involving γ^5 (*P*, *T*)~~

$$H^C = \sum_i \left(\overline{\psi}_n(x)\,\hat{O}_i\,\psi_p(x)\right)\left(\overline{\psi}_\nu(x)(C_i + C_i'\gamma_5)\hat{O}_i\psi_e(x)\right) +$$
$$+\left(\overline{\psi}_p(x)\,\hat{O}_i\,\psi_n(x)\right)\left(\overline{\psi}_e(x)\hat{O}_i(C_i^* - C_i'^*\gamma_5)\psi_\nu(x)\right)$$

- invariance (means =) if $C_i = C_i^*$ & $C_i' = -C_i'^*$
  $\rightarrow$ C_i real & C_i' imaginary

$$H_\beta = \sum_i \left(\overline{\psi}_p(x)\,\hat{O}_i\,\psi_n(x)\right)\left(\overline{\psi}_e(x)\hat{O}_i(C_i + C_i'\gamma_5)\psi_\nu(x)\right) +$$
$$+\left(\overline{\psi}_n(x)\,\hat{O}_i\,\psi_p(x)\right)\left(\overline{\psi}_\nu(x)(C_i^* - C_i'^*\gamma_5)\hat{O}_i\psi_e(x)\right)$$

Some tracks on theory: from Golden rule to events distributions…

Transformation of H_β under P, T or C → Invariance conditions on C_i, C_i' ?

SUMMARY

Transformation	Invariance conditions
P	$C_i' = 0$
T	C_i, C_i' real
C	C_i real, C_i' imaginary

- invariance under 2 operations → invariance under the 3rd (CPT theorem !!)
- violation of one symmetry → violation of another one !

→ *The study of correlations in β decay enables*

to test the violation of fundamental symmetries

… and the existence of "exotic" currents

Some tracks on theory: from Golden rule to events distributions…

"Progress report"

$$d^2\lambda = \mathrm{N(variables)} = \frac{2\pi}{\hbar}\left|V_{fi}\right|^2 \mathrm{d}\left(\frac{dn_e dn_\nu}{dE_0}\right)$$

transition probability

(< f | H | i >)²

density of final states

II. Interaction is here !
It is necessary to go deeper in theory :
- How managing hadrons & leptons ?
- How involving *Dirac formalism*, the fundamental relativistic wave equation ?

I. Constrain the open space to reach correlations (angular correlations !)

β–ν correlations

$\sim\ p_e^2 (Q - T_e)^2 \sin(\theta)\, dp_e d\theta$

$$\boxed{H_\beta = \sum_i \left(\overline{\psi}_p(x)\, \hat{O}_i\, \psi_n(x)\right)\left(\overline{\psi}_e(x) \hat{O}_i (C_i + C_i' \gamma_5) \psi_\nu(x)\right) + h.c.}$$

with i = V, A, S, T, P

Some tracks on theory: from Golden rule to events distributions…

$$d^2\lambda = \mathrm{N(variables)} = \frac{2\pi}{\hbar}\left|V_{fi}\right|^2 \mathrm{d}\left(\frac{dn_e dn_\nu}{dE_0}\right)$$

β–ν correlations

$$\boxed{d^2\lambda = N(p_e,\theta)dp_e d\theta = \frac{32\pi^4V^2}{h^7c^3}p_e^2(Q-T_e)^2\left|V_{fi}\right|^2\sin(\theta)dp_e d\theta}$$

here "V" is a normalization volume
which cancels with normalized wave functions
chosen in V_{fi} !!

$V_{fi} = H_\beta$

$$H_\beta = \sum_i \left(\overline{\psi}_p(x)\,\hat{O}_i\,\psi_n(x)\right)\left(\overline{\psi}_e(x)\hat{O}_i(C_i + C_i'\gamma_5)\psi_\nu(x)\right) + h.c.$$

Some tracks on theory: from Golden rule to events distributions…

$\boxed{V_{fi} = H_\beta}$

Framework and approximations

$$H_\beta = \sum_{i=V,A,S,T,P} \left(\overline{\psi}_p(x)\,\hat{O}_i\,\psi_n(x)\right)\left(\overline{\psi}_e(x)\hat{O}_i(C_i + C_i'\gamma_5)\psi_\nu(x)\right) + h.c.$$

Particles described with normalized plane waves

$$\psi(x) = \frac{1}{\sqrt{V}}u(\vec{q})\exp(-ixq) \quad \text{where} \quad xq = (Et\text{-}\vec{q}.\vec{r})$$

$$\& \quad u(\vec{q}) \text{ is a solution of Dirac equation} = \text{Dirac spinor}$$

- For leptons: allowed approximation (ℓ = 0) $\rightarrow$ exp (-ixq) ~ 1
- For nucleons: nonrelativistic approximation (NRA) $\rightarrow$ u(q) = u(0) $\rightarrow$ *Basic spinors*
  (at the particle level, p = n = nucleon)
- Nucleus decay → superposition of plane waves
  each term $\overline{u}_p O_i u_n$ is computed in the frame of NRA

$$u^{(1)} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad u^{(2)} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

Some tracks on theory: from Golden rule to events distributions…

$\boxed{V_{fi} = H_\beta}$

Framework and approximations

Important consequences due to NRA *(at the particle level, p = n = nucleon)*

i	Relativistic expression	NRA
S	$\bar{u}_p u_n$	1 for $u_p = u_n$
V	$\bar{u}_p \gamma^\mu u_n$	$\delta_{\mu 0}$ for $u_p = u_n$
T	$\bar{u}_p \gamma^\mu \gamma^\nu u_n$	$<\sigma^j>$ for $\mu \neq 0$, $\nu \neq 0$ j, μ, ν cycl. 0 for μ or $\nu = 0$
A	$\bar{u}_p \gamma^5 \gamma^\mu u_n$	$-<\sigma^\mu>$ for $\mu \neq 0$ 0 for $\mu = 0$
P	$\bar{u}_p \gamma^5 u_n$	0

Computed explicitly

limited to $\gamma^0 \rightarrow$ no spin !

σ: Pauli matrix $\rightarrow$ spin involved !

no pseudoscalar term !

Nuclear part not easy to compute precisely $\rightarrow$ nuclear matrix element *M*
(in nuclear physics, p ≠ n and Isospin operator makes the job! See slide 16)

S, V: no spin $\rightarrow$ only Fermi transitions: M_F

T, A: spin $\rightarrow$ only Gamow-Teller transitions: M_{GT}

Some tracks on theory: from Golden rule to events distributions…

$\boxed{V_{fi} = H_\beta}$

Framework and approximations

Important consequences due to NRA: example of computation (V) $\bar{u}_p \gamma^\mu u_n$

- μ = 1

$$\gamma^1 = \begin{pmatrix} 0 & \sigma_1 \\ -\sigma_1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \qquad \gamma^1 u_n^{(1)} = \gamma^1 \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ -1 \end{pmatrix}$$

$$\bar{u}_p^{(r)} \gamma^1 u_n^{(1)} = 0 \quad \text{whatever "r" (1 or 2)}$$

- μ = 0

$$\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \qquad \gamma^0 u_n^{(1)} = \gamma^0 \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\bar{u}_p^{(r)} \gamma^0 u_n^{(1)} = 1 \quad \text{if } r = 1$$

Some tracks on theory: from Golden rule to events distributions…

particle *antiparticle*

$$V_{fi} = \frac{1}{V}\sum_i M_i \left[\overline{u}_e^{(+)}(\vec{q}_e) O_i \left[C_i + C_i^{'}\gamma^5\right] u_\nu^{(-)}(-\vec{q}_\nu)\right]$$

where M_i is the nuclear matrix element in the frame of NRA

$$\boxed{d^2\lambda = N(p_e,\theta)dp_e d\theta = \frac{32\pi^4}{h^7 c^3} p_e^2 (Q - T_e)^2 X \sin(\theta) dp_e d\theta}$$

where $X = \left| \sum_i M_i \overline{u}_e^{(+)}(\vec{q}_e) F_i u_\nu^{(-)}(-\vec{q}_\nu) \right|^2$

$$= \sum_{i,j} M_i M_j^* \overline{u}_e^{(+)}(\vec{q}_e) F_i u_\nu^{(-)}(-\vec{q}_\nu) \overline{u}_\nu^{(-)}(-\vec{q}_\nu) \gamma^0 F_j^+ \gamma^0 u_e^{(+)}(\vec{q}_e)$$

$$F_i = O_i \,(C_i + C_i{'}\gamma^5)$$

Some tracks on theory: from Golden rule to events distributions…

$$X = \sum_{i,j} M_i M_j^* \bar{u}_e^{(+)}(\vec{q}_e) F_i u_\nu^{(-)}(-\vec{q}_\nu) \bar{u}_\nu^{(-)}(-\vec{q}_\nu) \gamma^0 F_j^+ \gamma^0 u_e^{(+)}(\vec{q}_e)$$

$$F_i = O_i \, (C_i + C_i' \gamma^5)$$

Explicit computation of X depends on the type of correlation investigated

Example: "pure" $\beta-\nu$ correlation

- unpolarized radioactive nucleus
- no spin detection

→ *Sum over all possible spin values & average value on possible directions of nucleus polarization*

$$X_{NP} = \sum_{i,j} [(M_i M_j^*)_{average} \sum_{spins} (\bar{u}_e^{(+)}(\vec{q}_e) F_i u_\nu^{(-)}(-\vec{q}_\nu) \bar{u}_\nu^{(-)}(-\vec{q}_\nu) \gamma^0 F_j^+ \gamma^0 u_e^{(+)}(\vec{q}_e))]$$

S, V $\rightarrow$ no spin $\qquad (M_i M_j^*)_{average} = |M_F|^2$

T, A → spin involved $\qquad (M_i M_j^*)_{average} = \pm \frac{1}{3} |M_{GT}|^2$

average on 3 Pauli matrices

Some tracks on theory: from Golden rule to events distributions…

Example: "pure" $\beta-\nu$ correlation

$$X_{NP} = \sum_{i,j}[(M_i M_j^*)_{average} \sum_{spins} (\bar{u}_e^{(+)}(\vec{q}_e) F_i u_\nu^{(-)}(-\vec{q}_\nu) \bar{u}_\nu^{(-)}(-\vec{q}_\nu) \gamma^0 F_j^+ \gamma^0 u_e^{(+)}(\vec{q}_e))]$$

Computation based on:

1. Relations for Dirac spinors (*"completeness" relations*) which lead to *traces* computation
2. Specific properties of traces of γ matrices products

This computation is very long and totally beyond the scope of this course
$\rightarrow$ details in *E. Liénard, Habilitation à Diriger des Recherches (in french), Hal Id: tel-00577620*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Examples

Tous ces termes peuvent être calculés grâce aux propriétés des traces des [...] démontrées à la section I.2.4. En particulier, tous les termes contenant un produit d'une matrice γ^5 et de moins de quatre matrices γ^μ différentes sont nuls. C'est le cas pour les termes III, IV, VI, IX et X. Pour le terme IX par exemple, les matrices σ^{ij}, produits de deux matrices γ^μ, sont identiques et dès lors, par le jeu des anticommutations, elles n'interviennent pas dans le compte final du nombre de matrices γ^μ : γ^5 est alors accompagné soit de deux matrices γ^μ (terme $\gamma q_e \sigma^{ij}\gamma^5\gamma q_\nu \sigma^{ij}$) soit d'une seule matrice γ^μ (terme $m\sigma^{ij}\gamma^5\gamma q_\nu\sigma^{ij}$).
Les traces restantes sont calculées explicitement ci-dessous.

I. $\mathrm{Tr}[(\gamma^\mu q_{e\mu}+m_e)(\gamma^\lambda q_{\nu\lambda})] = 4[E_eE_\nu - \vec{q}_e\cdot\vec{q}_\nu]$

$\mathrm{Tr}[\gamma^\mu q_{e\mu}\gamma^\lambda q_{\nu\lambda}] + \mathrm{Tr}[m_e\gamma^\lambda q_{\nu\lambda}] = 4q_eq_\nu + 0$ par (I.2.22) et (I.2.23)

II. $\mathrm{Tr}[(\gamma^\mu q_{e\mu}+m_e)\gamma^0(\gamma^\lambda q_{\nu\lambda})\gamma^0] = 4[E_eE_\nu + \vec{q}_e\cdot\vec{q}_\nu]$

$\mathrm{Tr}[(\gamma^\mu q_{e\mu})\gamma^0(\gamma^\lambda q_{\nu\lambda})\gamma^0] + \mathrm{Tr}[m_e\gamma^0\gamma^\lambda\gamma^0 q_{\nu\lambda}] = \mathrm{Tr}[(\gamma^0E_e-\gamma^k q_{ek})\gamma^0(\gamma^0E_\nu-\gamma^l q_{\nu l})\gamma^0] + 0$ par (I.2.23)
$= \mathrm{Tr}[(\gamma^0E_e-\gamma^k q_{ek})(\gamma^0E_\nu+\gamma^l q_{\nu l})] = \mathrm{Tr}[-(\gamma^0E_e-\gamma^k q_{ek})(\gamma^0E_\nu-\gamma^l q_{\nu l}-2\gamma^0E_\nu)]$
$= \mathrm{Tr}[-(\gamma^\mu q_{e\mu}\gamma^\lambda q_{\nu\lambda}) + (\gamma^0E_e-\gamma^k q_{ek})2\gamma^0E_\nu)] = \mathrm{Tr}[-(\gamma^\mu q_{e\mu}\gamma^\lambda q_{\nu\lambda}) + (2\gamma^0\gamma^0E_eE_\nu) - (2\gamma^k\gamma^0 q_{ek}E_\nu)]$
$= -4q_eq_\nu + 8E_eE_\nu - 0$ par (I.2.22)

V. $\mathrm{Tr}[(\gamma^\mu q_{e\mu}+m_e)(\gamma^\lambda q_{\nu\lambda})\gamma^0] = 4m_eE_\nu$

$\mathrm{Tr}[(\gamma^\mu q_{e\mu})(\gamma^\lambda q_{\nu\lambda})\gamma^0] + \mathrm{Tr}[m_e\gamma^\lambda\gamma^0 q_{\nu\lambda}] = 0 + m_e4g^{\lambda 0}q_{\nu\lambda} = 4m_eE_\nu$ par (I.2.22) et (I.2.23)

VII. $\mathrm{Tr}[(\gamma^\mu q_{e\mu}+m_e)\sigma^{ij}(\gamma^\lambda q_{\nu\lambda})\sigma^{ij}] = 12[E_eE_\nu + \frac{1}{3}\vec{q}_e\cdot\vec{q}_\nu]$

$\sigma^{ij} = i(\gamma^i\gamma^j - \gamma^j\gamma^i)/2$ i,j définis par l'approximation non relativiste
$\underbrace{\mathrm{Tr}[(\gamma^\mu q_{e\mu})\sigma^{ij}(\gamma^\lambda q_{\nu\lambda})\sigma^{ij}]} + \underbrace{\mathrm{Tr}[m_e\sigma^{ij}(\gamma^\lambda q_{\nu\lambda})\sigma^{ij}]}_{=0 \text{ par (I.2.23)}}$
$\mathrm{Tr}[(\gamma^\mu q_{e\mu})i(\gamma^i\gamma^j - \gamma^j\gamma^i)(\gamma^\lambda q_{\nu\lambda})i(\gamma^i\gamma^j - \gamma^j\gamma^i)/4]$

Trois termes sont à calculer, correspondant aux couples (i,j) = (1,2), (2,3), (3,1).
Explicitons le premier cas :
$-1/4\ \mathrm{Tr}[(\gamma^\mu q_{e\mu})\underbrace{(\gamma^1\gamma^2 - \gamma^2\gamma^1)(\gamma^\lambda q_{\nu\lambda})(\gamma^1\gamma^2 - \gamma^2\gamma^1)}_{\text{se décompose en 4 termes :}}]$

(a) $\gamma^1\gamma^2\gamma^\lambda\gamma^1\gamma^2 = \gamma^1\gamma^2(-2\delta_{\lambda 1} - \gamma^1\gamma^\lambda)\gamma^2 = -2\gamma^1\gamma^2\delta_{\lambda 1}\gamma^2 - \gamma^1\gamma^2\gamma^1\gamma^\lambda\gamma^2 = 2\gamma^1\delta_{\lambda 1} - \gamma^2\gamma^\lambda\gamma^2$ par (I.2.13)
$= 2\delta_{\lambda 1}\gamma^1 - \gamma^2(-2\delta_{\lambda 2} - \gamma^2\gamma^\lambda) = 2\delta_{\lambda 1}\gamma^1 + 2\delta_{\lambda 2}\gamma^2 - \gamma^\lambda$ et (I.2.14)
(b) $-\gamma^1\gamma^2\gamma^\lambda\gamma^2\gamma^1 = \gamma^1\gamma^2\gamma^\lambda\gamma^1\gamma^2 = 2\delta_{\lambda 1}\gamma^1 + 2\delta_{\lambda 2}\gamma^2 - \gamma^\lambda$ par analogie avec (a)
(c) $-\gamma^2\gamma^1\gamma^\lambda\gamma^1\gamma^2 = \gamma^1\gamma^2\gamma^\lambda\gamma^1\gamma^2 = 2\delta_{\lambda 1}\gamma^1 + 2\delta_{\lambda 2}\gamma^2 - \gamma^\lambda$ idem
(d) $\gamma^2\gamma^1\gamma^\lambda\gamma^2\gamma^1 = \gamma^1\gamma^2\gamma^\lambda\gamma^1\gamma^2 = 2\delta_{\lambda 1}\gamma^1 + 2\delta_{\lambda 2}\gamma^2 - \gamma^\lambda$ idem

Au total, ces 4 termes donnent : $8\delta_{\lambda 1}\gamma^1 + 8\delta_{\lambda 2}\gamma^2 - 4\gamma^\lambda$ que l'on insère dans la trace ci-dessus :
$-1/4\ \mathrm{Tr}[(\gamma^\mu q_{e\mu})(8\gamma^1 q_{\nu 1} + 8\gamma^2 q_{\nu 2} - 4\gamma^\lambda q_{\nu\lambda})] = -1/4\ [-32q_{e1}q_{\nu 1} - 32q_{e2}q_{\nu 2} - 16q_eq_\nu]$ par (I.2.22)

Les deux autres expressions en (i,j) donneront des résultats analogues :
(i,j) = (2,3) → $-1/4\ [-32q_{e2}q_{\nu 2} - 32q_{e3}q_{\nu 3} - 16q_eq_\nu]$
(i,j) = (3,1) → $-1/4\ [-32q_{e3}q_{\nu 3} - 32q_{e1}q_{\nu 1} - 16q_eq_\nu]$

L'ensemble des termes conduit finalement à l'expression suivante :
$1/4\ [64q_{e1}q_{\nu 1} + 64q_{e2}q_{\nu 2} + 64q_{e3}q_{\nu 3} + 48q_eq_\nu] = 4\ [4\vec{q}_e\cdot\vec{q}_\nu + 3q_eq_\nu] = 4\ [\vec{q}_e\cdot\vec{q}_\nu + 3E_eE_\nu]$

VIII. $\mathrm{Tr}[(\gamma^\mu q_{e\mu}+m_e)\gamma^k(\gamma^\lambda q_{\nu\lambda})\gamma^k] = 12[E_eE_\nu - \frac{1}{3}\vec{q}_e\cdot\vec{q}_\nu]$ avec $k \neq 0$ (A.N.R.)[1]

[1] A.N.R. : Approximation non relativiste.

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$\underbrace{\mathrm{Tr}[(\gamma^\mu q_{e\mu})\gamma^k(\gamma^\lambda q_{\nu\lambda})\gamma^k]} + \underbrace{\mathrm{Tr}[m_e\gamma^k(\gamma^\lambda q_{\nu\lambda})\gamma^k]}_{=0 \text{ par (I.2.23)}}$
$\gamma^1\gamma^\lambda\gamma^1 = \gamma^1(-2\delta_{\lambda 1} - \gamma^1\gamma^\lambda) = -2\delta_{\lambda 1}\gamma^1 + \gamma^\lambda$
$\gamma^2\gamma^\lambda\gamma^2 = -2\delta_{\lambda 2}\gamma^2 + \gamma^\lambda$
$\gamma^3\gamma^\lambda\gamma^3 = -2\delta_{\lambda 3}\gamma^3 + \gamma^\lambda$
→ $-2(\delta_{\lambda 1}\gamma^1 + \delta_{\lambda 2}\gamma^2 + \delta_{\lambda 3}\gamma^3) + 3\gamma^\lambda$
→ $\mathrm{Tr}[(\gamma^\mu q_{e\mu})(-2(q_{\nu 1}\gamma^1 + q_{\nu 2}\gamma^2 + q_{\nu 3}\gamma^3) + 3\gamma^\lambda q_{\nu\lambda})] = 8(q_{e1}q_{\nu 1} + q_{e2}q_{\nu 2} + q_{e3}q_{\nu 3}) + 12q_eq_\nu$
$= 8\,\vec{q}_e\cdot\vec{q}_\nu + 12(E_eE_\nu - \vec{q}_e\cdot\vec{q}_\nu)$

XI. $\mathrm{Tr}[(\gamma^\mu q_{e\mu}+m_e)\sigma^{ij}(\gamma^\lambda q_{\nu\lambda})\gamma^5\gamma^k] = -12m_eE_\nu$

Dans ce cas, i, j et k prennent les valeurs 1, 2 et 3 de manière cyclique. Regardons le premier cas :
$\mathrm{Tr}[(\gamma^\mu q_{e\mu}+m_e)i(\gamma^1\gamma^2 - \gamma^2\gamma^1)(\gamma^\lambda q_{\nu\lambda})\gamma^5\gamma^3/2] =$
$\mathrm{Tr}[(\gamma^\mu q_{e\mu})i(\gamma^1\gamma^2 - \gamma^2\gamma^1)(\gamma^\lambda q_{\nu\lambda})\gamma^5\gamma^3/2] + \mathrm{Tr}[im_e\underbrace{(\gamma^1\gamma^2 - \gamma^2\gamma^1)(\gamma^\lambda q_{\nu\lambda})\gamma^5\gamma^3}/2]$
→ 0 par (I.2.24)
$\gamma^1\gamma^2\gamma^\lambda\gamma^5\gamma^3 - \gamma^2\gamma^1\gamma^\lambda\gamma^5\gamma^3 = 2\gamma^1\gamma^2\gamma^\lambda\gamma^5\gamma^3$
Les traces de ces produits de matrices ne sont pas nulles si $\lambda = 0$ (I.2.24).
→ $\mathrm{Tr}[im_e\gamma^1\gamma^2(\gamma^0 q_{\nu 0})\gamma^5\gamma^3] = \mathrm{Tr}[-im_e\gamma^5\gamma^0\gamma^1\gamma^2\gamma^3E_\nu] = -im_e(-4i)E_\nu$ par (I.2.25)
Un résultat analogue est obtenu pour les deux autres cas, conduisant au total à $-12m_eE_\nu$ pour la valeur de cette trace.

XII. $\mathrm{Tr}[(\gamma^\mu q_{e\mu}+m_e)\sigma^{ij}(\gamma^\lambda q_{\nu\lambda})\gamma^k] = 0$

Les valeurs de i, j et k sont cycliques comme dans le cas précédent. Le développement du produit des matrices conduit à une valeur nulle pour le premier terme par (I.2.23) et il reste seulement le terme incluant la masse m_e : $\mathrm{Tr}[m_e\sigma^{ij}(\gamma^\lambda q_{\nu\lambda})\gamma^k]$. Développons le cas correspondant à (i,j,k) = (1,2,3) :
$i(\gamma^1\gamma^2 - \gamma^2\gamma^1)(\gamma^\lambda q_{\nu\lambda})\gamma^3/2$ → $2\gamma^1\gamma^2\gamma^\lambda\gamma^3$
si $\lambda = 0$, $\gamma^1\gamma^2\gamma^0\gamma^3 \sim \gamma^5$ → $\mathrm{Tr}() = 0$ par (I.2.24)
si $\lambda \neq 0$, $\gamma^1\gamma^2\gamma^\lambda\gamma^3 \sim \gamma^{\mu}\gamma^{\nu}$ → $\mathrm{Tr}() = 0$ par (I.2.22)
En conclusion, la trace de l'élément XII est nulle dans tous les cas.

Les expressions suivantes fournissent les formes des termes A et B de (I.3.11) après inclusion des valeurs explicites des traces calculées ci-dessus :

$$A = \{(|C_S|^2+|C_S'|^2)(1-\frac{\vec{q}_e\cdot\vec{q}_\nu}{E_eE_\nu}) + (|C_V|^2+|C_V'|^2)(1+\frac{\vec{q}_e\cdot\vec{q}_\nu}{E_eE_\nu}) + 2\mathrm{Re}(C_SC_V^*+C_S'C_V'^*)\frac{m_e}{E_e}\}$$

$$B = \{(|C_T|^2+|C_T'|^2)3(1+\frac{\vec{q}_e\cdot\vec{q}_\nu}{3E_eE_\nu}) + (|C_A|^2+|C_A'|^2)3(1-\frac{\vec{q}_e\cdot\vec{q}_\nu}{3E_eE_\nu}) + 6\mathrm{Re}(C_TC_A^*+C_T'C_A'^*)\frac{m_e}{E_e}\}$$

En posant :

$$\xi = |M_F|^2(|C_S|^2+|C_S'|^2+|C_V|^2+|C_V'|^2) + |M_{GT}|^2(|C_T|^2+|C_T'|^2+|C_A|^2+|C_A'|^2)$$
$$a = [|M_F|^2(-|C_S|^2-|C_S'|^2+|C_V|^2+|C_V'|^2) + |M_{GT}|^2(|C_T|^2+|C_T'|^2-|C_A|^2-|C_A'|^2)/3] / \xi$$
$$b = [|M_F|^2\mathrm{Re}(C_SC_V^*+C_S'C_V'^*) + |M_{GT}|^2\mathrm{Re}(C_TC_A^*+C_T'C_A'^*)] / \xi ,$$

l'équation (I.3.11) s'écrit :

$$X_{NP} = \xi(1 + a\frac{\vec{q}_e\cdot\vec{q}_\nu}{E_eE_\nu} + b\frac{2m_e}{E_e}) ,$$

et la distribution du taux d'événements (I.3.7) :

$$N(p_e,\theta)dp_e d\theta d\Omega_e = \frac{8\pi^3}{h^7c^3}p_e^2(E_0 - E_e)^2\xi(1 + a\frac{\vec{q}_e\cdot\vec{q}_\nu}{E_eE_\nu} + b\frac{2m_e}{E_e})\sin(\theta)dp_e d\theta d\Omega_e$$

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Some tracks on theory: from Golden rule to events distributions…

Example: "pure" β–ν correlation

$$N(p_e,\theta)dp_e d\theta = \frac{32\pi^4}{h^7 c^3} p_e^2 (Q-T_e)^2 \xi (1 + a\frac{v_e}{c}\cos(\theta) + b\frac{m_e c^2}{E_e})\sin(\theta)dp_e d\theta$$

$$\xi = |M_F|^2(|C_S|^2+|C_S'|^2+|C_V|^2+|C_V'|^2) + |M_{GT}|^2(|C_T|^2+|C_T'|^2+|C_A|^2+|C_A'|^2)$$

$$a = [|M_F|^2(-|C_S|^2-|C_S'|^2+|C_V|^2+|C_V'|^2) + |M_{GT}|^2(|C_T|^2+|C_T'|^2-|C_A|^2-|C_A'|^2)/3] / \xi$$

β⁻ / β⁺

$$b = \pm 2[|M_F|^2 Re(C_S C_V^* + C_S' C_V'^*) + |M_{GT}|^2 Re(C_T C_A^* + C_T' C_A'^*)] / \xi$$

a : *β-ν* angular correlation parameter   *b* : Fierz interference term (cross-terms)

	Current	a	ξ
Fermi	Scalar	-1	$\|M_F\|^2\ (\|C_S\|^2+\|C_S'\|^2)$
	Vector	+1	$\|M_F\|^2\ (\|C_V\|^2+\|C_V'\|^2)$
G-T	Axial	-1/3	$\|M_{GT}\|^2\ (\|C_A\|^2+\|C_A'\|^2)$
	Tensor	+1/3	$\|M_{GT}\|^2\ (\|C_T\|^2+\|C_T'\|^2)$

Standard Model: *V - A*

- *The distribution of events drastically depends on "a".*
- *A measurement sensitive to its shape enables to test the V-A theory !!*

Some tracks on theory: from Golden rule to events distributions…

Any distribution can be deduced thanks to integration and/or average values leaving adequate parameters variable

Tremendous job performed by Jackson at al in 1957…

PHYSICAL REVIEW VOLUME 106, NUMBER 3 MAY 1, 1957

Possible Tests of Time Reversal Invariance in Beta Decay

J. D. JACKSON,* S. B. TREIMAN, AND H. W. WYLD, JR.
Palmer Physical Laboratory, Princeton University, Princeton, New Jersey

(Received January 28, 1957)

!

old γ matrices definition !!

<u>Example</u>

Polarized nucleus (J)
&
$\beta - \nu$ correlation

β-ν angular parameter

$$\omega(\langle J\rangle | E_e,\Omega_e,\Omega_\nu)dE_e d\Omega_e d\Omega_\nu$$
$$=\frac{1}{(2\pi)^5}p_e E_e(E^0-E_e)^2 dE_e d\Omega_e d\Omega_\nu \xi \Bigg\{ 1 + a\frac{\mathbf{p}_e\cdot\mathbf{p}_\nu}{E_eE_\nu} + b\frac{m}{E_e}$$
$$+c\left[\frac{1}{3}\frac{\mathbf{p}_e\cdot\mathbf{p}_\nu}{E_eE_\nu} - \frac{(\mathbf{p}_e\cdot\mathbf{j})(\mathbf{p}_\nu\cdot\mathbf{j})}{E_eE_\nu}\right]\left[\frac{J(J+1)-3\langle(\mathbf{J}\cdot\mathbf{j})^2\rangle}{J(2J-1)}\right]$$
$$+\frac{\langle\mathbf{J}\rangle}{J}\cdot\left[A\frac{\mathbf{p}_e}{E_e} + B\frac{\mathbf{p}_\nu}{E_\nu} + D\frac{\mathbf{p}_e\times\mathbf{p}_\nu}{E_eE_\nu}\right]\Bigg\}.$$

Fierz

β asymmetry

"Triple" correlation

Which term for which physics ?

Key: behaviour of involved vectors under *P* and *T* operations

p *changes sign* — ***p, J*** *change sign*

Terms of type	$\boldsymbol{p}_1.\boldsymbol{p}_2$	$\boldsymbol{J}.\boldsymbol{p}$	$\boldsymbol{J}.(\boldsymbol{p}_1 x \boldsymbol{p}_2)$
	Invariant under *P* & *T*	Invariant under *T* but not *P* !	Invariant under *P* but not *T* !
Examples:	$\boxed{a\dfrac{\vec{p}_e.\vec{p}_\nu}{E_e E_\nu}}$	$A\dfrac{\vec{J}.\vec{p}_e}{JE_e}$	$D\dfrac{\vec{J}.(\vec{p}_e \times \vec{p}_\nu)}{J(E_e E_\nu)}$

$$\boxed{a\xi = |M_F|^2(-|C_S|^2+|C_V|^2-|C_S'|^2+|C_V'|^2)+\frac{|M_{GT}|^2}{3}(|C_T|^2-|C_A|^2+|C_T'|^2-|C_A'|^2)}$$

$a \propto \left|C_i^{(\prime)}\right|^2$ ➡ not sensitive to the character real or imaginary of the constants !

➡ sensitive to the types of currents involved in weak interaction

Experimental setup: sensitive to the shape of the $\beta-\nu$ distribution

Which term for which physics ?

Terms of type	$\mathbf{p}_1.\mathbf{p}_2$	$\mathbf{J}.\mathbf{p}$	$\mathbf{J}.(\mathbf{p}_1 x \mathbf{p}_2)$
	Invariant under *P* & *T*	Invariant under *T* but not *P* !	Invariant under *P* but not *T* !
Examples:	$a\frac{\vec{p}_e.\vec{p}_\nu}{E_eE_\nu}$	$\boxed{A\frac{\vec{J}.\vec{p}_e}{JE_e}}$	$D\frac{\vec{J}.(\vec{p}_e\times\vec{p}_\nu)}{J(E_eE_\nu)}$

$$\lambda_{J'J}=\begin{cases}1, & J\to J'=J-1\\ 1/(J+1), & J\to J'=J\\ -J/(J+1), & J\to J'=J+1;\end{cases}$$

$$\boxed{A\xi=2\,\mathrm{Re}\left[\pm|M_{GT}|^2\lambda_{J'J}(C_TC_T'^*-C_AC_A'^*)+\delta_{J'J}|M_F||M_{GT}|\left(\frac{J}{J+1}\right)^{\frac{1}{2}}(C_SC_T'^*+C_S'C_T^*-C_VC_A'^*-C_V'C_A^*)\right]}$$

$A\propto|M_{GT}|C_iC_j'$ ➡

- not accessible in pure F !
- $A\neq 0\Rightarrow C_i'\neq 0$ ➡ $\boxed{\text{Test of } P \text{ violation}}$

Parameter measured by Wu

Experimental setup: *1. Radioactive nucleus must be polarized in controlled direction*

2. Detection of e^- in optimized direction vs J (0° and/or 180°)

Which term for which physics ?

Terms of type	$\mathbf{p}_1.\mathbf{p}_2$	$\mathbf{J}.\mathbf{p}$	$\mathbf{J}.(\mathbf{p}_1 x \mathbf{p}_2)$
	Invariant under *P* & *T*	Invariant under *T* but not *P* !	Invariant under *P* but not *T* !
Examples:	$a\,\dfrac{\vec{p}_e.\vec{p}_\nu}{E_e E_\nu}$	$A\,\dfrac{\vec{J}.\vec{p}_e}{J E_e}$	$\boxed{D\,\dfrac{\vec{J}.(\vec{p}_e \times \vec{p}_\nu)}{J(E_e E_\nu)}}$

$$\boxed{D\xi = 2\,\mathrm{Im}\left\{\delta_{J'J}|M_F||M_{GT}|\left(\frac{J}{J+1}\right)^{\frac{1}{2}}(C_S C_T^* - C_V C_A^* + C_S' C_T'^* - C_V' C_A'^*)\right\},}$$

$D \propto Im(|M_F||M_{GT}|C_i C_j^*)$ ➡

- not accessible in pure F nor in pure GT !
  $\Rightarrow$ *Mirror transitions*
- $D \neq 0 \Rightarrow C_i$ not purely real (due to "Im")
  $\Rightarrow$ T Reversal Violation (TRV) !

Experimental setup: 1. Radioactive nucleus must be polarized in controlled direction

2. Sensitive to β–ν correlation in optimized direction vs J (0° and/or 180°)

Illustration: Discovery of *P* violation by "Madam Wu"

I. History

- Before 1955, symmetry ok for all interactions but W.I.
- Puzzle $\theta - \tau$
  - 2 mesons seen by their decay: $\theta^+ \rightarrow \pi^+ \pi^0$
    $\tau^+ \rightarrow \pi^+ \pi^+ \pi^-$
  - Same mass, same half-life → same particle ?
    meson K with spin 0 ?!
  - Problem: decay to systems with 2π and 3π → different parity !!

- Lee et Yang, PR104 (1956)254

PHYSICAL REVIEW VOLUME 104, NUMBER 1 OCTOBER 1, 1956

Question of Parity Conservation in Weak Interactions*

...suggest experiment with polarized ^{60}Co

of the question of parity conservation.) To decide unequivocally whether parity is conserved in weak interactions, one must perform an experiment to determine whether weak interactions differentiate the right from the left. Some such possible experiments will be

$$I(\theta)d\theta = (\text{constant})(1+\alpha\cos\theta)\sin\theta d\theta,$$

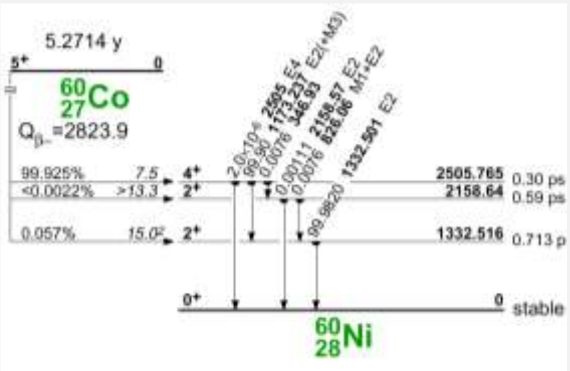
where $\alpha = A\, p_e / E_e$, A : β asymmetry parameter
θ = angle between nucleus spin and e^- momentum

Discovery of *P* violation by "Madam Wu"

II. Mme Wu experiment, PR105 (1957)1413

Experimental Test of Parity Conservation in Beta Decay*

C. S. Wu, *Columbia University, New York, New York*
AND
E. Ambler, R. W. Hayward, D. D. Hoppes, and R. P. Hudson, *National Bureau of Standards, Washington, D. C.*
(Received January 15, 1957)



- ^{60}Co source:

Main decay: $5^+ \rightarrow 4^+$ selected by γ detection (1173 keV & 1332 keV)

Pure GT transition: $M_F = 0$

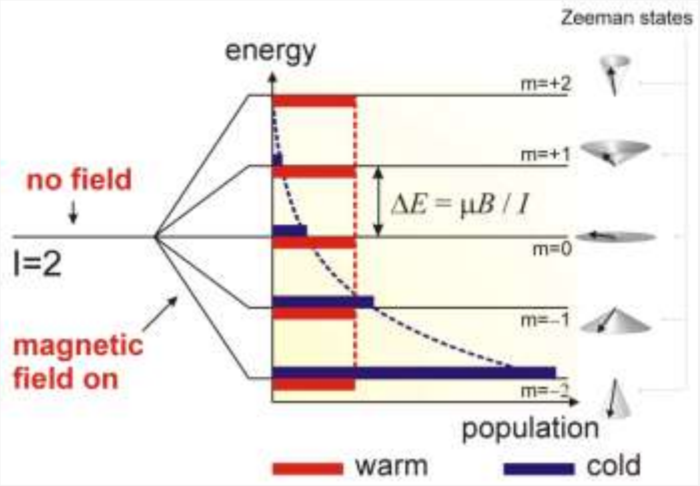
- Nucleus polarization:

Source put on a ferromagnetic support in an external magnetic field

→ High field in the support for a weak external field

- negligible effect on the β's
- orientation of radioactive nuclei at low T° (~0.01K!!) thanks to different populations of hyperfine states

Example: ^{114}In

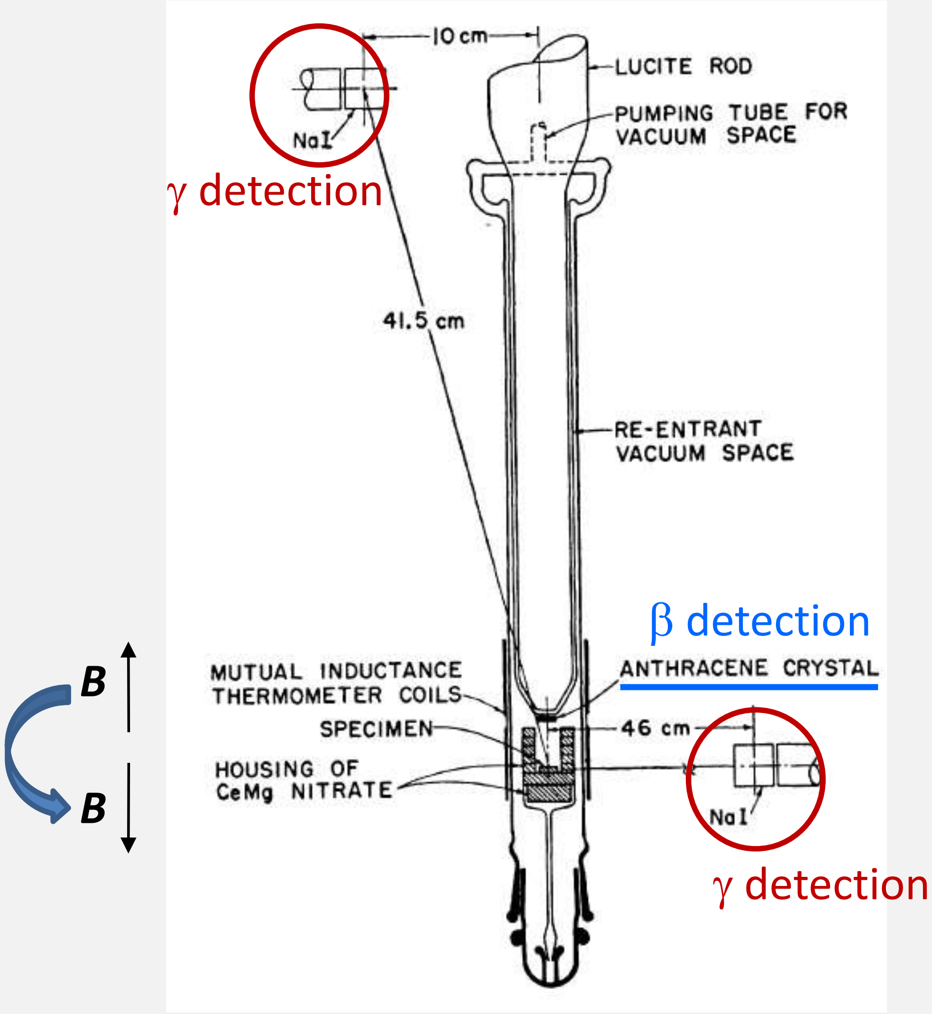


Population → Maxwell-Boltzmann

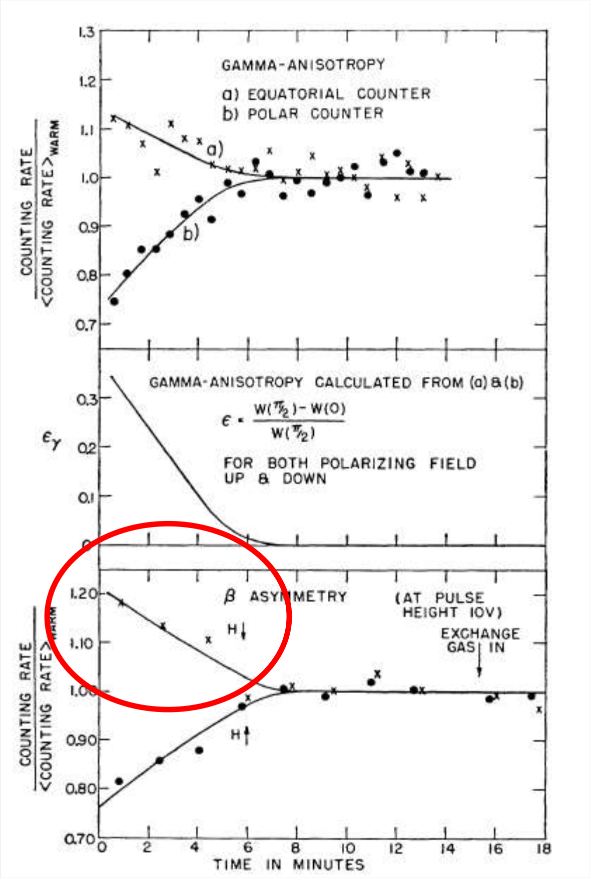
$$p(m_i) \sim \exp(-m_i(\Delta E/kT))$$

Discovery of *P* violation by "Madam Wu"

- Nucleus polarization measurement: from γ anisotropy
- β detection at 0° & 180° (switch of magnetic field orientation)



*If P invariance, then same β counting rates is expected for the two directions of **B***



Rates higher at 180° than at 0° !

Discovery of *P* violation by "Madam Wu"

- Nucleus polarization measurement: from γ anisotropy
- β detection at 0° & 180° (switch of magnetic field orientation)

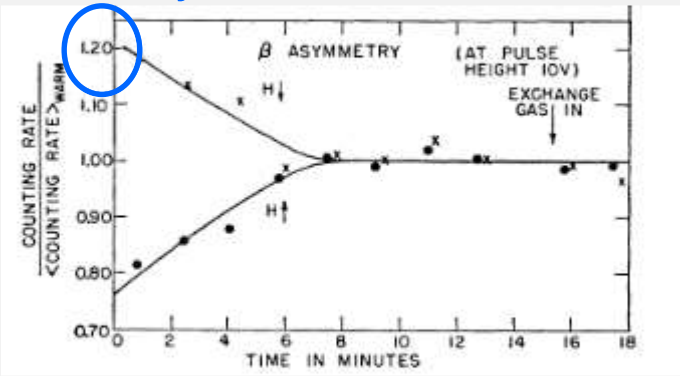
✓ β asymmetry: mainly emitted at 180° vs ***J***

✓ $W(\theta) \propto 1 + \alpha \frac{\hat{J}.\vec{p}_e}{E_e} = 1 + AP\frac{v}{c}\cos(\theta)$

Asym. ~ 0.2
v/c ~ 0.6 } → α ~ 0.4
(Asym. = α v/c)

Polarisation degree:
(γ) → P ~ 0.6

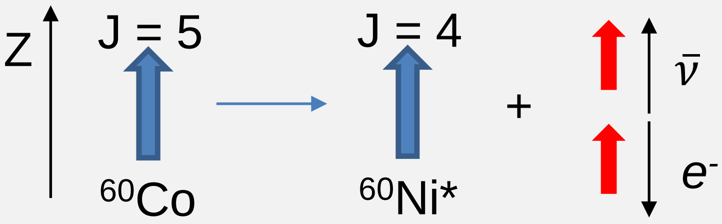
Asym. ~ 0.2



⇒ $\boxed{A < 0 \text{ et } |A| > 0.7\ (\alpha/P)}$

without uncertainty ! but clear evidence of parity violation in W.I. !!

- First information on "helicity"



1. e^- preferentially emitted at 180° vs J
2. ΔJ = 1 → lepton spins aligned in J direction

   → *electron is left-handed !*

   (and *anti-neutrino is right-handed*)

Discovery of *P* violation by "Madam Wu"

I. Mme Wu experiment

$\boxed{A < -0.7}$

II. 1980: Chirovsky et al., PL94B(1980)127

DIRECTIONAL DISTRIBUTIONS OF BETA-RAYS EMITTED FROM POLARIZED ^{60}Co NUCLEI ☆

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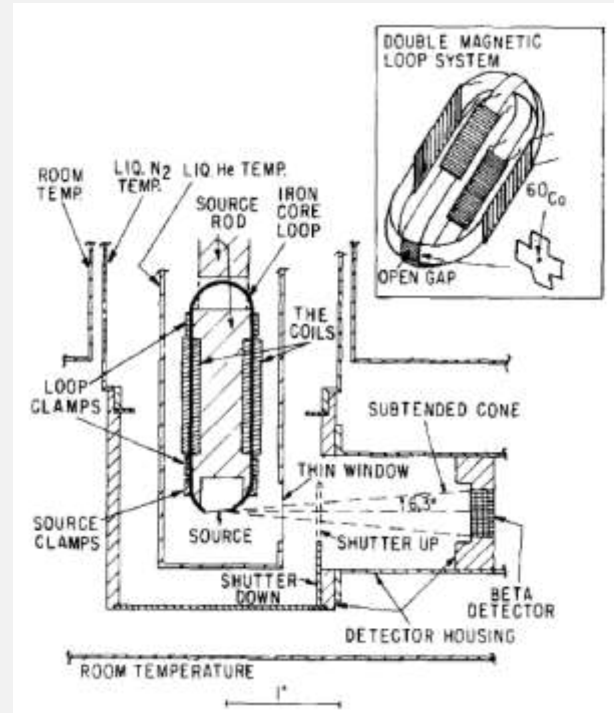
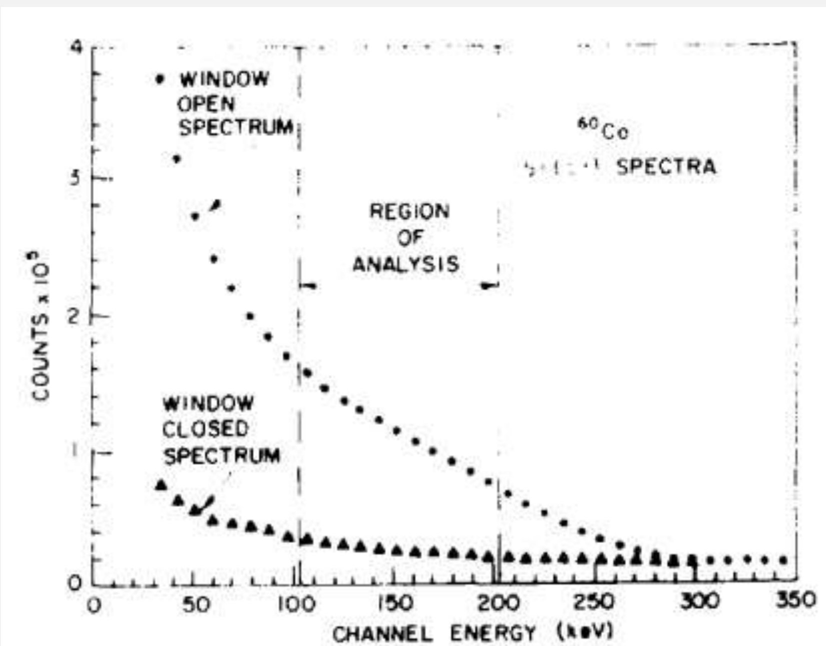
Department of Physics, Columbia University, New York, NY 10027, USA



Measurement from 10° to 170° vs *J*

- with shutter (γ only) and without ($\beta + \gamma$)
- with cold and warm source



$$W(\theta) = \frac{Spectrum(\beta+\gamma)_{cold} - Spectrum(\gamma)_{cold}}{Spectrum(\beta+\gamma)_{warm} - Spectrum(\gamma)_{warm}}$$

$\boxed{A_\beta = -1.01\ (2)}$

Discovery of *P* violation by "Madam Wu"

A_β = -1.01 (2)   Why does this value imply a *Maximal Parity Violation* ?

$$\lambda_{J'J} = \begin{cases} 1, & J \to J' = J-1 \\ 1/(J+1), & J \to J' = J \\ -J/(J+1), & J \to J' = J+1; \end{cases}$$

$J = 5 \rightarrow J' = 4$

$\rightarrow \lambda_{J'J} = 1$

β^-

$$A\xi = 2\,\mathrm{Re}\left[\pm |M_{GT}|^2 \lambda_{J'J} (C_T C_T'^* - C_A C_A'^*) + \delta_{J'J} |M_F|\,|M_{GT}| \left(\frac{J}{J+1}\right)^{\frac{1}{2}} (C_S C_T'^* + C_S' C_T^* - C_V C_A'^* - C_V' C_A^*)\right]$$

= 0 (M$_F$ = 0)

= 0

$$\xi = |M_F|^2 (|C_S|^2 + |C_V|^2 + |C'_S|^2 + |C'_V|^2) + |M_{GT}|^2 (|C_T|^2 + |C_A|^2 + |C'_T|^2 + |C'_A|^2)$$

Standard Model : 1. Time Reversal Invariance → Real coupling constants

2. V-A theory : $C_T = 0$

$$A = \frac{2(-C_A C_A')}{(|C_A|^2 + |C_A'|^2)}$$

= -1 if $C_A = C_A'$ *Maximal Parity Violation !*

! *Sign reversed in new notation …*

A word on some approximations and needed corrections

At 10^{-3} – 10^{-4} precision level

I. Nonrelativistic approximation (nucleons)

Expressions deduced without recoil energy…

"Recoil" corrections

B.R. Holstein, Phys. Rev. C 4 (1971) 740

B.R. Holstein, Rev. Mod. Phys. 46 (1974) 789

II. Nuclei basically described

Expressions deduced without strong interaction effects…

"Nuclear" corrections

III. Charged particles radiate

Effects of radiation not taken into account…

"Radiative" corrections

F. Glück, Computer Phys. Comm. 101 (1997) 223

F. Glück, Nucl. Phys. A 628 (1998) 493

IV. Final state interaction

"Coulomb" corrections

Coulomb interaction between β and recoil ion (*Fermi function*)

J. D. Jackson et al., Nucl. Phys. 4 (1957) 206

J. C. Brodine, Phys. Rev. D 1 (1970) 100

A special case: the Fierz term

JD Jackson et al PR106 (1957)

Events distribution
for
Polarized nucleus (J)
&
$\beta - \nu$ correlation

$$\omega(\langle J\rangle | E_e,\Omega_e,\Omega_\nu)dE_e d\Omega_e d\Omega_\nu$$
$$= \frac{1}{(2\pi)^5} p_e E_e (E^0 - E_e)^2 dE_e d\Omega_e d\Omega_\nu \xi \Big\{ 1 + a\frac{\mathbf{p}_e\cdot\mathbf{p}_\nu}{E_e E_\nu} + b\frac{m}{E_e}$$
$$+ c\left[\frac{1}{3}\frac{\mathbf{p}_e\cdot\mathbf{p}_\nu}{E_e E_\nu} - \frac{(\mathbf{p}_e\cdot\mathbf{j})(\mathbf{p}_\nu\cdot\mathbf{j})}{E_e E_\nu}\right]\left[\frac{J(J+1)-3\langle(\mathbf{J}\cdot\mathbf{j})^2\rangle}{J(2J-1)}\right]$$
$$+ \frac{\langle \mathbf{J}\rangle}{J}\cdot\left[A\frac{\mathbf{p}_e}{E_e} + B\frac{\mathbf{p}_\nu}{E_\nu} + D\frac{\mathbf{p}_e\times\mathbf{p}_\nu}{E_e E_\nu}\right]\Big\}.$$

Fierz

$$\xi = |M_F|^2(|C_S|^2+|C_S'|^2+|C_V|^2+|C_V'|^2) + |M_{GT}|^2(|C_T|^2+|C_T'|^2+|C_A|^2+|C_A'|^2)$$
$$b = \pm 2[|M_F|^2 \mathrm{Re}(C_S C_V^* + C_S' C_V'^*) + |M_{GT}|^2 \mathrm{Re}(C_T C_A^* + C_T' C_A'^*)] / \xi$$

- Always present, even in β energy distribution, due to cross-terms (S-V, T-A)
  → *$b = 0$ in SM !*
- "Ideal" to test *V-A* (linear dependence in C_i) but difficult to measure directly in β spectrum because of scattering

A special case: the Fierz term

How is it managed in correlation measurements ?

In the framework of *V-A* theory, $b = 0$ → *no problem !*

Otherwise: $b \simeq 0 \;\rightarrow\;$ b is "included" in the measured correlation parameter

Example: β–ν angular correlation measurement

$$N(p_e,\theta)dp_e d\theta = N(p_e)(1 + a\frac{\mathrm{v_e}}{c}\cos(\theta) + b\frac{\mathrm{m_e}c^2}{\mathrm{E_e}})\sin(\theta)dp_e d\theta$$

⬇

$$N(p_e,\theta)dp_e d\theta = N(p_e)(1 + \tilde{a}\frac{\mathrm{v_e}}{c}\cos(\theta))\sin(\theta)dp_e d\theta$$

where $\tilde{a} = a / (1 + b \underbrace{\langle m_e c^2/E_e \rangle}_{\text{Mean value computed from real values accessible to experiment}})$

Mean value computed from real values accessible to experiment

A special case: the Fierz term

Consequences example: pure GT transition (β^-)

$a_{exp} = a_{GT} / (1+b_{GT} <m_e c^2/E_e>)$ [1]

measured value

$$a_{GT} = -\frac{1}{3}\frac{C_A^2+C_A'^2-C_T^2-C_T'^2}{C_A^2+C_A'^2+C_T^2+C_T'^2}$$

$$b_{GT} = 2\frac{Re(C_T C_A^*+C_T' C_A'^*)}{C_A^2+C_A'^2+C_T^2+C_T'^2}$$

MPV (V-A) $\rightarrow$ $C_A = C_A'$

TRI $\rightarrow$ C_i, C_i' real

$$a_{GT} = -\frac{1}{3}\frac{2-C_T^2/C_A^2-C_T'^2/C_A^2}{2+C_T^2/C_A^2+C_T'^2/C_A^2}$$

$$b_{GT} = 2\frac{C_T/C_A+C_T'/C_A}{2+C_T^2/C_A^2+C_T'^2/C_A^2}$$

in [1]

$$\frac{C_T^2}{C_A^2}\left(a_{exp}-\tfrac{1}{3}\right)+2a_{exp}<\frac{m_e c^2}{E_e}>\frac{C_T}{C_A}+\frac{C_T'^2}{C_A^2}\left(a_{exp}-\tfrac{1}{3}\right)+2a_{exp}<\frac{m_e c^2}{E_e}>\frac{C_T'}{C_A}+2a_{exp}+\tfrac{2}{3}=0$$

Circle equation

$$\boxed{\left[\frac{C_T}{C_A}+\frac{a_{exp}<\frac{m_e c^2}{E_e}>}{(a_{exp}-\frac{1}{3})}\right]^2+\left[\frac{C_T'}{C_A}+\frac{a_{exp}<\frac{m_e c^2}{E_e}>}{(a_{exp}-\frac{1}{3})}\right]^2=\frac{2a_{exp}^2<\frac{m_e c^2}{E_e}>^2}{(a_{exp}-\frac{1}{3})^2}-\frac{2(a_{exp}+\frac{1}{3})}{(a_{exp}-\frac{1}{3})}}$$

centered at: $X_0 = Y_0 = -\dfrac{a_{exp}<\frac{m_e c^2}{E_e}>}{(a_{exp}-\frac{1}{3})}$ with radius: $R = \sqrt{\dfrac{2a_{exp}^2<\frac{m_e c^2}{E_e}>^2}{(a_{exp}-\frac{1}{3})^2}-\dfrac{2(a_{exp}+\frac{1}{3})}{(a_{exp}-\frac{1}{3})}}$

A special case: the Fierz term

Consequences   example: pure GT transition

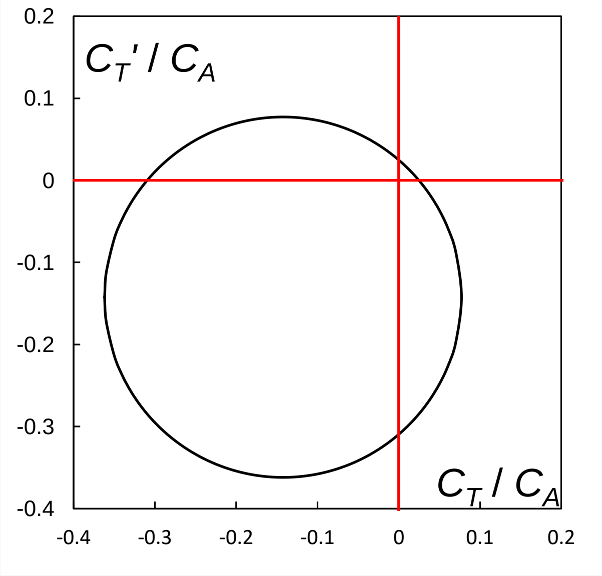
Center:
$$X_0 = Y_0 = -\frac{a_{exp} < \frac{m_e c^2}{E_e} >}{(a_{exp} - \frac{1}{3})} \approx -\frac{< \frac{m_e c^2}{E_e} >}{2}$$

Radius:
$$R = \sqrt{\frac{2a_{exp}^2 < \frac{m_e c^2}{E_e} >^2}{(a_{exp} - \frac{1}{3})^2} - \frac{2(a_{exp} + \frac{1}{3})}{(a_{exp} - \frac{1}{3})}} \approx \frac{< \frac{m_e c^2}{E_e} >}{\sqrt{2}}$$

because a_{exp} ~ -1/3

⇒ *Circle position and its radius are dominated by the factor $<m_e c^2/E_e>$*

Example: Measurement in ^{6}He decay by
Johnson et al Phys. Rev. 132 (1963)
corrected for radiative effects by
F. Glück, Nucl. Phys. A 628 (1998)



a_{exp} = -0.3308 (30)   et $<m_e c^2/E_e>$ = 0.286

$X_0 = Y_0$ = - 0.142   ≈ - 0.286/2 = - 0.143

R = 0.22   ≈ 0.286/√2 = 0.20

A special case: the Fierz term

Consequences   example: pure GT transition

Center: $$X_0 = Y_0 = -\frac{a_{exp} < \frac{m_e c^2}{E_e} >}{(a_{exp} - \frac{1}{3})} \approx -\frac{< \frac{m_e c^2}{E_e} >}{2}$$

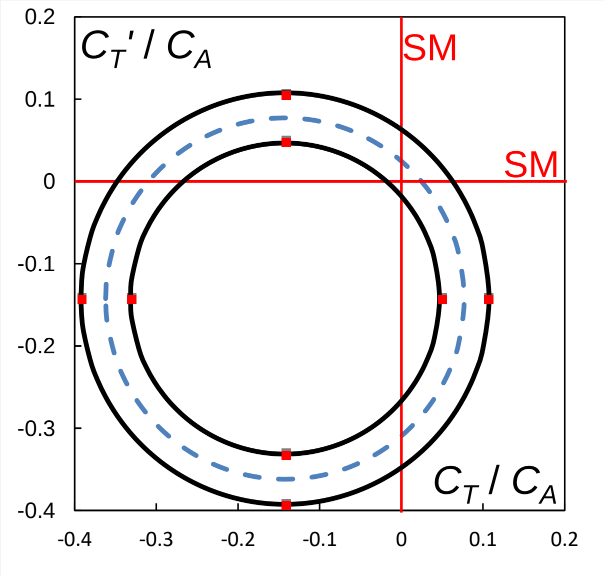
Radius: $$R = \sqrt{\frac{2a_{exp}^2 < \frac{m_e c^2}{E_e} >^2}{(a_{exp} - \frac{1}{3})^2} - \frac{2(a_{exp} + \frac{1}{3})}{(a_{exp} - \frac{1}{3})}} \approx \frac{< \frac{m_e c^2}{E_e} >}{\sqrt{2}}$$

because $a_{exp} \sim -1/3$

⇨ *Circle position and its radius are dominated by the factor* $< m_e c^2/E_e >$

Limits $\Delta a_{exp} \rightarrow \Delta R \; (+ \; \Delta X_0, \Delta Y_0)$

⇨ ~ spherical layer



Conclusion: *the most sensitive candidates are the ones with the highest* $< m_e / E_e >$ *…!!*

The Standard Model (SM) and beyond

Experiments (>1955):

V-A theory : $C_S = C_T = 0$, $C_A/C_V < 0$

Time Reversal Invariance : C_i, C_i' real

Maximal Parity Violation : $C_i = - C_i'$

!

Sign depends on γ matrices choice

$$H_{MS} = [\overline{\psi}_p(x)\gamma^\mu(C_V + C_A\gamma^5)\psi_n(x)][\overline{\psi}_e(x)\gamma^\mu(1-\gamma^5)\psi_\nu(x)] + h.c.$$

This expression gives information on particles helicities !!

What is helicity ?

= projection of particle spin on its momentum

- $h < 0$ left-handed particle **s** anti-// **p**
- $h > 0$ right-handed particle **s** // **p**

The Standard Model (SM) and particles helicity

Case of particles with $m = 0$ (~ ν) → defined helicity

- Dirac equation $\vec{\alpha}.\vec{p}\,\psi = E\psi$ *(Independent of β)*

  Solutions:
  - 2x2 Pauli matrices sufficient: $\alpha_i = \pm\sigma_i$
  - Basis of 2-componants spinors, ϕ *and* χ

$\vec{\sigma}.\vec{p}\,\phi = -E\phi \leftrightarrow \vec{\sigma}.\hat{p}\,\phi = -\phi \quad \Rightarrow$ left-handed ν (p > 0) or right-handed $\bar{\nu}$ (p< 0)

$\vec{\sigma}.\vec{p}\,\chi = E\chi \leftrightarrow \vec{\sigma}.\hat{p}\,\chi = \chi \quad \Rightarrow$ right-handed ν (p > 0) or left-handed $\bar{\nu}$ (p< 0)

- Weyl representation for γ matrices:
  (γ^0 is different and then γ^5 too !)

$$\gamma^0 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} \rightarrow \gamma^5 = \begin{pmatrix} -I & 0 \\ 0 & I \end{pmatrix}$$

- Projection factors:

$$\frac{1+\gamma^5}{2} = \begin{pmatrix} 0 & 0 \\ 0 & I \end{pmatrix} \qquad \frac{1-\gamma^5}{2} = \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix}$$

$$\boxed{\left(\frac{1+\gamma^5}{2}\right)\psi = \left(\frac{1+\gamma^5}{2}\right)\begin{pmatrix} \phi \\ \chi \end{pmatrix} = \begin{pmatrix} 0 \\ \chi \end{pmatrix}} \qquad \boxed{\left(\frac{1-\gamma^5}{2}\right)\psi = \left(\frac{1-\gamma^5}{2}\right)\begin{pmatrix} \phi \\ \chi \end{pmatrix} = \begin{pmatrix} \phi \\ 0 \end{pmatrix}}$$

ν state projected on its right component
$\bar{\nu}$ state projected on its left component

ν state projected on its left component
$\bar{\nu}$ state projected on its right component

The Standard Model (SM) and particles helicity

$$H_{MS} = [\overline{\psi}_p(x)\gamma^\mu(C_V + C_A\gamma^5)\psi_n(x)]\underline{[\overline{\psi}_e(x)\gamma^\mu(1-\gamma^5)\psi_\nu(x)]} + h.c.$$

equivalent to: $\overline{\psi}_e(x)\gamma^\mu \frac{(1-\gamma^5)}{2}(1-\gamma^5)\psi_\nu(x) = \underline{\overline{\psi}_e(x)\frac{(1+\gamma^5)}{2}}\,\underline{\gamma^\mu(1-\gamma^5)\psi_\nu(x)}$

$\boxed{\overline{\psi}_e(x)\frac{(1+\gamma^5)}{2} \text{ equivalent to } \frac{(1-\gamma^5)}{2}\psi_e(x)}$ → left-handed e^- right-handed $\bar{\nu}$

➡ *consistent with Mme Wu experiment !*

…and beyond

$$H_{ST} = C_S[\overline{\psi}_p(x)\psi_n(x)][\overline{\psi}_e(x)(1-\gamma^5)\psi_\nu(x)] +$$

$$+ C_T[\overline{\psi}_p(x)\gamma^\mu\gamma^\nu\psi_n(x)]\underline{[\overline{\psi}_e(x)\gamma^\mu\gamma^\nu(1-\gamma^5)\psi_\nu(x)]} + h.c.$$

equivalent to: $\underline{\overline{\psi}_e(x)\frac{(1-\gamma^5)}{2}}\,\underline{\gamma^\mu\gamma^\nu(1-\gamma^5)\psi_\nu(x)}$

right-handed e^- right-handed $\bar{\nu}$

➡ Helicity measurements enable to determine main currents in W. I.

→ famous experiment of Goldhaber in 1958 with ^{152m}Eu *Phys. Rev. 109 (1958) 1015*

The Standard Model (SM) and the values of C_V and C_A

A direct access through *ft* values

$$d^2\lambda = KW(p_e)\xi(1 + a\frac{v_e}{c}\cos(\theta) + b\frac{m_e c^2}{E_e})\sin(\theta)dp_e d\theta$$

1. $\int d\theta$ → $N(p_e)$ independent of *a*
2. SM → $b = 0$ ⚠ *and what happens if we remove this constrain ? → see later*
3. $\int dp_e$ → λ :

$$\lambda = K'\xi\underbrace{\int W(p_e)dp_e}_{\longrightarrow f(Z',E_0)} = \frac{\ln 2}{t_{1/2}}$$

⚠ *This is a partial half-life*

et $\xi = 2(C_V)^2|M_F|^2 + 2(C_A)^2|M_{GT}|^2$

These factors "2" come from C_i'

ft value

$$\boxed{ft_{1/2} = \frac{4.794\,10^{-5}}{2(C_V^2|M_F|^2 + C_A^2|M_{GT}|^2)}\,s}$$

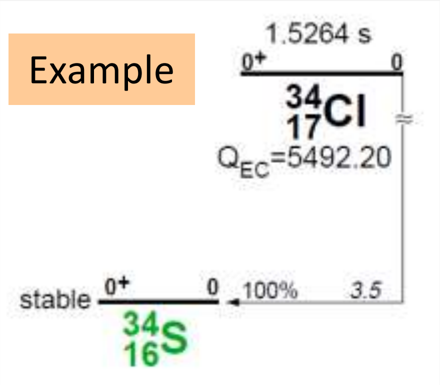
www.nndc.bnl.gov/logft/

The Standard Model (SM) and the value of C_V

Pure Fermi transitions: $0^+ \rightarrow 0^+$

$$ft_{1/2} = \frac{4.794\ 10^{-5}}{2C_V^2 \left|M_F\right|^2} = K''$$

constant



$M_{GT} = 0$
$M_F = \sqrt{2}$

for all superallowed pure F transitions (Isospin = β decay operator)

C_V ($\sqrt{2}\ C_V$ …) is a constant !!

$\sqrt{2}\ C_V = 8.8336\ 10^{-5}$ MeV fm^3 (*ft* ~ 3070 s)

- CVC (*Conserved Vector Current*) "hypothesis"
- Is "$\sqrt{2}\ C_V$" the weak interaction fundamental constant ?

The Standard Model (SM) and the value of C_V

$\sqrt{2}\, C_V \longleftrightarrow G_F$ (from pure leptonic μ decay: $\mu^- \to e^- + \nu_\mu + \bar{\nu}_e$)

8.8336 10^{-5} < 8.9618 10^{-5} MeV fm^3

In weak interaction, quarks eigenstates are a mixing of their mass eigenstates

Cabibbo-Kobayashi-Maskawa matrix (*built on 3 particles generations*)

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

unitarity condition:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 \ \ ??$$

β decay

$$\boxed{\sqrt{2}\, C_V = G_F V_{ud} \quad \text{or} \quad V_{ud} = \sqrt{2}\, C_V / G_F}$$

energy not sufficient to produce s & b …!!

Precise measurements of C_V → tests of CVC & CKM unitarity

(V_{us} & V_{ub} from K, B mesons decays)

The Standard Model (SM) and the value of C_V

Precise measurements of C_V → tests of CVC & CKM unitarity

which parameters to be measured ?

$$\boxed{ft_{1/2} = \frac{4.794\,10^{-5}}{2C_V^2 \left|M_F\right|^2} = K''}$$

1. $f \propto \int_0^{p_e^{max}} W(p_e)dp_e = F(Q_\beta)$ → Drastic dependence on nuclei masses !

2. $t_{1/2} = T_{1/2}$ / BR *partial half-life* → Measurements of half-lives & branching ratios !

⚠ *In case of β^+ decays, electronic capture process must be taken into account*

All superallowed pure Fermi decays…!!

$t_{1/2} = T_{1/2}\,(1+P_{EC})$ / BR

P_{EC} is computed with sufficient precision

3. ⚠ At high precision (*10^{-3} – 10^{-4}*), theoretical corrections are needed

→ 2 types: radiative and isospin symmetry breaking (ISB) → $\boxed{f_{corr}\, t = Ft}$

corrected "Ft" values

The Standard Model (SM) and the value of C_A

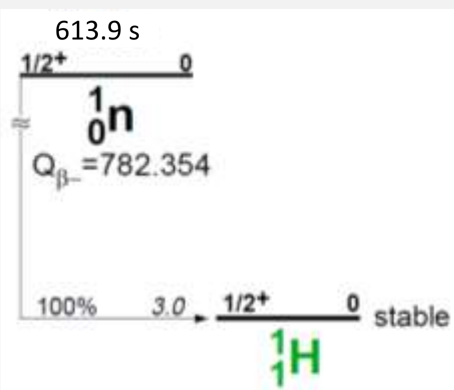
Pure Gamow-Teller transitions:

$$Ft_{1/2} = \frac{4.794\,10^{-5}}{2C_A^2 \left|M_{GT}\right|^2} \neq K" \text{ !!}$$

M_{GT} is not a constant &
is not easy to compute…!!

➡ Axial current is not conserved ~~CAC~~

C_A measured in neutron decay (F + GT) ➡ Isospin doublet: T = ½ → $M_F = 1$
+ 3 Pauli matrices → $M_{GT} = \sqrt{3}$



$$Ft_{1/2} = \frac{4.794\,10^{-5}}{2(C_V^2 + 3C_A^2)}\, s \approx 1040\ s$$

is compared to $(Ft)_{0^+ \to 0^+} \approx 3070\ s$

$$\alpha \approx \frac{3070}{1040} \approx 2.95$$

$$\alpha = \frac{C_V^2 + 3C_A^2}{(\sqrt{2})^2 C_V^2} = \frac{(1 + 3C_A^2 / C_V^2)}{2} = \frac{(1+\rho^2)}{2}$$

ρ is the mixing ratio

➡ $$\boxed{\frac{C_A}{C_V} = \sqrt{\frac{(2\alpha - 1)}{3}} \approx 1.27}$$

The Standard Model (SM) and the value of C_A

Sign of C_A/C_V : A_β measurements in decay of polarized neutrons

β^-

$$A\xi = 2\,\mathrm{Re}\Big[\pm|M_{GT}|^2\lambda_{J'J}(C_TC_T'^* - C_AC_A'^*) + \delta_{J'J}|M_F||M_{GT}|\left(\frac{J}{J+1}\right)^{\frac{1}{2}}(C_SC_T'^* + C_S'C_T^* - C_VC_A'^* - C_V'C_A^*)\Big]$$

$$\lambda_{J'J} = \begin{cases} 1, & J\to J'=J-1 \\ 1/(J+1), & J\to J'=J \\ -J/(J+1), & J\to J'=J+1; \end{cases}$$

$\rho = C_A|M_{GT}| / C_V|M_F| = \pm\, 1.27\ \sqrt{3} \simeq \pm\, 2.2$

$C_A = C_A'$ real, $C_S = C_T = 0$ $\quad J = J' = 1/2$

$$\Rightarrow\quad A\xi = 2\Big[\,|M_{GT}|^2\left(\frac{1}{J+1}\right)(-C_A^2) + |M_F||M_{GT}|\left(\frac{J}{J+1}\right)^{1/2}(-2C_VC_A)\Big]$$

$$\xi = 2|M_F|^2|C_V|^2 + 2|M_{GT}|^2|C_A|^2 = 2|M_F|^2|C_V|^2\,(1+\rho^2)$$

$$\Rightarrow\quad \boxed{A = \frac{-\rho^2 - 2\rho\sqrt{J(J+1)}}{(J+1)(1+\rho^2)}}$$

$\Rightarrow$ $\rho > 0$: $\quad A = -\,0.9875$

$\Rightarrow$ $\rho < 0$: $\quad A = -\,0.1174$

consistent with measured values $\Rightarrow \rho < 0$!!

$\Rightarrow$ **$V - A$ theory**

Special issue on "mirror" transitions

1. $(\mathcal{F}t)_{\text{mirror}}$ compared to $(\mathcal{F}t)_{0^+\to 0^+}$ → ρ & %F, %GT in decays

$$\alpha = \frac{(\mathcal{F}t)_{0^+\to 0^+}}{(\mathcal{F}t)_{mirror}} = \frac{(1+\rho^2)}{2} \qquad \%F = \frac{C_V^2|M_F|^2}{C_V^2|M_F|^2 + C_A^2|M_{GT}|^2} = \frac{1}{1+\rho^2} = \frac{1}{2\alpha}$$

$$\%GT = \frac{C_A^2|M_{GT}|^2}{C_V^2|M_F|^2 + C_A^2|M_{GT}|^2} = \frac{\rho^2}{1+\rho^2} = 1 - \%F$$

All transitions are $T = ½$ isomultiplet

→ $M_F = 1$

www.nndc.bnl.gov/logft/

Noyau	$T_{1/2}$	Q_{ec} (keV)	BR (%)	log(ft)	α	%F	%GT
n	613.9 s	782.354	100	3.017	2.954	17	83
^{13}N	9.965 m	2220.5	100	3.665	0.665	75	25
^{19}Ne	17.25 s	3238.8	99.986	3.231	1.804	28	72
^{35}Ar	1.775 s	5966.1	98.36	3.755	0.545	92	8
^{39}Ca	0.861 s	6532.6	99.998	3.63	0.715	70	30

β^+

→ **Large Fermi component !!**

Special issue on "mirror" transitions

2. Large Fermi component !! → sensitivity to CVC hypothesis & V_{ud} …

$$\boxed{(ft)_{mirror} = \frac{4.794\,10^{-5}}{2(C_V^2|M_F|^2 + C_A^2|M_{GT}|^2)} = \frac{4.794\,10^{-5}}{2C_V^2|M_F|^2(1+\rho^2)}\,s}$$

… if ρ can be determined independently !!

Measurement of a correlation parameter

- Already shown in the neutron case (β^-): $$A_n = \frac{-\rho^2 - 2\rho\sqrt{J(J+1)}}{(J+1)(1+\rho^2)}$$
- Applicable to "nuclear" mirror transitions (β^+): $$A_m = \frac{\rho^2 - 2\rho\sqrt{J(J+1)}}{(J+1)(1+\rho^2)}$$
- Nucleus polarization not mandatory…: $$a_m = \frac{(1 - \rho^2/3)}{(1+\rho^2)}$$

Correlations study in mirror decays → tests of CVC hypothesis and CKM unitarity !!

A last point before illustrations…

Some slides ago …

$$d^2\lambda = KW(p_e)\xi\left(1 + a\frac{v_e}{c}\cos(\theta) + b\frac{m_e c^2}{E_e}\right)\sin(\theta)dp_e d\theta$$

1. $\int d\theta$ → $N(p_e)$ independent of a
2. SM → $b = 0$ ⚠ *and what happens if we remove this constrain ? → see later*
3. $\int dp_e$ → λ :

$$\lambda = K'\xi \underbrace{\int W(p_e)dp_e}_{\longrightarrow f(Z',E_0)} = \frac{\ln 2}{t_{1/2}}$$

This is now !

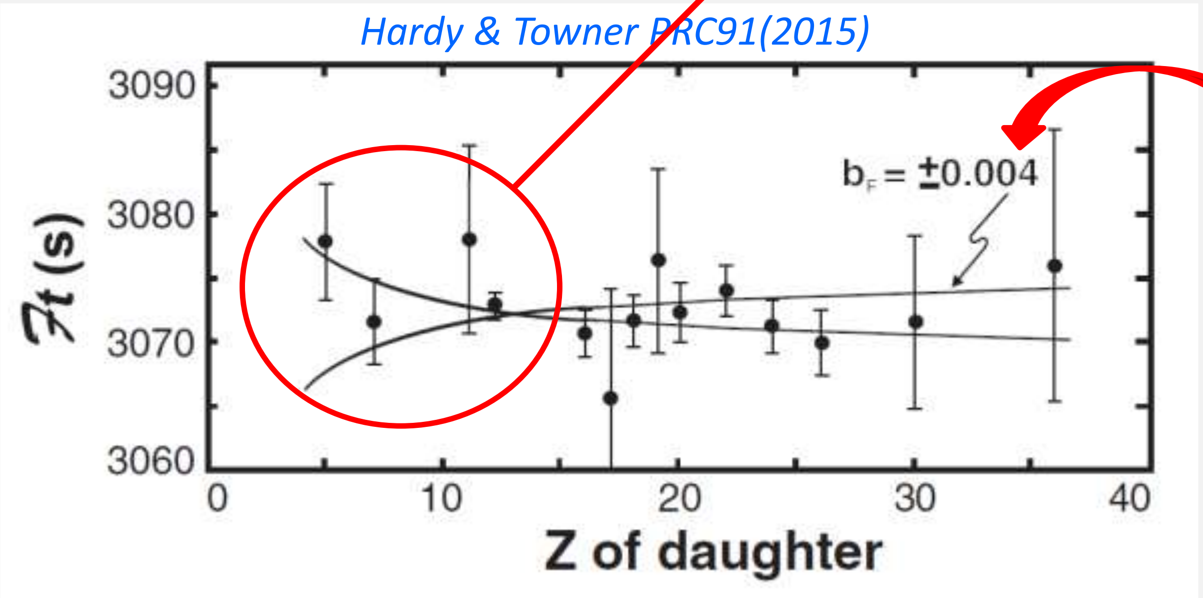
If $b \neq 0$, the shape of f is modified: $\int W(p_e)\left(1 + b\frac{m_e c^2}{E_e}\right)dp_e$

… and a dependence of Ft vs $<1/E_e>$ should be observed !

as $Z \downarrow$ → $<1/E_e> \uparrow$ ➡ dependence of Ft vs Z …

A last point before illustrations…

If $b \neq 0$ … ➡ … dependence of $\mathcal{F}t$ vs Z …



Test of fixed b values

b_F = -0.0028 (26) (best χ^2) at 1 σ ➡ $\boxed{C_S / C_V = 0.0014\ (13)}$

"Best" constraint got on C_S … under certain conditions …

Exercise: *In which conditions $C_S/C_V = -b_F/2$?*

➡ *Let's go to the "Illustrations" part …*

Outline

I. Introduction (13 slides)
- Why and How (LE vs HE)?
- Current questions and goals of the lectures
- A quick reminder on beta decay (Prerequisites)

II. Nuclear beta decay: How testing the weak interaction? (61 slides)
- Some tracks on theory: from Golden rule to events distributions
- Which terms for which physics?
- A word on some approximations and consequences…
- A special case: the Fierz term
- The Standard Model (SM) and beyond (helicity, "ft" values,…)

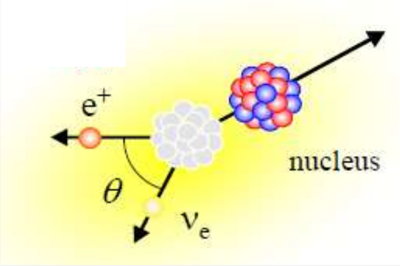
III. From theoretical rates to correlation experiments (21 slides)
- Beta-neutrino correlations
- Correlations involving polarized decaying nuclei

IV. Last section: CVC, V_{ud} & CKM (20 slides)
- Pure Fermi decays
- Other sources: nuclear mirror decays
- Other sources: the neutron case

From theoretical rates to correlation experiments

β–ν angular correlation

$$N(p_e,\theta)dp_e d\theta = N(p_e)(1+\tilde{a}\frac{v_e}{c}\cos(\theta))\sin(\theta)dp_e d\theta$$

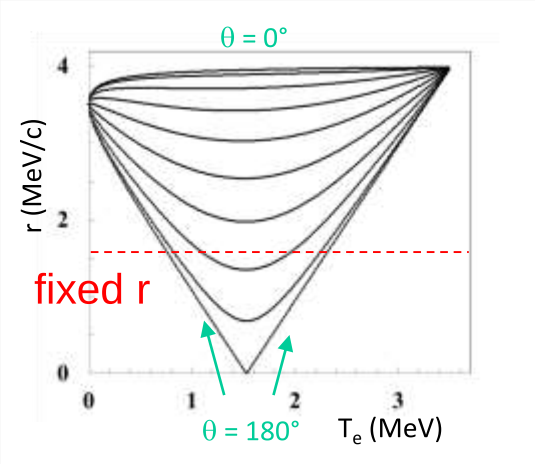


- This theoretical rate supposes the detection of the ν !!
- Fortunately the recoil motion is sensitive to θ

➡ *Change of kinematic variables*

1. $p_e \rightarrow T_e$: $N(p_e) \rightarrow N(T_e)$
2. $\theta \rightarrow r$: $r^2 = p_e^2 + p_\nu^2 + 2p_e p_\nu \cos(\theta)$

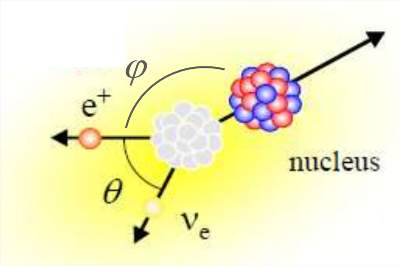
➡ $$\sin(\theta)\, d\theta = |d\cos(\theta)| = \frac{r dr}{p_e p_\nu}$$



$$\boxed{N(T_e,r)dT_e dr = N(T_e)r(1+\tilde{a}c\frac{(r^2-p_e^2-p_\nu^2)}{2E_e p_\nu})dT_e dr}$$

From theoretical rates to correlation experiments

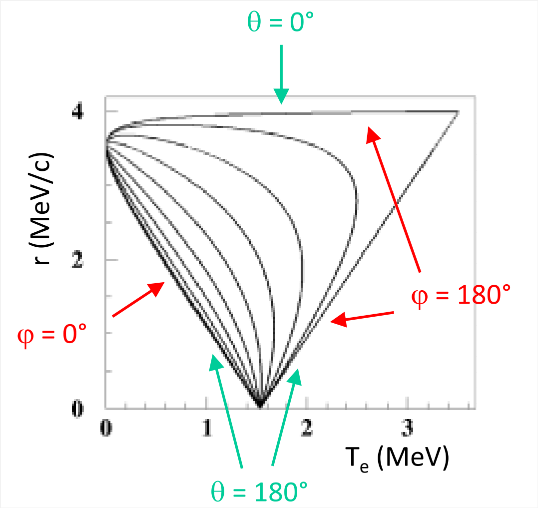
β–ν angular correlation



Coincidences not mandatory, recoil motion sufficient !

➡ *Integration of the rate formula on T_e*

At each "r", the 2 limits T_{min} & T_{max} are given by range in angle φ or θ



	r c< Q	r c> Q
T_{min}	φ = 0°	θ = 0°
T_{max}	φ = 180°	θ = 180°

$$T_{min} = \frac{(rc - Q)^2}{2(m_e c^2 + Q - rc)} \qquad T_{max} = \frac{(rc + Q)^2}{2(m_e c^2 + Q + rc)}$$

➡ $\boxed{N(r)dr = C\, r\, \{f + g + a\, [f + h(r)]\}dr}$

C : constant, f,g (T_{min} ,T_{max})

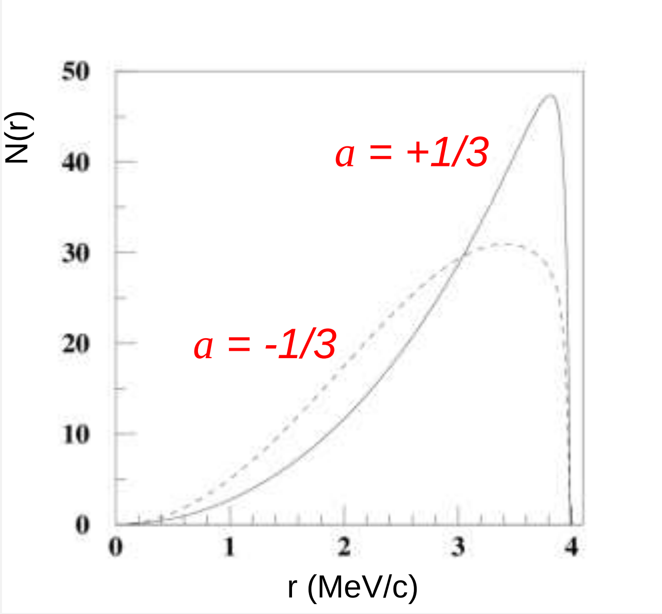
From theoretical rates to correlation experiments

I. $\beta-\nu$ angular correlation from recoil energy direct measurement

$$\boxed{N(r)dr = C\, r\, \{f + g + a\, [f + h(r)]\}dr}$$

C : constant, f,g (T_{min} ,T_{max})

Example : pure GT transition with Q = 3.5 MeV (^{6}He decay)



Problem: energy range 0 keV → 1.4 keV …

- Traditional "Si" detectors are not useable
- µ_channel plate (or channeltron) are efficient counters, but non-sensitive to ion energy…

➡ Ion energy must be defined or measured before detection → analysis by E-M fields

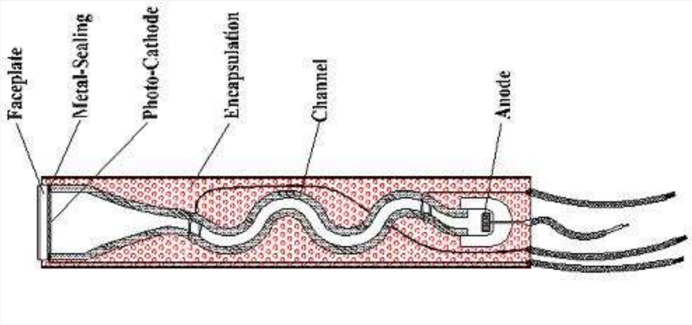
➡ Such energy range requires a "transparent" source !

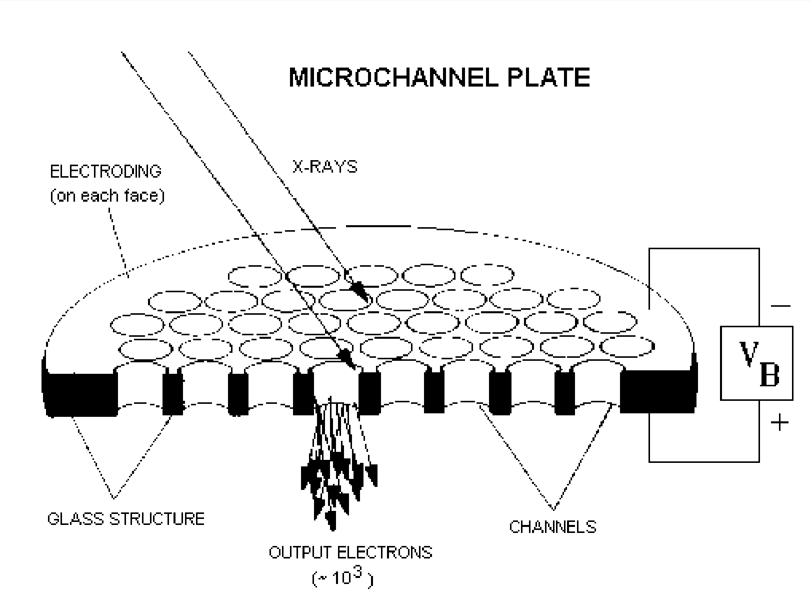
From theoretical rates to correlation experiments

I. $\beta-\nu$ angular correlation from recoil energy direct measurement

Conditions : T_{recoil} ~ 1 keV at best …

- Decay between GS (recoil not perturbed by secondary particle emission)
- "Transparent" source, ideal = vacuum
- Energy analysis by E-M fields
- Detection with "channeltron" or µ-channel plate



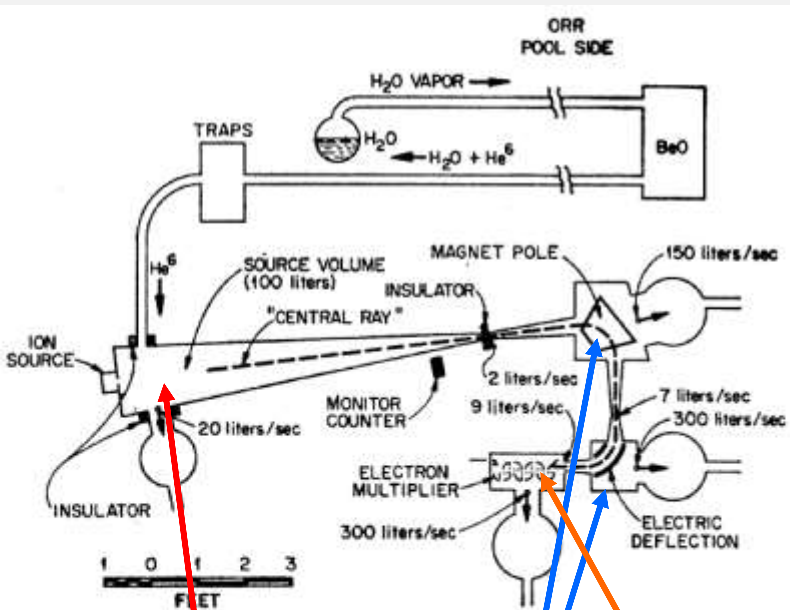


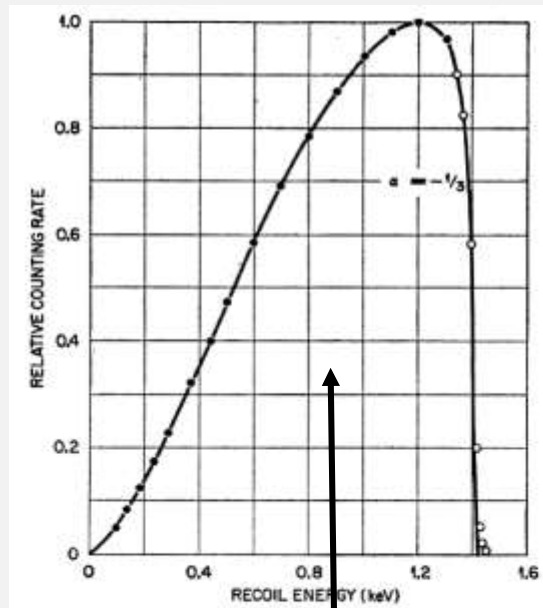
From theoretical rates to correlation experiments

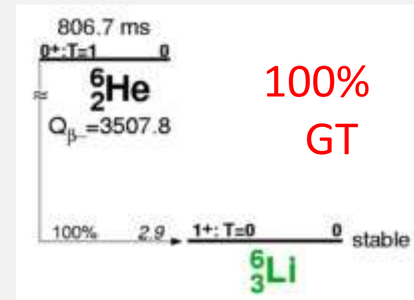
I. $\beta-\nu$ angular correlation from recoil energy direct measurement

Example : recoil spectrometer used by Johnson et al. (1963) with ^{6}He (Oak Ridge)

Johnson et al, PR132(1963)







100%
GT

$a = -0.3343\ (30)$

Most precise value measured in a pure GT transition !

Gaseous source (not well defined!)

E x M analysis

Channeltron detection

Discrete spectrum:

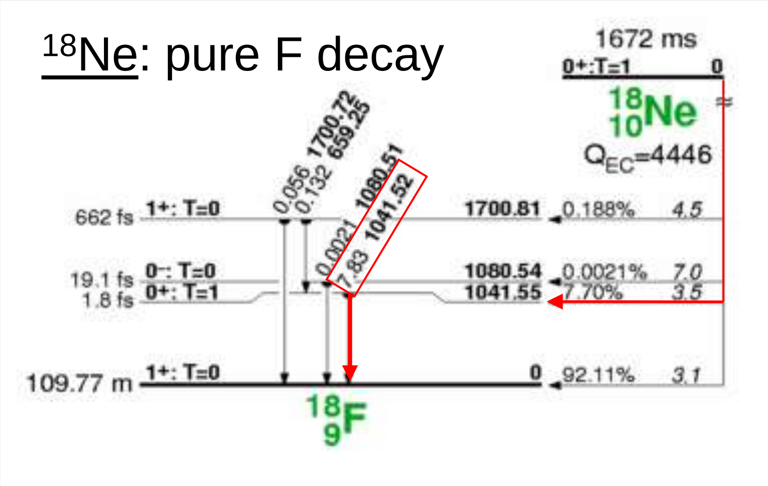
- requires a *very good knowledge* of spectrometer response !
- only 13 DoF and $\chi^2 = 1.69$ …

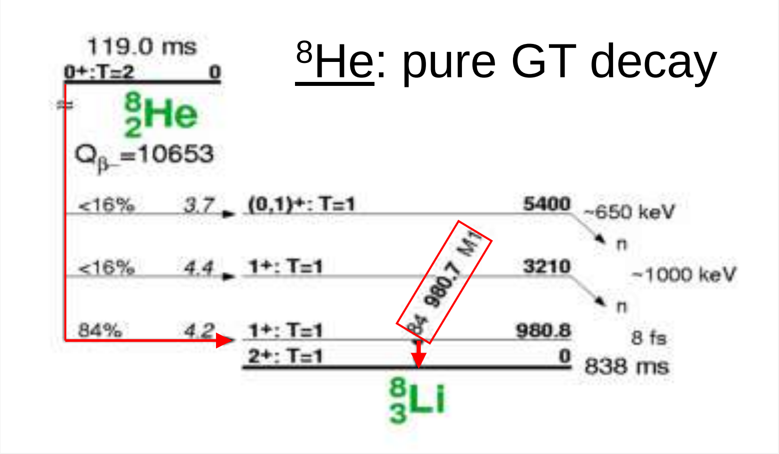
From theoretical rates to correlation experiments

II. β–ν angular correlation from recoil energy indirect measurement

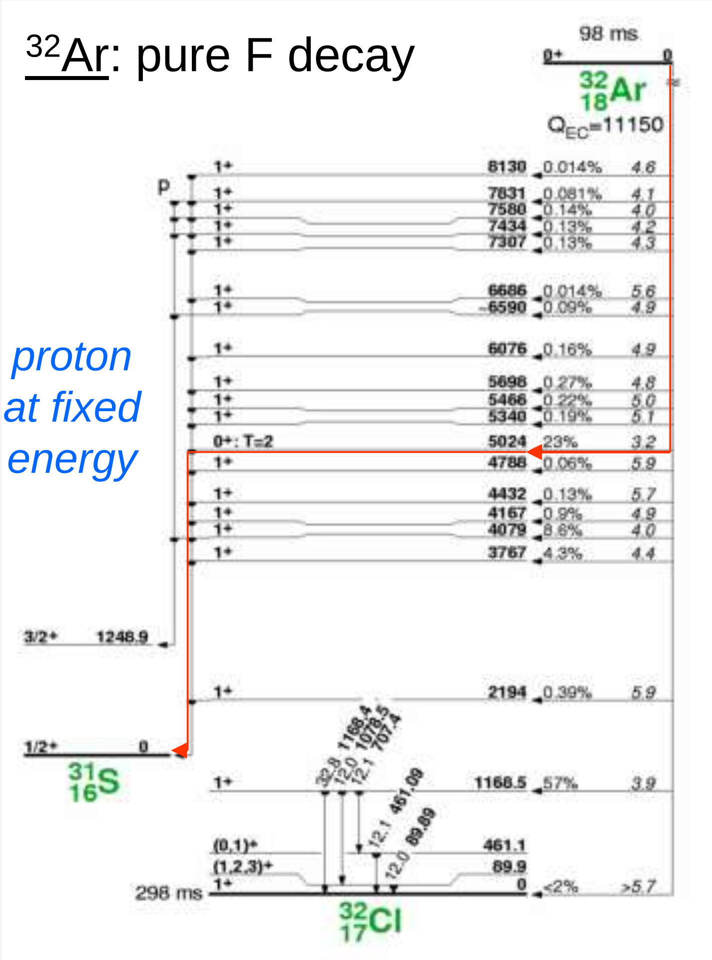
Alternative method: detection of a delayed particle emitted during recoil

gamma at fixed energy





proton at fixed energy



From theoretical rates to correlation experiments

II. β–ν angular correlation from recoil energy indirect measurement

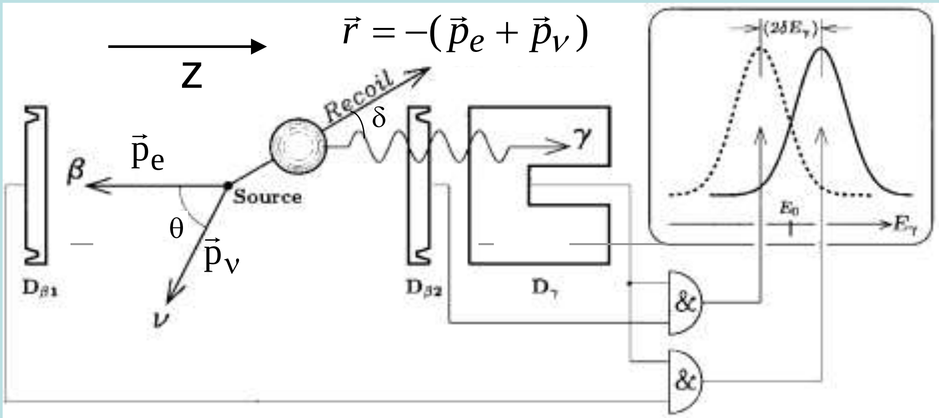
- γ case: Doppler shift measurement

$$E_\gamma' = E_\gamma\,(1 + v_S/c\cos(\delta))$$

↓

source speed: $v_s = r / M_{ion}$

δ : angle between γ & ion



Vorobel et al EPJA16(2003) (^{14}O)

β–γ coincidence at

0° : $E_\gamma' < E_\gamma$

180° : $E_\gamma' > E_\gamma$

on a average, β >< recoil ion

"Double" Doppler shift

$$\langle \delta E \rangle(0°,180°) = E_\gamma\,(\pm \langle r_z \rangle / M_{ion}c) \longrightarrow 2\langle \delta E \rangle = 2E_\gamma\,(\langle r_z \rangle / M_{ion}c)$$

where $\langle r_z \rangle$ is a weighted mean of Z-component of $\vec{r}$ $\qquad r_z = p_e + p_\nu\cos(\theta)$

$$\boxed{\langle r_Z \rangle = p_e + \frac{p_\nu \int_0^\pi \cos(\theta)\left(1+\tilde{a}\,{}^{v_e}\!/_c\cos(\theta)\right)\sin(\theta)d\theta}{\int_0^\pi \left(1+\tilde{a}\,{}^{v_e}\!/_c\cos(\theta)\right)\sin(\theta)d\theta} = p_e\left(1 + \tilde{a}\,\frac{p_\nu c}{3E_e}\right)}$$

Illustration

From theoretical rates to correlation experiments

II. β–ν angular correlation from recoil energy indirect measurement

- γ case: Doppler shift measurement

$$2 < \delta E > = 2E_\gamma \, p_e (1 + \tilde{a} \, E_\nu / 3E_e) / M_{ion} c$$

Example: Experiment performed by Egorov et al (1997) with ^{18}Ne (Orsay)

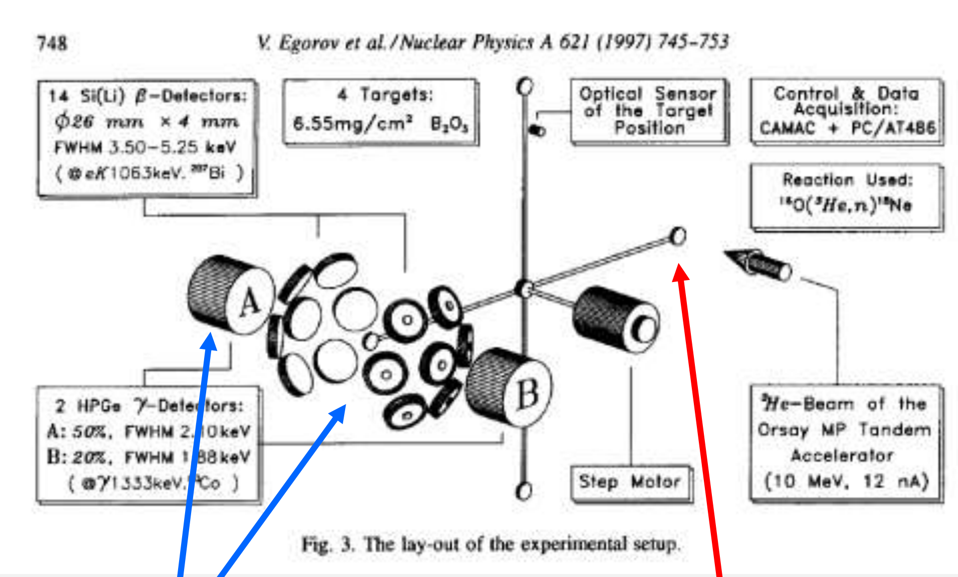
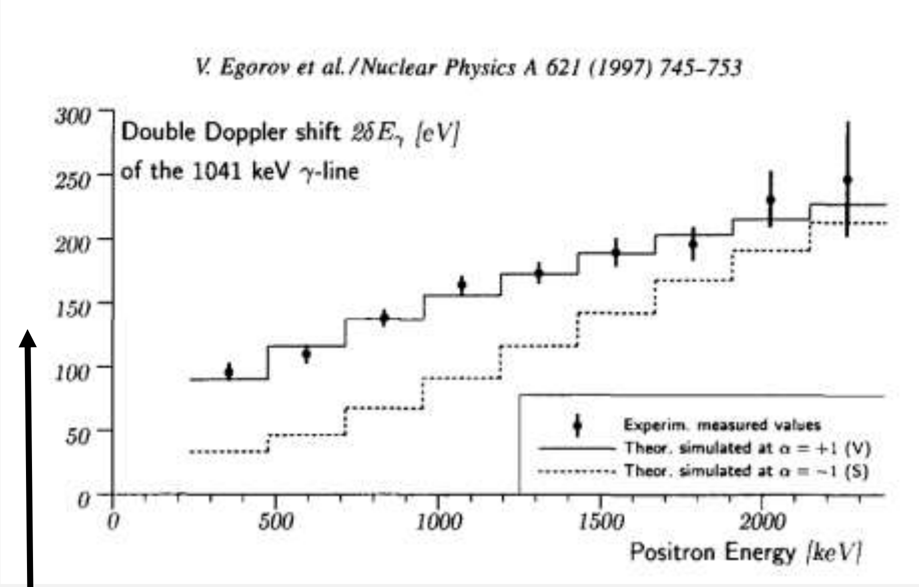


Fig. 3. The lay-out of the experimental setup.



Detection setup:
- isolated from the beam
- Si(Li) → β / HPGe → γ

Solid target
→ solid source

Typical value:
- 2< δE > ~ 200 eV !
- Ge resolution: 1-2 keV !

→ "subtle" analysis …

$\tilde{a}_F = 1.06\ (1)$

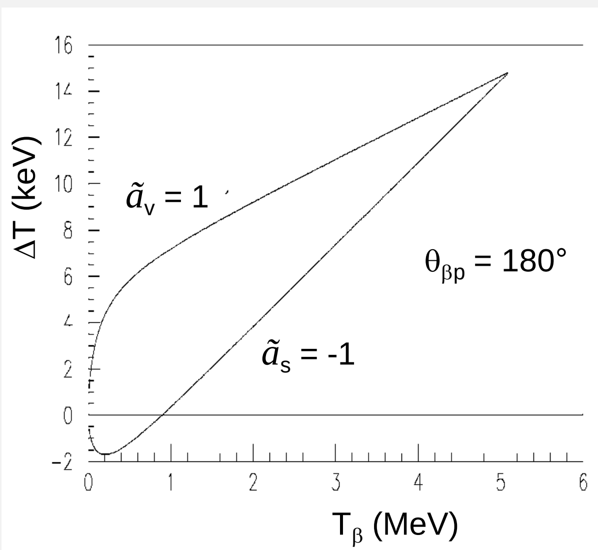
From theoretical rates to correlation experiments

II. β–ν angular correlation from recoil energy indirect measurement

- Charged particle case: kinetic shift measurement

$p_{shift}\,(180°) = p + \langle r_Z \rangle\, m_p / M_{ion}$

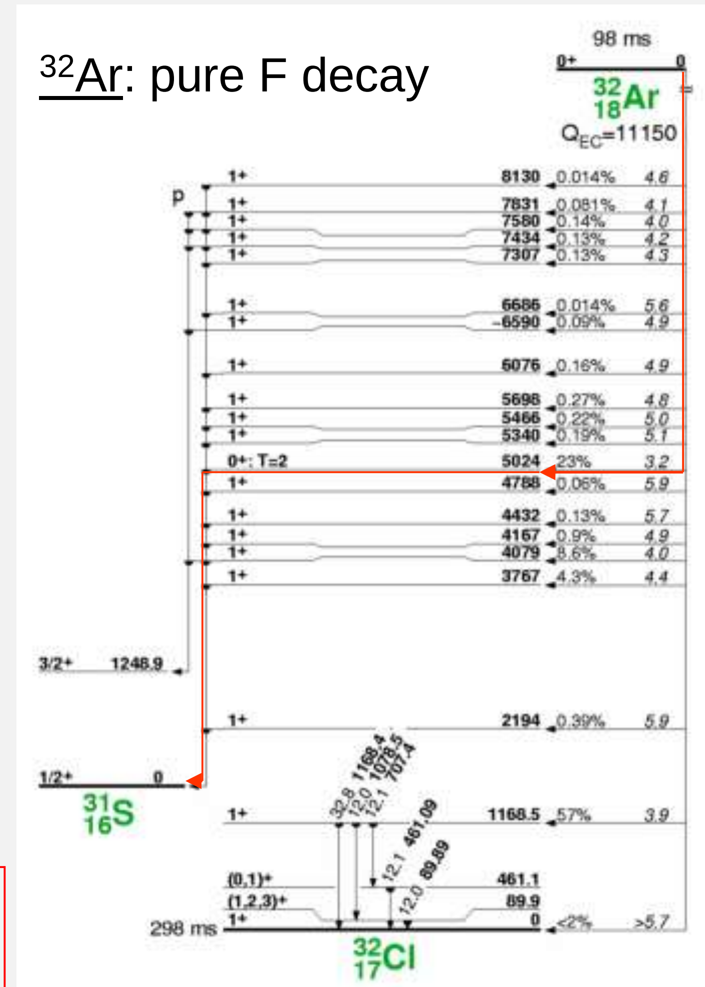
$$\Delta T = \frac{(p_{shift}^2 - p^2)}{2m_p} = \frac{\langle r_z \rangle}{2M_{ion}}\left(\frac{\langle r_z \rangle m_p}{M_{ion}} + 2p\right)$$



$T_p = 3.3$ MeV
$T_R^{max} = 520$eV

Typical value:

$\Delta T \sim 10$ keV



Shift larger than in γ case !
Kinetic broadening directly measurable !

From theoretical rates to correlation experiments

II. β–ν angular correlation from recoil energy indirect measurement

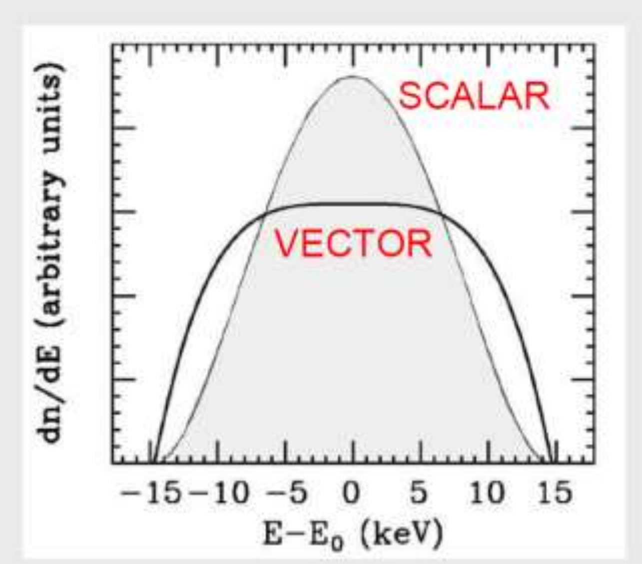
- Charged particle case: kinetic shift measurement

Example: Experiment performed by Adelberger et al (1999) with ^{32}Ar (ISOLDE)

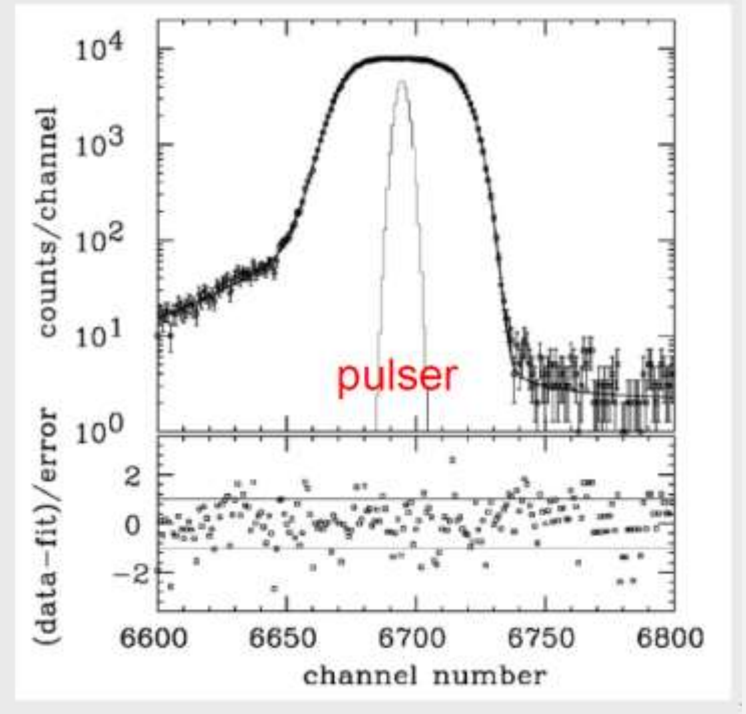
Adelberger et al. PRL 83 (1999) 1299

Experimental setup

- ^{32}Ar beam implanted in a thin C foil inclined at 45°
- p detected by 2 p-i-n diodes located at 1.6 cm
- β eliminated by a strong magnetic field



Theoretical curves



Experimental spectrum

$\boxed{a_F = 0.9989(65)}$

The second most precise value measured in a pure F transition !

Illustration

From theoretical rates to correlation experiments

III. β–ν angular correlation from β–recoil coincidences measurement

$$\boxed{N(T_e,r)dT_e dr = N(T_e)r\left(1+\tilde{a}c\frac{(r^2-p_e^2-p_\nu^2)}{2E_e p_\nu}\right)dT_e dr}$$

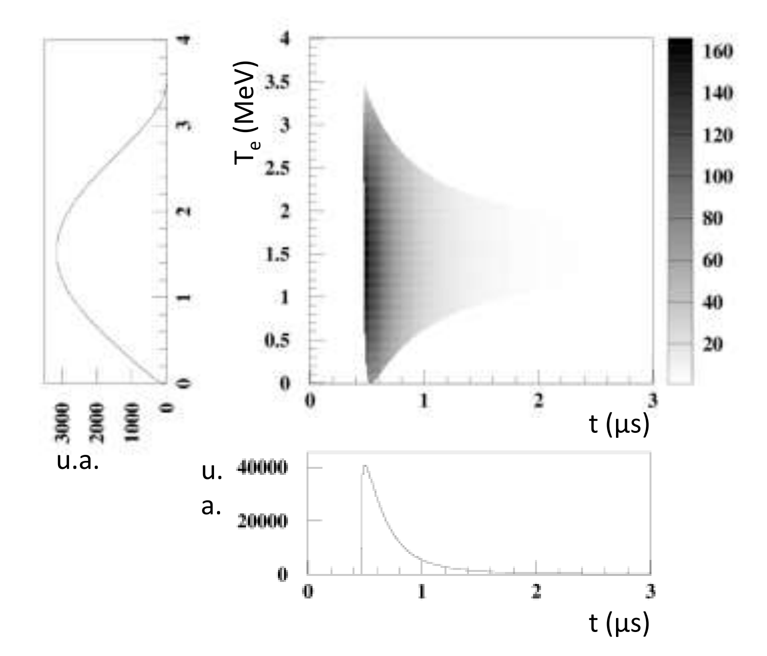
<u>Easiest method</u>: measurement of a time-of-flight between β and recoil ion

➡ *Change of kinematic variable: $r \rightarrow t$*

$r = M_{ion}\, v_{ion} = M_{ion}\, d_{SD} / t$

$\rightarrow dr \sim dt/t^2 \sim r^2\, dt$

$$\boxed{N(T_e,t)dT_e dt = N(T_e)r^3\left(1+\tilde{a}c\frac{(r^2-p_e^2-p_\nu^2)}{2E_e p_\nu}\right)dT_e dt}$$



From theoretical rates to correlation experiments

III. β–ν angular correlation from β–recoil coincidences measurement

Measurement of a time-of-flight between β and recoil ion

$$\boxed{N(T_e,t)dT_e dt = N(T_e)r^3\left(1+\tilde{a}c\,\frac{(r^2-p_e^2-p_\nu^2)}{2E_e p_\nu}\right)dT_e dt}$$

Conditions : T_{recoil} ~ 1 keV at best …

- Decay between GS (recoil not perturbed by secondary particle emission)
- "Transparent" source, ideal = vacuum
- Detection of β using plastic scintillators (fast start detector)
- Detection of recoil ions with μ-channel plate (fast stop detector)

From theoretical rates to correlation experiments

III. β–ν angular correlation from β–recoil coincidences measurement

Measurement of a time-of-flight between β and recoil ion

Example: Experiment performed by Gorelov et al (2005) with ^{38m}K (TRIUMF)

Gorelov et al. PRL 94 (2005)

recoil ion detector

recoil ion spectrometer (~ 4π)

β telescope

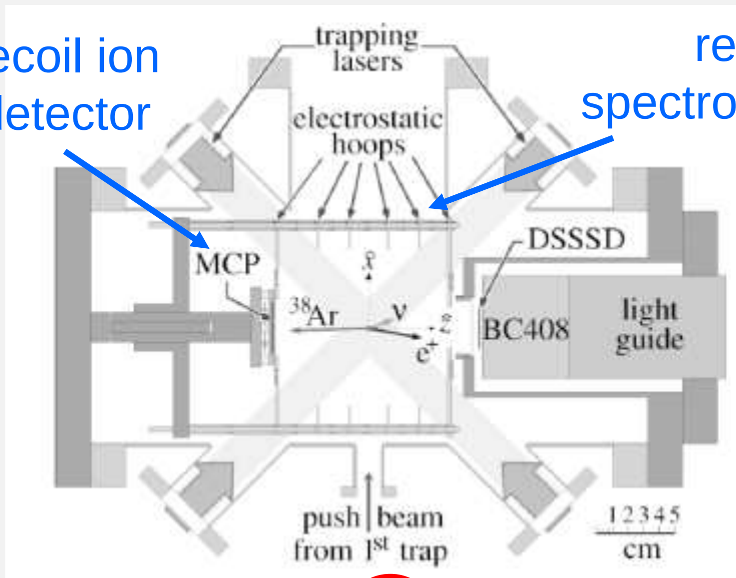
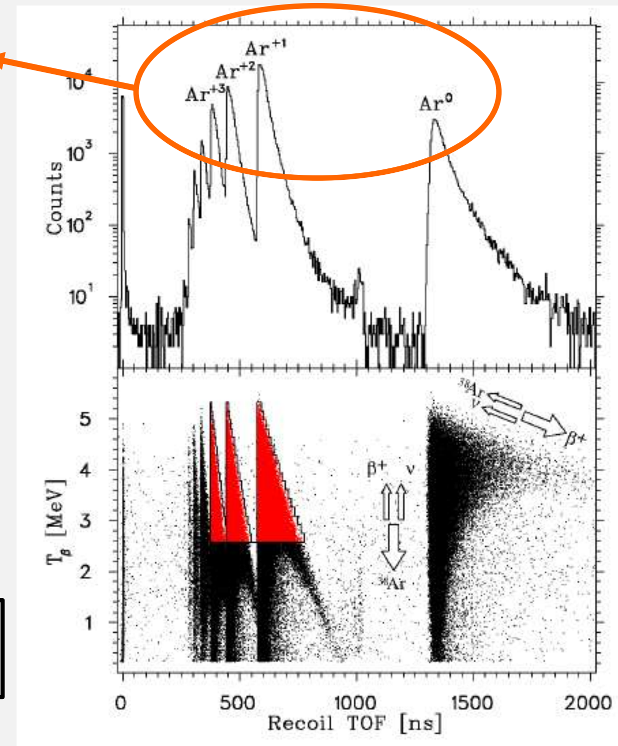


FIG. 1. Top view of the 2nd MOT apparatus with the recoil and β detectors.

Shake-off process…



$\tilde{a}_F$ = *0.9981(48)*

Magneto-Optical Trap: very well defined source in vacuum (R ~ 10 µm) !!

The most precise value measured in a pure F transition !

From theoretical rates to correlation experiments

Relative precision

β–ν angular correlation: the best results

- GT: ^{6}He (Johnson *et al.* PRC 1963) $\rightarrow \tilde{a}_{GT}$ = -0.3308 (30)
  corrected for radiative and recoil corrections (Glück NPA 1998)

  ^{8}Li (Sternberg *et al.* PRL 2015) $\rightarrow \tilde{a}_{GT}$ = -0.3342 (39)

  ~ 1%

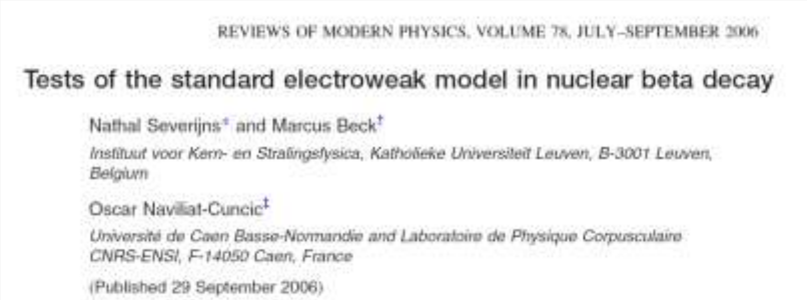
- F: ^{32}Ar (Adelberger *et al.* PRL 1999) $\rightarrow \tilde{a}_{F}$ = 0.9989 (65)

  ^{38m}K (Gorelov *et al.* PRL 2005) $\rightarrow \tilde{a}_{F}$ = 0.9981 (48)

  ~ 0.5%

Results used in a global analysis including all available data

Reviews:

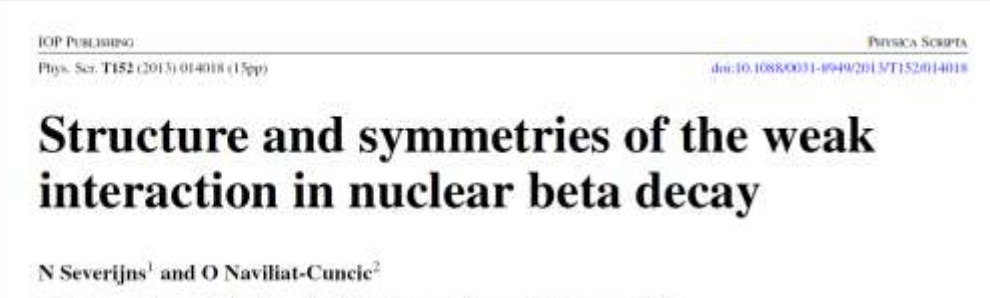

REVIEWS OF MODERN PHYSICS, VOLUME 78, JULY–SEPTEMBER 2006

Tests of the standard electroweak model in nuclear beta decay

Nathal Severijns* and Marcus Beck†
Instituut voor Kern- en Stralingsfysica, Katholieke Universiteit Leuven, B-3001 Leuven, Belgium

Oscar Naviliat-Cuncic‡
Université de Caen Basse-Normandie and Laboratoire de Physique Corpusculaire CNRS-ENSI, F-14050 Caen, France

(Published 29 September 2006)


IOP Publishing Physica Scripta

Phys. Scr. T152 (2013) 014018 (15pp) doi:10.1088/0031-8949/2013/T152/014018

Structure and symmetries of the weak interaction in nuclear beta decay

N Severijns[1] and O Naviliat-Cuncic[2]

+ Boothroyd *et al.* PRC 1984, Severijns *et al.* ARNPS 2011, Severijns JPG 2014, Wauters *et al.* PRC 2014 …

From theoretical rates to correlation experiments

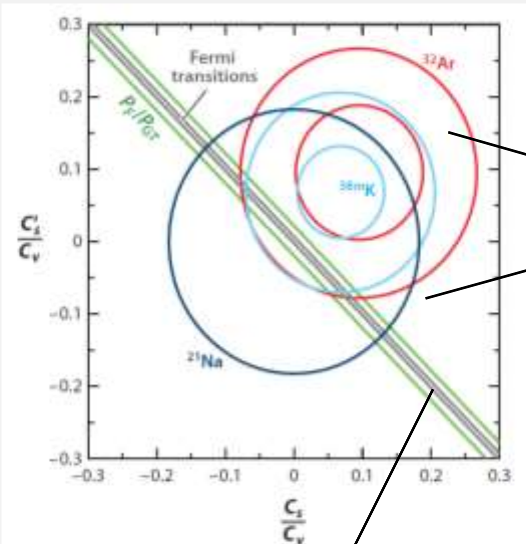
β–ν angular correlation: the status

Severijns & Naviliat ARNPS61(2011)

SCALAR

$$\tilde{a} = \frac{a}{1 + b\langle m_e / E_e \rangle}$$

TENSOR

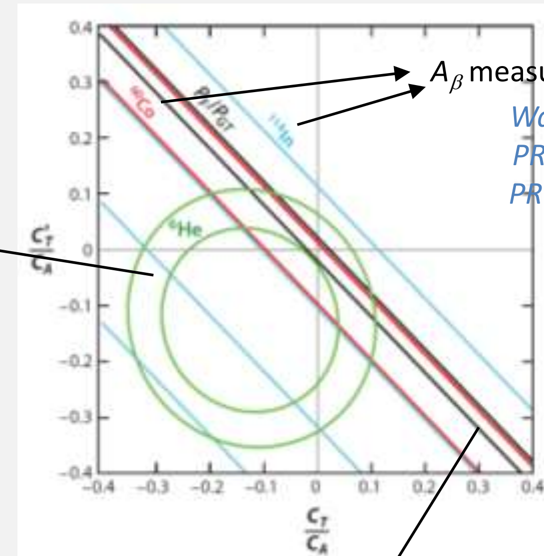


see slide 65

Circles because

$a\,(C^2_{S,T}, C^2_{V,A})$

$b\,(C_{S,T}, C_{V,A})$



A_β measurements

Wauters et al PRC80(2009) PRC 82(2010)

see slide 78

$$Ft \propto \left(1 + \langle m/E \rangle b_F\right)^{-1}$$

Hardy et al PRC79(2009)

$$\left|C_S^{(\prime)} / C_V^{(\prime)}\right| < 0.07$$

$$\left|C_T^{(\prime)} / C_A^{(\prime)}\right| < 0.09$$

$$P_F / P_{GT} \propto b_F - b_{GT}$$

Wichers et al PRC58(1987)

Carnoy et al PRC43(1991)

- Best constraints from "*b*", but "*a*" adds limits... ("*b*" unsensitive to right-handed ν !)
- Measurements of "*b*" requires "precise" detection of β particles

⇒ *Enough room for measurements of "a"...*

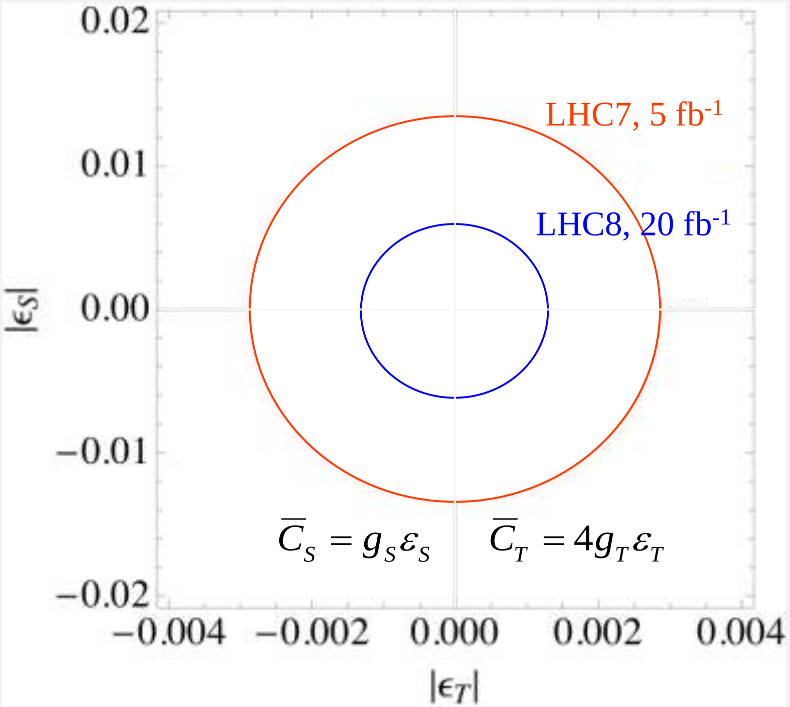
From theoretical rates to correlation experiments

β–ν angular correlation: the status

adapted from Severijns & Naviliat PST152(2013)

Parent	Technique	Team, laboratory	Remarks	
^{6}He	Spectrometer	ORNL	$a = -0.3308(30)$	1963
^{32}Ar	Foil: p recoil	UW-Seattle, ISOLDE	$\tilde{a} = 0.9989(52)(39)$	1999
^{38m}K	MOT	SFU, TRIUMF	$\tilde{a} = 0.9981(30)(34)$	2005
^{21}Na	MOT	Berkeley, BNL	$a = 0.5502(38)(46)$	2008
^{6}He	Paul trap	LPC-Caen, GANIL	$\tilde{a} = -0.3335(73)(75)$	2011
^{6}He	Paul trap	LPC-Caen, GANIL	Analysis under way	
^{8}Li	Paul trap; $\beta\alpha$	ANL	$\tilde{a} = -0.3342(26)(29)$	2015
^{35}Ar	Paul trap	LPC-Caen, GANIL	Analysis under way	
^{32}Ar	Foil; β-p coinc	CENBG, ISOLDE	In preparation	
^{19}Ne	Paul trap	LPC-Caen, GANIL	Analysis under way	
^{6}He	EIBT	Weizmann, SOREQ	In progress	
^{6}He	MOT	ANL, CENPA	In progress	
Ne	MOT	Weizmann, SOREQ	In progress	
~~^{21}Na~~	~~MOT~~	~~KVI-Groningen~~	~~In progress~~	
^{32}Ar	Penning trap	Texas A&M	In preparation	
^{8}He	Foil: $\beta\gamma$	NSCL	In preparation ?	

Comparison with LHC (CMS)
channel: $pp \rightarrow e$+MET+X



Naviliat & González ADP525(2013)
Cirigliano et al PPNP71(2013)

- Many projects (a & b) with precision < 0.5 %
- Competitive with LHC results

⇒ *Better constraints on exotic currents expected in the "coming" years*

From theoretical rates to correlation experiments

$\beta-\nu$ angular correlation: needs to reach a relative precision better than 0.1%

1. Why is it difficult ?
   - Sometimes statistics are limited due to:
     - low production rates of radioactive beams
     - bad events, background, …
     - the loss of ~ 80% of statistics when β^+ decays (recoils are neutral !!)
   - Systematic effects have to be investigated at the same level of precision
     - in particular, in direct measurements (recoil energy or ToF), any process modifying the kinematics (electric field, scattering, …) must be identified and precisely controlled…!

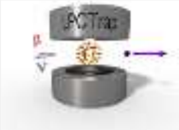
2. Why is it feasible today (or "early" tomorrow…)?
   - Development of new sources and techniques → significant increasing of beam intensities
   - Decaying sources are cleaner (use of ion and atom traps)
   - Simulation tools are more and more sophisticated (GEANT4) and hardware enables to run the most realistic simulations (GPU: Graphics Processing Unit)
   - DAQ systems are faster (signal digitization) allowing high rates of data and reducing drastically the deadtime during data taking …

From theoretical rates to correlation experiments

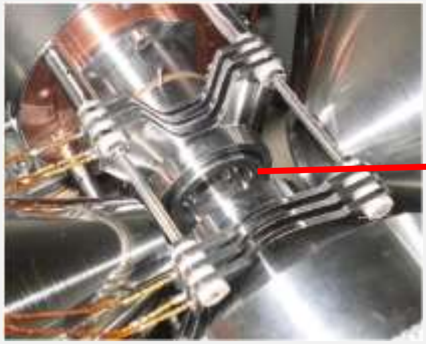
$\beta - \nu$ angular correlation: needs to reach a relative precision better than 0.1%

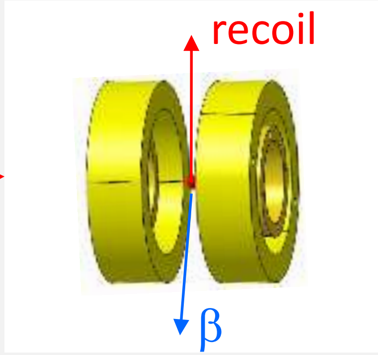
Example of main systematic effect and its management in LPCTrap experiment

- *Decay source confined in a transparent Paul trap*

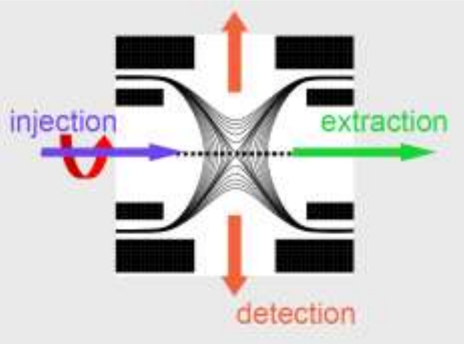


beam



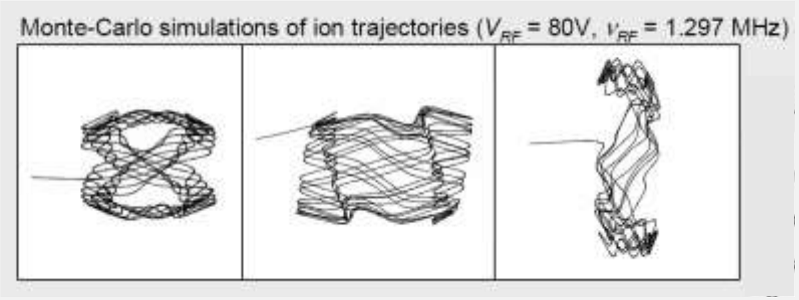


Scheme: Ideal ponctual source in vacuum



In real life, ions:

- describe specific trajectories in the confinement field
- undergo collisions with residual gas
- suffer charge repulsion from colleagues (typical load of some 10k ions in 1 mm^3)

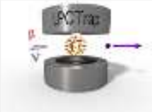


Illustration

From theoretical rates to correlation experiments

$\beta-\nu$ angular correlation: needs to reach a relative precision better than 0.1%

Example of main systematic effect and its management in LPCTrap experiment



In real life, ions:

- describe specific trajectories in the confinement field
  - high precision probe register the real RF potential put on electrodes & a realistic field map is computed
- undergo collisions with residual gas
  - ion-atom interaction potentials are computed by atomic physicists (theoreticians…)
- suffer charge repulsion from colleagues (typical load of some 10k ions in 1 mm^3)
  - "simple" Coulomb interaction: at each step, each ion interacts with all others…

Such realistic simulation requires extremely large memories and parallel programming methods allowed by GPU systems

The whole procedure takes much time, and globally such a project (experiment preparation, data taking and analysis) lasts *at least* 10 years…

From theoretical rates to correlation experiments

Correlations involving polarized decaying nuclei (A_β, D, …)

- Parameter deduced from a difference in couting rates between 2 orientations
  → not sensitive to the shape of events distribution

see slides 54-59

M^me Wu experiment

- Main difficulty: nucleus orientation → degree, conservation and estimation…
  "New" trend: optical pumping method using lasers

→ Thanks to multiple interaction with lasers @ adequate frequencies, hyperfine states are populated, which correspond to the needed nucleus polarization

3 methods:

Examples

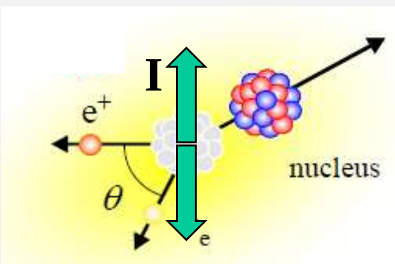
- In a Magneto-Optical Trap → lasers are "naturally" present ➡ A_β in ^{37}K (TRIUMF)
  Fenker et al arXiv 2017
- In beam colinear polarization + implantation in a cold crystal ➡ A_β in ^{35}Ar (ISOLDE)
  Severijns et al, tests in progress
- Polarization in 3-D Paul trap → original, never tested… ➡ D in ^{23}Mg (GANIL)
  Delahaye et al, project submitted

From theoretical rates to correlation experiments

Correlations involving polarized decaying nuclei (A_β, D, …)

Example: Measurement of D in ^{23}Mg (GANIL) Delahaye et al, project MORA*

- Interest: T violation → CP violation: source of matter-antimatter asymmetry ?



$$D\,\frac{\vec{J}.(\vec{p}_e\times\vec{q})}{J(E_e E_\nu)}$$

- β- recoil coincidences
- $\vec{J}$ known

$$D=\frac{-2\rho\, Im(\delta_{JJ'}(\frac{J}{J+1})^{1/2}\frac{C_A^*}{C_A})}{(1+\rho^2)}$$

- $D \neq 0 \rightarrow \rho \neq 0$ → Mirror decay !

- Basic setup:
  - LPCTrap (Paul trap and detection system) for coincidences
  - Adequate lasers for ion cloud polarization: high degree expected (> 99% in 0.2ms) and continuously measured through A_β
- Beam production: ^{23}Mg produced with high intensity at GANIL (~ 2×10^8 pps)
- Expected precision < 1×10^{-4} $D \propto \frac{N^+-N^-}{N^++N^-}$ between 2 opposite polarization directions

*MORA: Matter's Origin from the RadioActivity of trapped and laser oriented ions

Outline

I. Introduction (13 slides)

- Why and How (LE vs HE)?
- Current questions and goals of the lectures
- A quick reminder on beta decay (Prerequisites)

II. Nuclear beta decay: How testing the weak interaction? (61 slides)

- Some tracks on theory: from Golden rule to events distributions
- Which terms for which physics?
- A word on some approximations and consequences…
- A special case: the Fierz term
- The Standard Model (SM) and beyond (helicity, "ft" values,…)

III. From theoretical rates to correlation experiments (21 slides)

- Beta-neutrino correlations
- Correlations involving polarized decaying nuclei

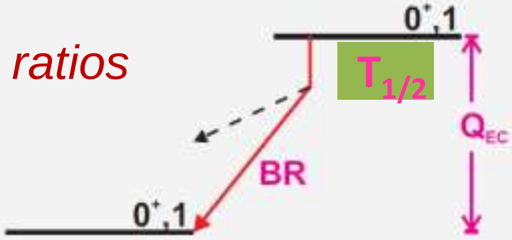
IV. Last section: CVC, V_{ud} & CKM (20 slides)

- Pure Fermi decays
- Other sources: nuclear mirror decays
- Other sources: the neutron case

Last section: CVC, V_{ud} & CKM

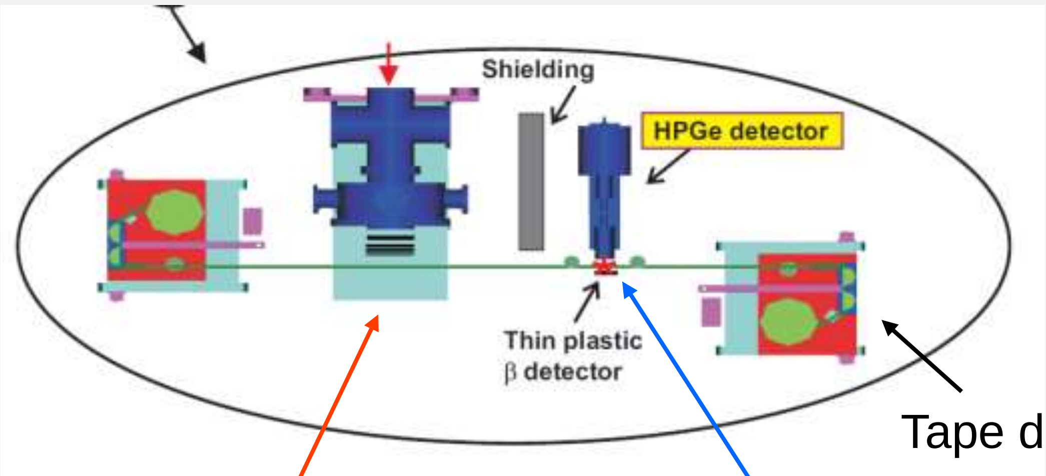
"ft" value in pure F decays: $$\boxed{ft_{1/2} = \frac{4.794\,10^{-5}}{2C_V^2 |M_F|^2}}$$ with $M_F^2 = 2$

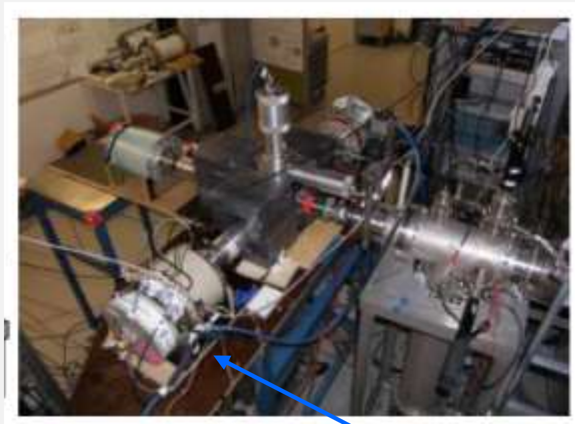
➡ *Measurements of* *masses* *half-lives & branching ratios*



EXPERIMENT

- *half-lives & branching ratios:* ~ *common setup*





1. Beam implantation on a tape

2. Detection setup: plastic scintillator (β) & HPGe (γ)

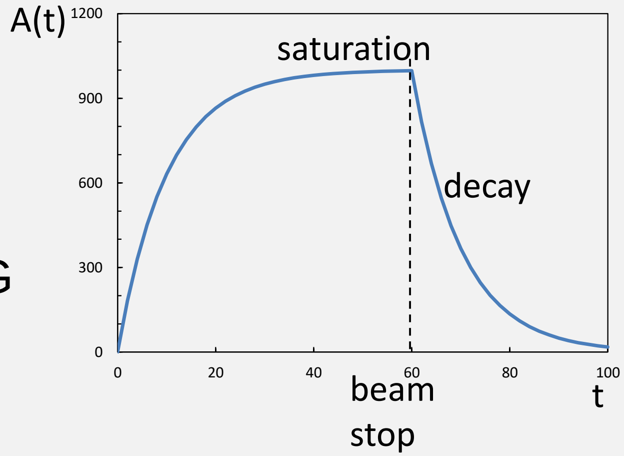
$\rightarrow$ shielded from the beam line

Last section: CVC, V_{ud} & CKM

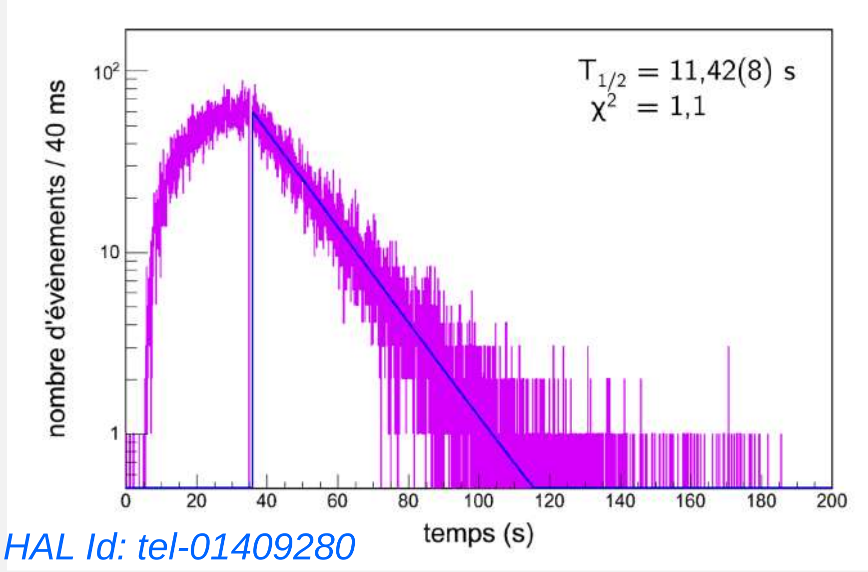
- *half-lives & branching ratios: ~ common setup*

Procedure:

1. Beam implantation (3-4 $T_{1/2}$) → reaching saturation
2. Beam stop and tape shift → detection setup
3. Decay measurement (10-15 $T_{1/2}$) → reaching the BG
   ⮡ β counting (with or without γ)
4. Tape shift and new cycle



Example: ^{23}Mg (*C. Magron, PhD CENBG*)



HAL Id: tel-01409280

Requirements and systematic effects:

- Beam purity
- Deadtime (depends on counting rates)
- BG management
- PM stability
- Evaporation from tape
- …

Relative precision reached ~ 10^{-3} – 10^{-4}

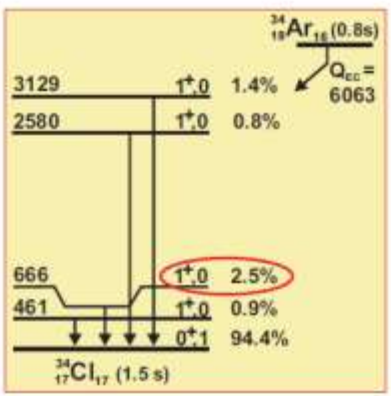
Last section: CVC, V_{ud} & CKM

- *half-lives & branching ratios: ~ common setup*

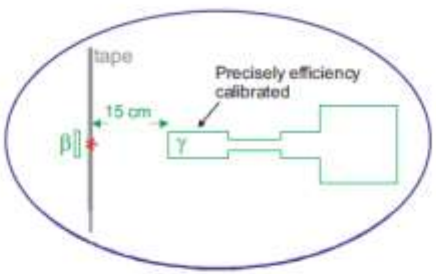
Measurement of $\beta-\gamma$ coincidences, BR deduced from a ratio with β in single

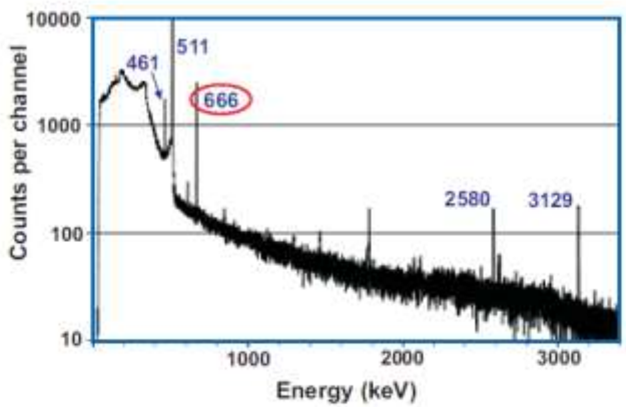
➡ depends on γ detection efficiency !

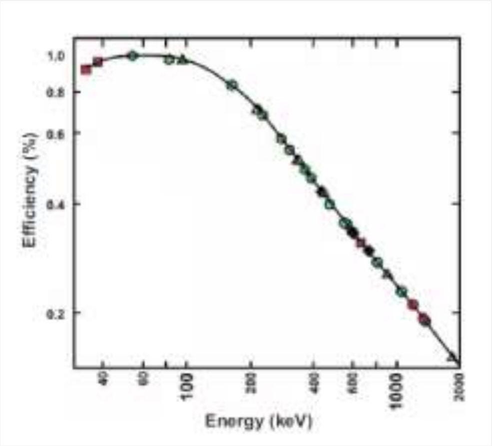
Example: ^{34}Ar



$$\frac{N_{\gamma_1\beta}}{N_\beta} = \frac{\cancel{N_0} R_1 \cancel{\varepsilon_\beta} \varepsilon_{\gamma_1}}{\cancel{N_0} \cancel{\varepsilon_\beta}}$$







Requirements & systematic effects:

- ε_γ to be determined precisely with calibrated sources
- Beam position and geometry must be well controlled
- γ peak fit (shape, BG …)

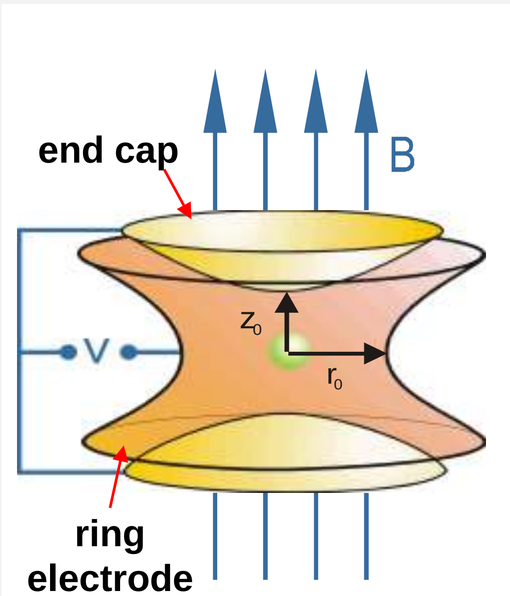
* *on BR of interest*

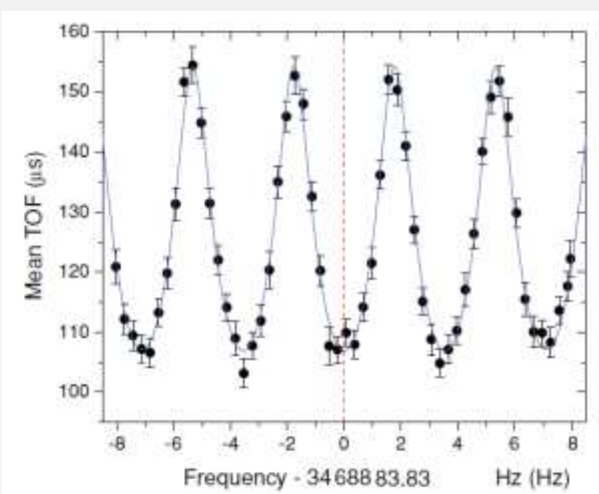
Relative precision reached* ~ 10^{-3} – 10^{-5}

Illustration

Last section: CVC, V_{ud} & CKM

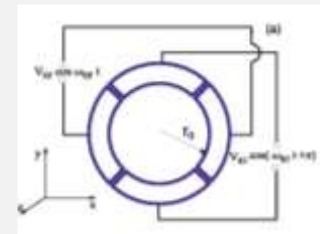
- *Masses: best setup = Penning trap* (*ref: course of S. Grévy, EJC 2015*)





<u>Ion trapping</u>:

- Thanks to static EM fields, E & B
- B → cyclotron motion $\omega_c = qB/M$



<u>Principle of mass measurement</u>:

- Excitation of ion motion = external RF signal (ω_{RF}) imposed on segmented ring electrode
- RF scanning: when $\omega_{RF} = \omega_c$ the transfered energy is maximal
- Ions extracted and ToF measured: minimal ToF corresponds to $\omega_{RF} = \omega_c$

<u>Example</u>: ^{31}S @ JYFL (*Kankainen et al. PRC 2010*)

<u>Limiting factors</u>:
Species production, purity, half-life, system stability,…

Relative precision reached ~ $10^{-5} – 10^{-8}$

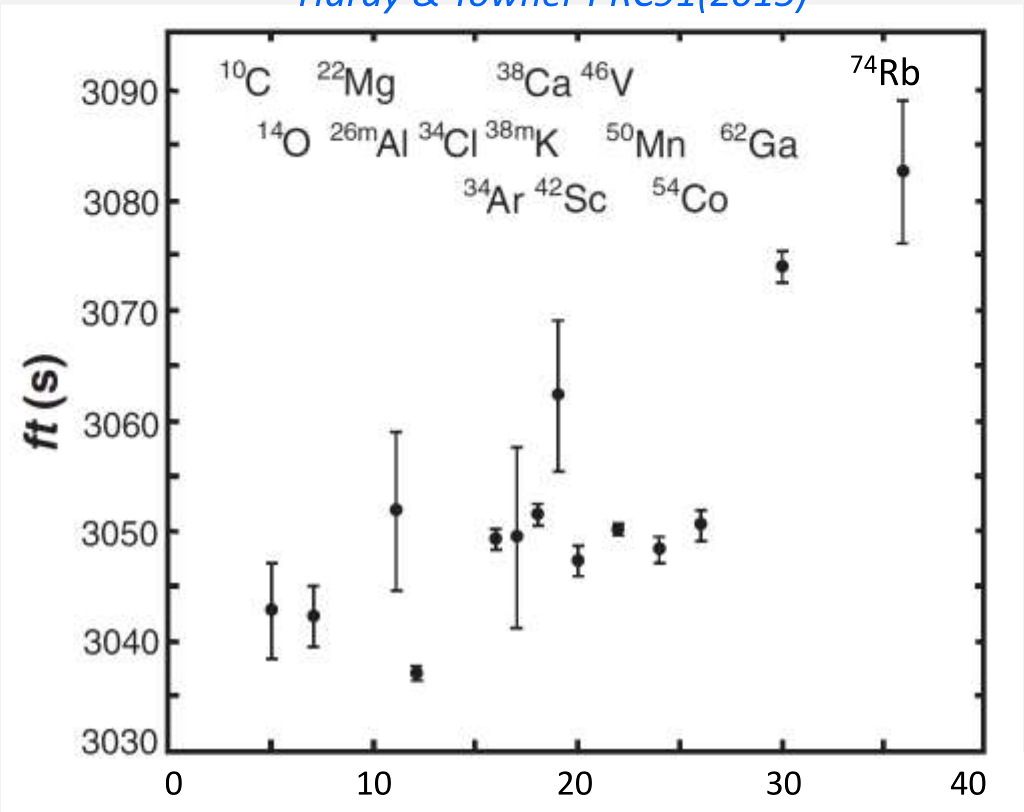
Last section: CVC, V_{ud} & CKM

"ft" value in pure F decays: status

Many reviews for ~ 30 years

JC Hardy & IS Towner …

Hardy & Towner PRC91(2015)



- 14 best candidates
- 8 ft values determined @ 10^{-4} precision level
- ~ *220* measurements included

"ft" values are not constant !! Vector Current not Conserved ??

Illustration

Last section: CVC, V_{ud} & CKM

"ft" value in pure F decays: status

At high precision (*10^{-3} – 10^{-4}*), theoretical corrections are needed !

$$Ft = ft_{1/2}(1+\underbrace{\delta_R)(1+\delta_{NS}-\delta_C}_{\text{Transition dependent}}) = \frac{4.794\,10^{-5}}{2C_V^2|M_F|^2\underbrace{(1+\Delta_R)}_{\text{Interaction dependent}}}$$

- Radiative corrections (virtual emission, Bremsstrahlung):
  δ_R : depends on global nucleus characteristics (Z, Q_β)
  δ_{NS} : depends on nuclear structure details
  Δ_R : common to all decays

- Isospin Symmetry Breaking (ISB) correction:
  δ_C : due to "Coulomb" and other charge dependent forces

Computed using different models and "validated" on independent parameters (R, M…)

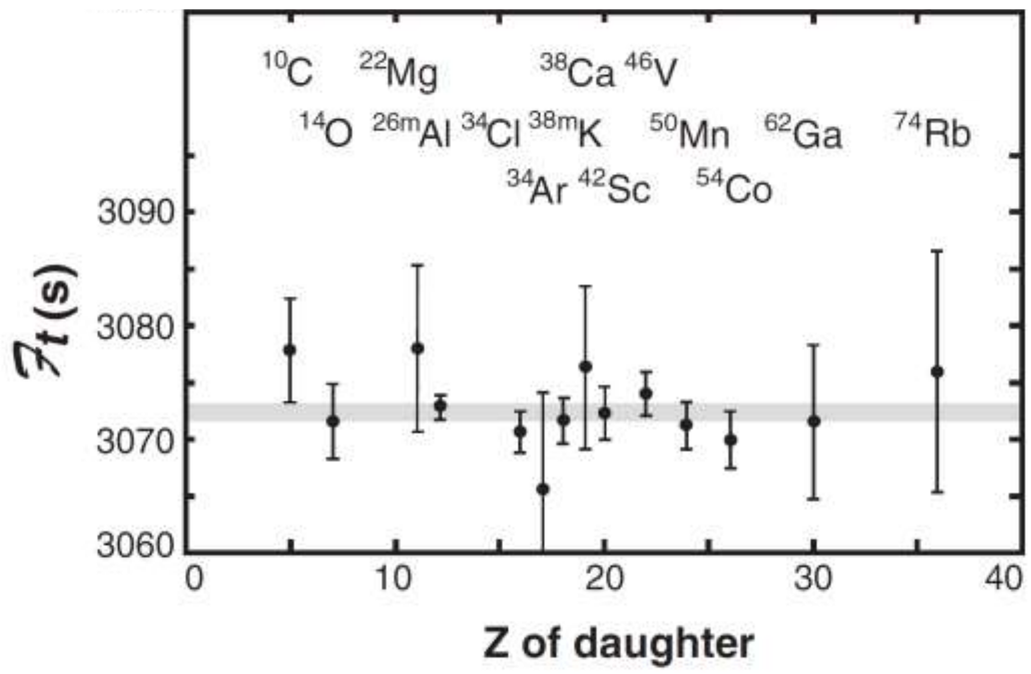
Error bars are even estimated

→ Total correction effect less than 1%

Last section: CVC, V_{ud} & CKM

"Ft" value in pure F decays: status

Hardy & Towner PRC91(2015)



$\mathcal{F}t$ constant at ~10^{-4}

→ $\boxed{\mathcal{F}t = 3072.27(72)\ \mathrm{s}}$

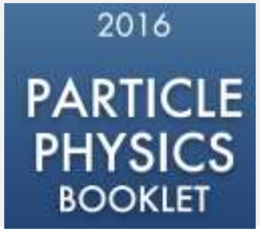
CVC hypothesis verified at ~10^{-4}

⇨ $\boxed{\sqrt{2}\, C_V = 8.7303\ 10^{-5}\ \mathrm{MeV\ fm^3}}$ → $\boxed{V_{ud} = \sqrt{2}\, C_V / G_F = 0.97417\ (21)}$

⇨ $$\boxed{|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.999\,78 \pm 0.000\,55}$$

K-decay : 0.22534 (93) B Meson-decay : 0.00393 (35)

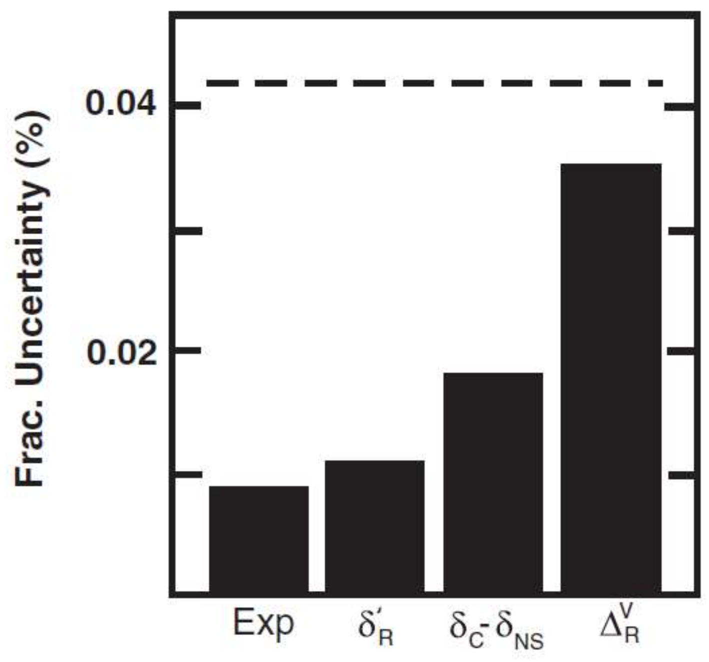
value in



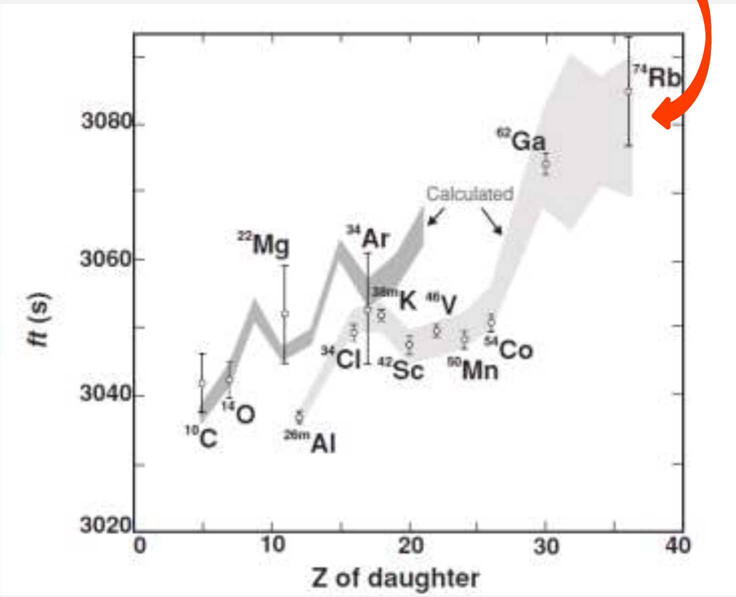
Last section: CVC, V_{ud} & CKM

"Ft" value in pure F decays: status & perspectives

Hardy & Towner PRC91(2015)



Example: measurements at high Z where corrections are larger

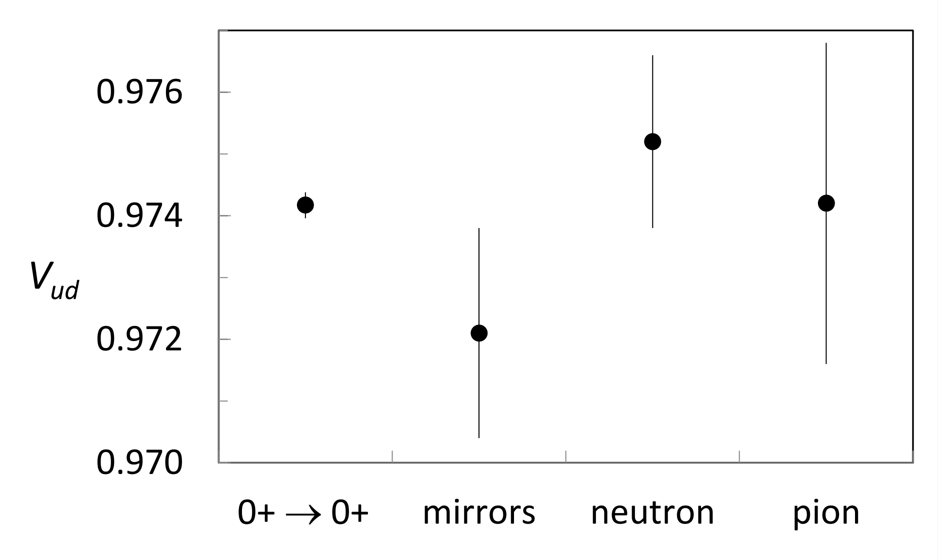


Bands → computed from $\overline{\mathcal{F}t}/[(1+\delta'_R)(1-\delta_C+\delta_{NS})]$.

Uncertainty dominated by theoretical corrections ! ⇒ *Crucial to perform measurements to improve them !*

Last section: CVC, V_{ud} & CKM

V_{ud}, CKM: other sources



Other sources	V_{ud}
Neutron	0.9752 (14)
Pion	0.9728 (30)
Mirror	0.9719 (17)

$$\pi^+ \rightarrow \pi^0 + e^+ + \nu_e$$

very rare decay

$$n \rightarrow p + e^- + \bar{\nu}_e$$

Mirror decays

$$^{A}_{Z_i}X_{N_i} \rightarrow {}^{A}_{Z_f=N_i}Y_{N_f=Z_i} + e^+ + \nu_e$$

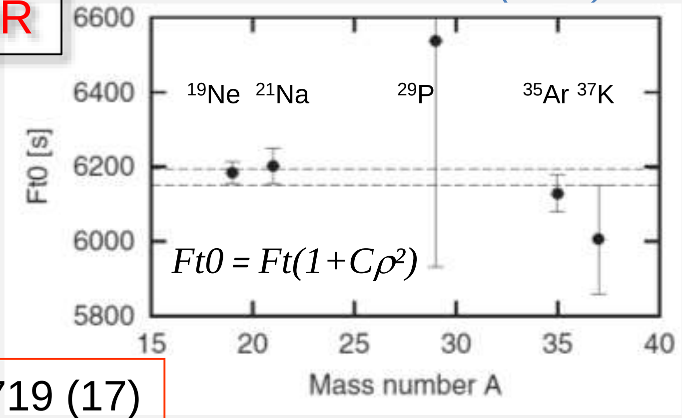
$$\boxed{(ft)_{mirror} = \frac{4.794\,10^{-5}}{2(C_V^2|M_F|^2 + C_A^2|M_{GT}|^2)} = \frac{4.794\,10^{-5}}{2C_V^2|M_F|^2(1+\rho^2)}\,s}$$

Last section: CVC, V_{ud} & CKM

V_{ud}, CKM: nuclear mirror vs $0^+ \rightarrow 0^+$ decays

MIRROR

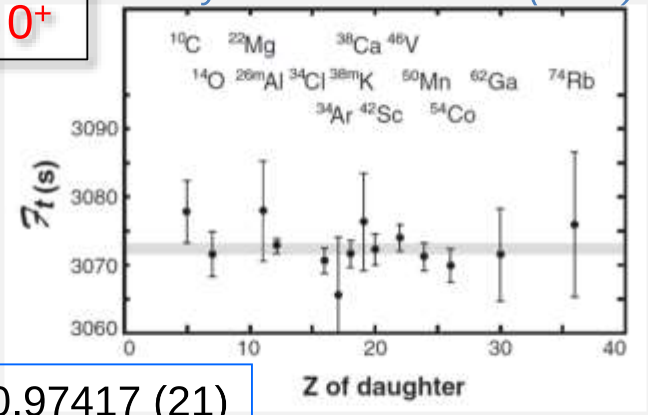
Naviliat et al. PRL102 (2009)



$V_{ud} = 0.9719\ (17)$

$0^+ \rightarrow 0^+$

Hardy & Towner PRC91 (2015)



$V_{ud} = 0.97417\ (21)$

$$(Ft)^{PF} = f_V t_{1/2}(1+\delta_R)(1+\delta_{NS} - \delta_C) = \frac{K}{V_{ud}^2(1+\Delta_R)}$$

($T_{1/2}$, BR, M) measurements

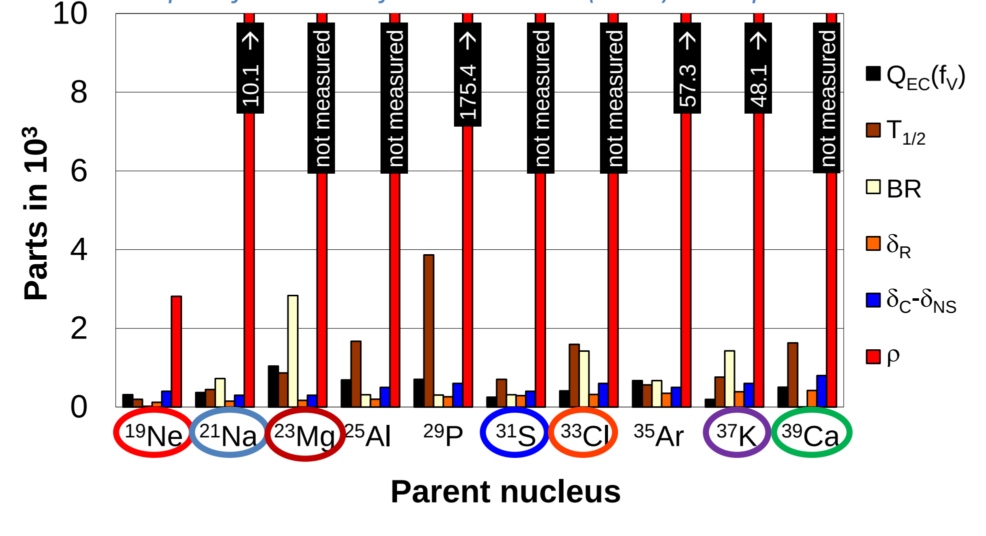
$$(Ft)^{mirror} = f_V t_{1/2}(1+\delta_R)(1+\delta_{NS} - \delta_C) = \frac{2K}{V_{ud}^2(1+\Delta_R)(1+\frac{f_A}{f_V}\rho^2)}$$

($T_{1/2}$, BR, M, ρ) measurements

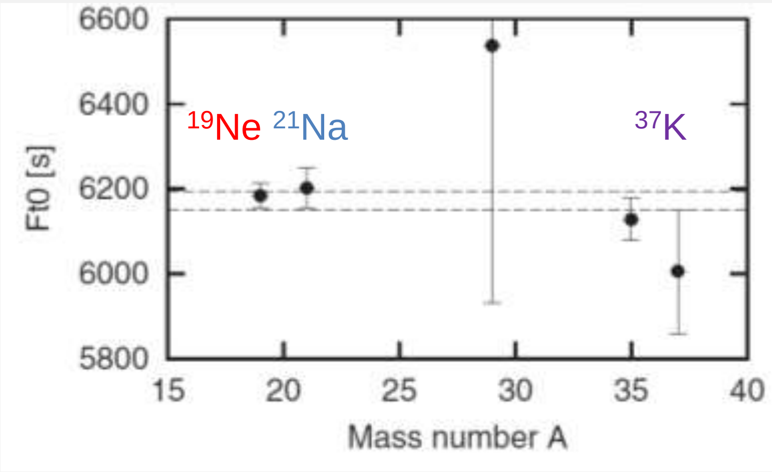
Last section: CVC, V_{ud} & CKM

V_{ud}, CKM: nuclear mirror decays

adapted from Severijns et al. PRC78 (2008) and updated



Naviliat et al. PRL102 (2009)



- ^{19}Ne — $T_{1/2}$: *Broussard et al. PRL112 (2014)*
- ^{21}Na — M: *Mukherjee et al. EPJA35 (2008)*; $T_{1/2}$: *Grinyer et al. PRC91 (2015)*
- ^{23}Mg — M: *Saastamoinen et al. PRC80 (2009)*
- ^{31}S — M: *Kankainen et al. PRC82 (2010)*; $T_{1/2}$: *Bacquias et al. EPJA48 (2012)*
- ^{33}Cl — $T_{1/2}$: *Grinyer et al. PRC92 (2015)*
- ^{37}K — $T_{1/2}$: *Shidling et al. PRC90 (2014)*
- ^{39}Ca — $T_{1/2}$: *Blank et al. EPJA44 (2010)*

The scientific community involved in this field... BUT

V_{ud} (2009) = 0.9719 (17)

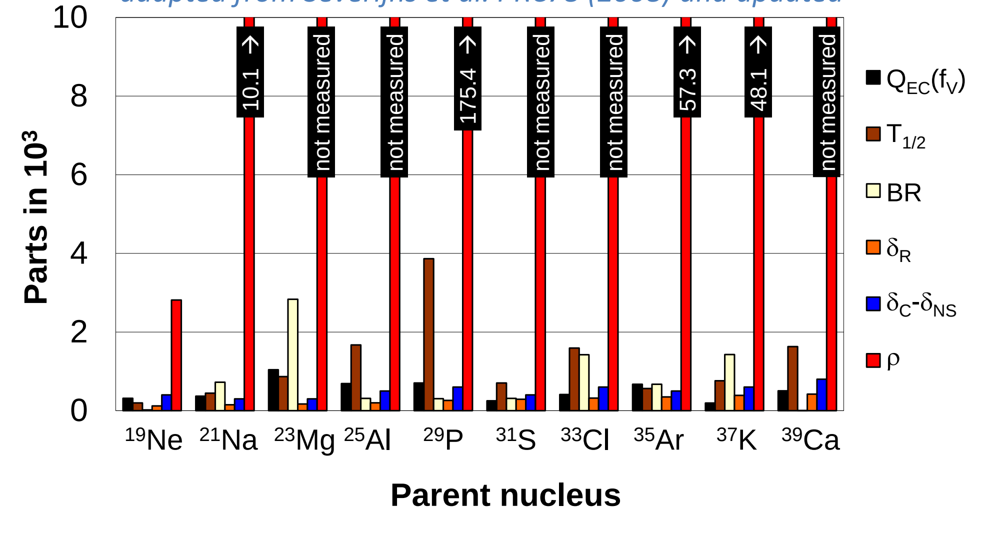
↓

V_{ud} (2017) = 0.9721 (17) !!

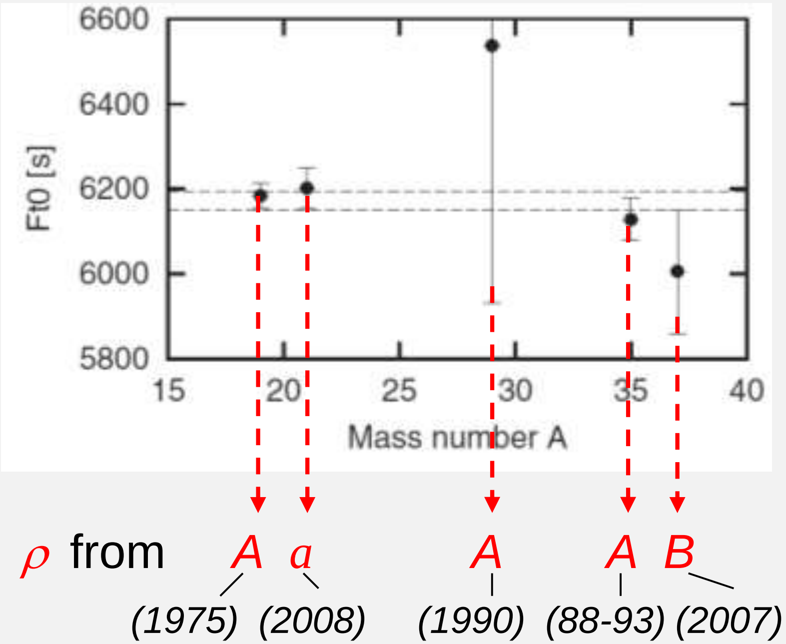
Last section: CVC, V_{ud} & CKM

V_{ud}, CKM: nuclear mirror decays

adapted from Severijns et al. PRC78 (2008) and updated



Naviliat et al. PRL102 (2009)



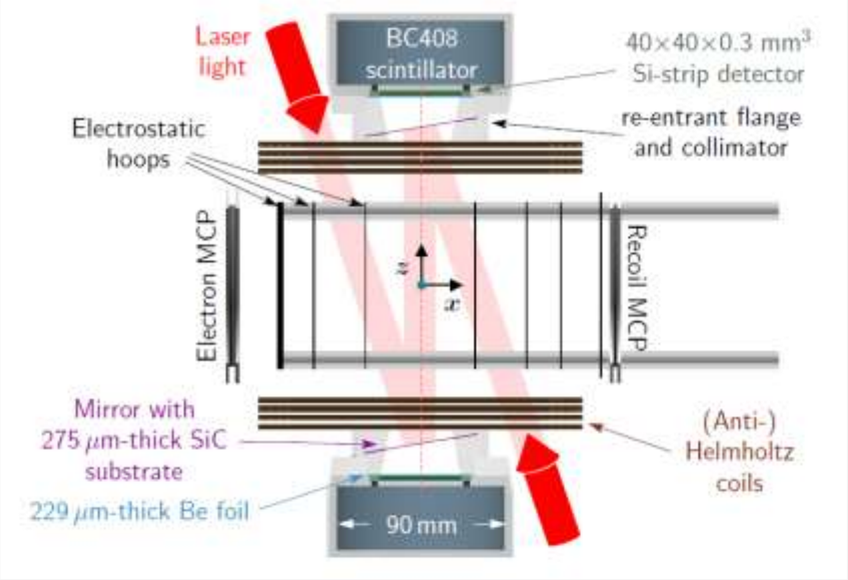
ρ = *GT/F :* the least or even not known quantity !
determined from a correlation measurement

⇒ *For V_{ud} determination, ρ improvements are necessary ...*

Last section: CVC, V_{ud} & CKM

V_{ud}, CKM: nuclear mirror decays

Recent result: Measurement of A_β in ^{37}K (TRIUMF) Fenker et al. arXiv:1706.00414v1 2017



- Source confined in MoT of TRINAT
- Detection of β in Z direction with nucleus polarization in ±Z
- Degree of P measured by laser probe & detection of photo-ions $\rightarrow P_{\sigma-} = 99.13(8)\%$ $\quad P_{\sigma+} = 99.12(9)\ \%$

⇨ $A_\beta = -0.5707(18)$ 0.3% relative precision

⇨ V_{ud} (2009) = 0.9719 (17) → V_{ud} (2017) = 0.9728 (14) !!

one single shot → significant improvement of V_{ud} & ^{37}K is not the most sensitive case …

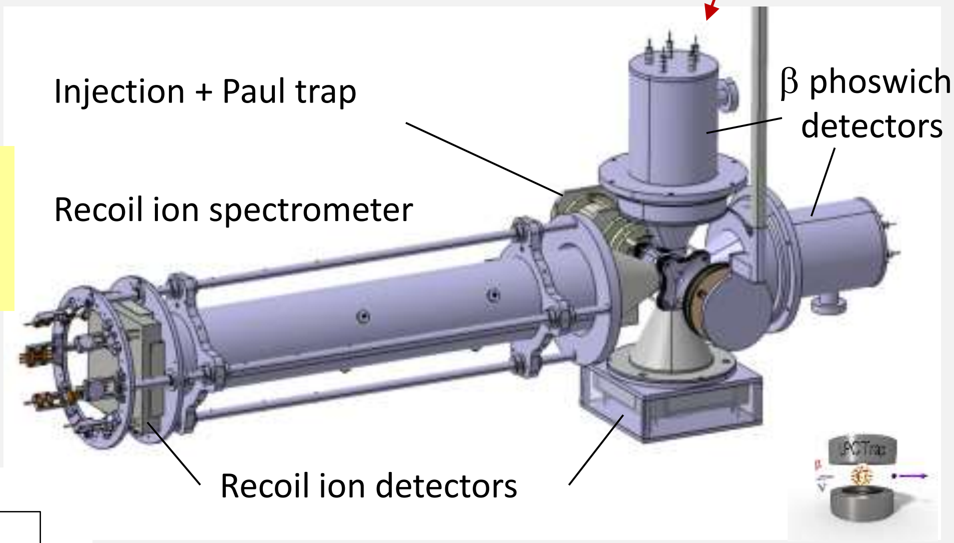
Last section: CVC, V_{ud} & CKM

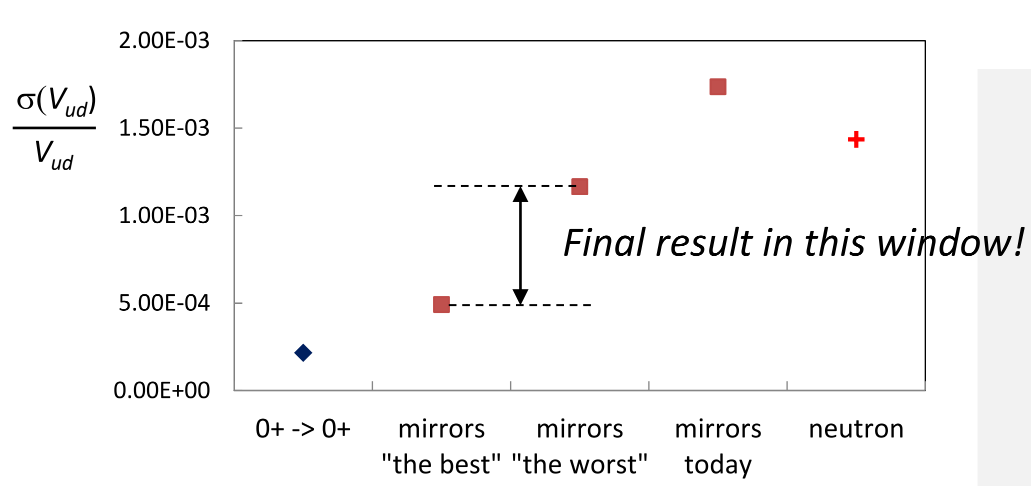
V_{ud}, CKM: nuclear mirror decays

Perspectives @ GANIL: Measurement of a in several mirror decays using LPCTrap2

Ion	$T_{1/2}$ (s)	Expected rate (pps)
^{21}Na	22.49	6.5E+08
^{23}Mg	11.32	2.1E+08
^{33}Cl	2.51	3.4E+07
^{37}K	1.22	7.4E+08

Production > 10^7 pps





In any case, a significant improvement on V_{ud} is reachable

Last section: CVC, V_{ud} & CKM

V_{ud}, CKM: nuclear mirror decays

Alternative interest: Test of models used to compute theoretical corrections (δ_C)

Konieczka et al. PRC93 (2016)

Severijns et al. PRC78 (2008)

	A	$\delta_C^{(NCCI)}$	$\delta_C^{(NSM)}$	
	17	0.166(17)	0.585(27)	
$^{19}Ne \rightarrow {}^{19}F$	19	0.339(34)	0.415(39)	
	21	0.300(30)	0.348(27)	
$^{23}Mg \rightarrow {}^{23}Na$	23	0.316(32)	0.293(22)	0.153
	25	0.413(41)	0.461(47)	
$^{27}Si \rightarrow {}^{27}Al$	27	0.439(44)	0.312(34)	0.266
	29	0.520(52)	0.976(53)	
	31	0.585(59)	0.715(36)	
	33	0.705(71)	0.865(59)	
$^{35}Ar \rightarrow {}^{35}Cl$	35	0.366(37)	0.493(46)	
$^{37}K \rightarrow {}^{37}Ar$	37	0.907(91)	0.734(61)	
	39	0.318(32)	0.855(81)	
	41	0.426(43)	0.821(63)	
	43	0.690(69)	0.50(10)	
	45	0.589(59)	0.87(12)	
	47	0.673(67)		
	49	0.646(65)		
	51	0.714(71)		
	53	0.898(90)		
	55	0.620(62)		

Smirnova et al. 2016
CENBG

- Different models give different results !
- Such study is also useful for $0^+ \rightarrow 0^+$ decays
- Recent interest in the community thanks to mirror decays study…

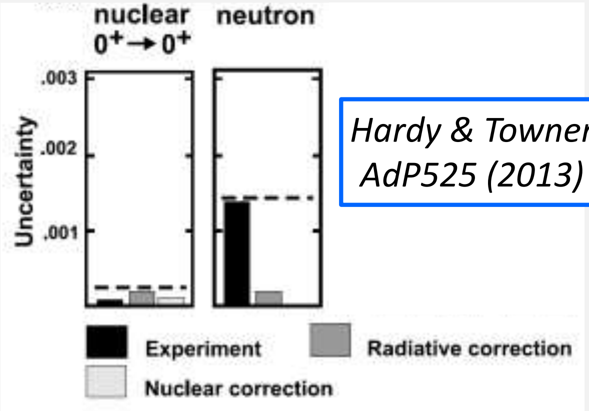
Last section: CVC, V_{ud} & CKM

V_{ud}, CKM: the neutron case $n \to p + e^- + \bar{\nu}_e$

= the basic mirror decay: *Why is it considered alone ?*

→ Very interesting feature: no nuclear correction !! $\delta_C = \delta_{NS} = 0$

studied during decades…



Hardy & Towner AdP525 (2013)

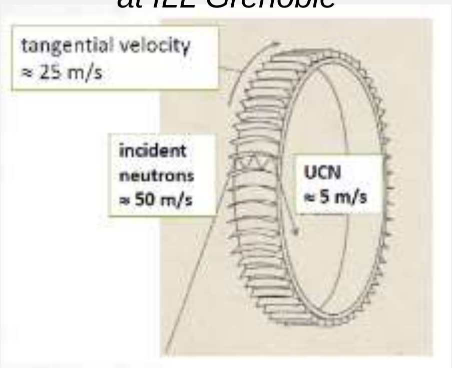
⇒ *Why results remain limited ?*

1. neutron manipulation is difficult

- free neutrons are produced by fission or spallation
- the slowest are the best

slowest distributions are favored by successive *moderators*

Scheme of a wheel used at ILL Grenoble



⇒ Production of CN, VCN and UCN

Cold Neutrons, Very Cold Neutrons and Ultra Cold Neutrons

Last section: CVC, V_{ud} & CKM

V_{ud}, CKM: the neutron case $\quad n \rightarrow p + e^{-} + \bar{\nu}_e$

Why results remain limited ?

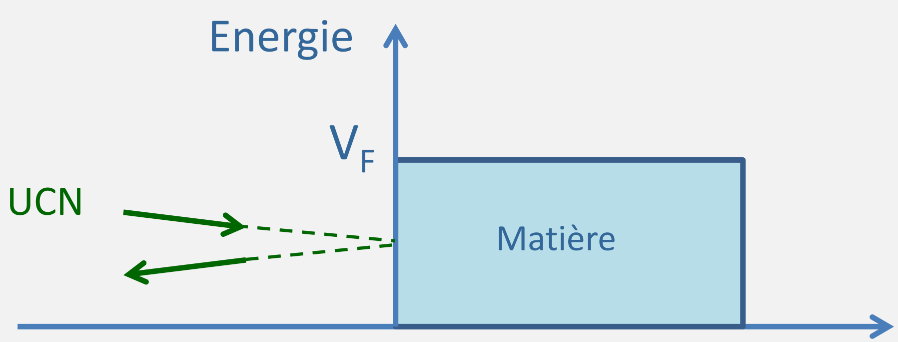
1. neutron manipulation is difficult

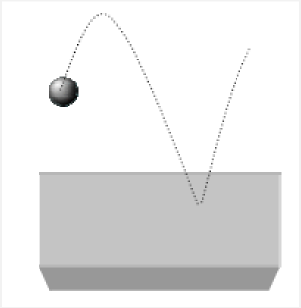
VCN, UCN $\rightarrow$ typical velocities: 10 m/s

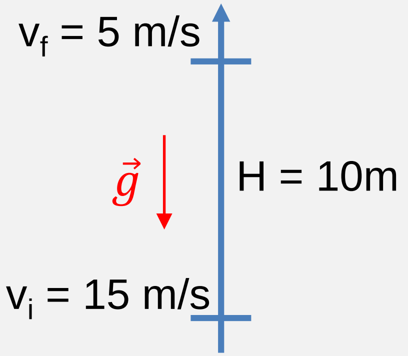
- n scatter on specific material like light
  $\rightarrow$ they can be guided and trapped !!
- kinetic energy can be modified using gravitational force…

Reference on the web:
Kirch et al. Nucl. Phys. News 20 (2010)
Recent review: Young et al., JPG 41 (2014)

$E_n < V_F$: Fermi potential







Illustration

Last section: CVC, V_{ud} & CKM

V_{ud}, CKM: the neutron case $\quad n \rightarrow p + e^{-} + \overline{\nu}_e$

Why results remain limited ?

1. neutron manipulation is difficult
2. discrepancy on $T_{1/2}$ results depending on method used

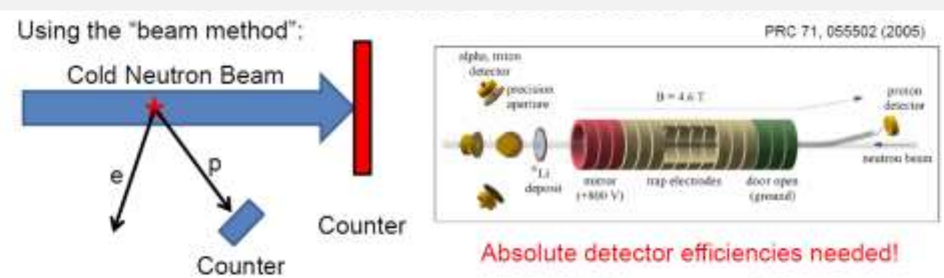
In flight: "beam" method

In trap: "bottle" method

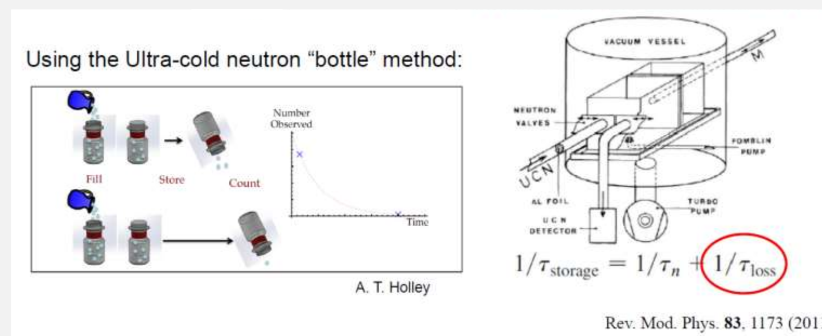
Decay rate: $dN/dt = -N/\tau_n$ $\longrightarrow$ "solution": $N(t) = N\exp(-t/\tau_{eff})$

- source badly defined
- requires n & p countings
- important parameters: Efficiencies, losses …

- source well defined
- requires only n countings
- but $\tau_{eff} = \tau_{storage} \neq \tau_n$ …!



Pictures from A. Saunders, Los Alamos Nat. Lab, LA-UR-15-24679



Last section: CVC, V_{ud} & CKM

V_{ud}, CKM: the neutron case $\quad n \rightarrow p + e^- + \bar{\nu}_e$

In trap: "bottle" method $\quad$ N(t) = N exp(-t/τ_{eff}) $\quad$ with $\tau_{eff} = \tau_{storage} \neq \tau_n$ …!

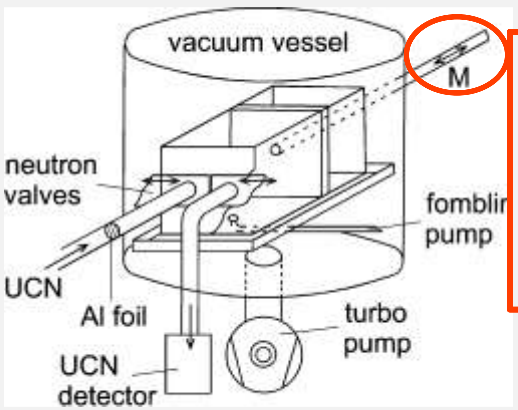
$$1/\tau_{\text{storage}} = 1/\tau_n + 1/\tau_{\text{loss}}$$

due to absorption in walls, neutron heating and many (still) unknown other reasons…

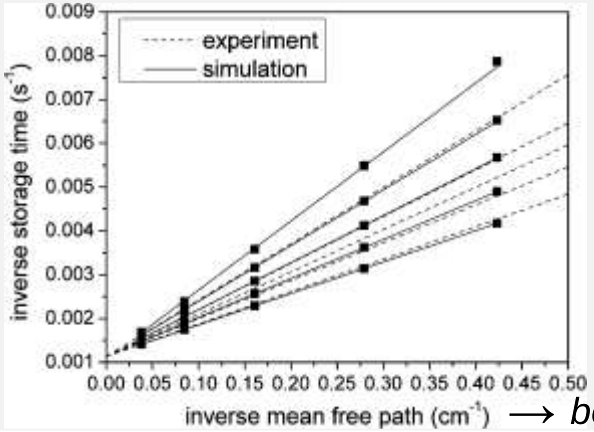
τ_{loss} depends on trap Volume/Surface ratio

$\rightarrow$ for ideal infinite trap: V/S $\rightarrow \infty$ $\Rightarrow$ losses $\rightarrow$ 0 !

$\Rightarrow$ Measurements are performed with variable trap volumes…



this device enables to modify the trap volume



$\longrightarrow$ *between two collisions on walls*

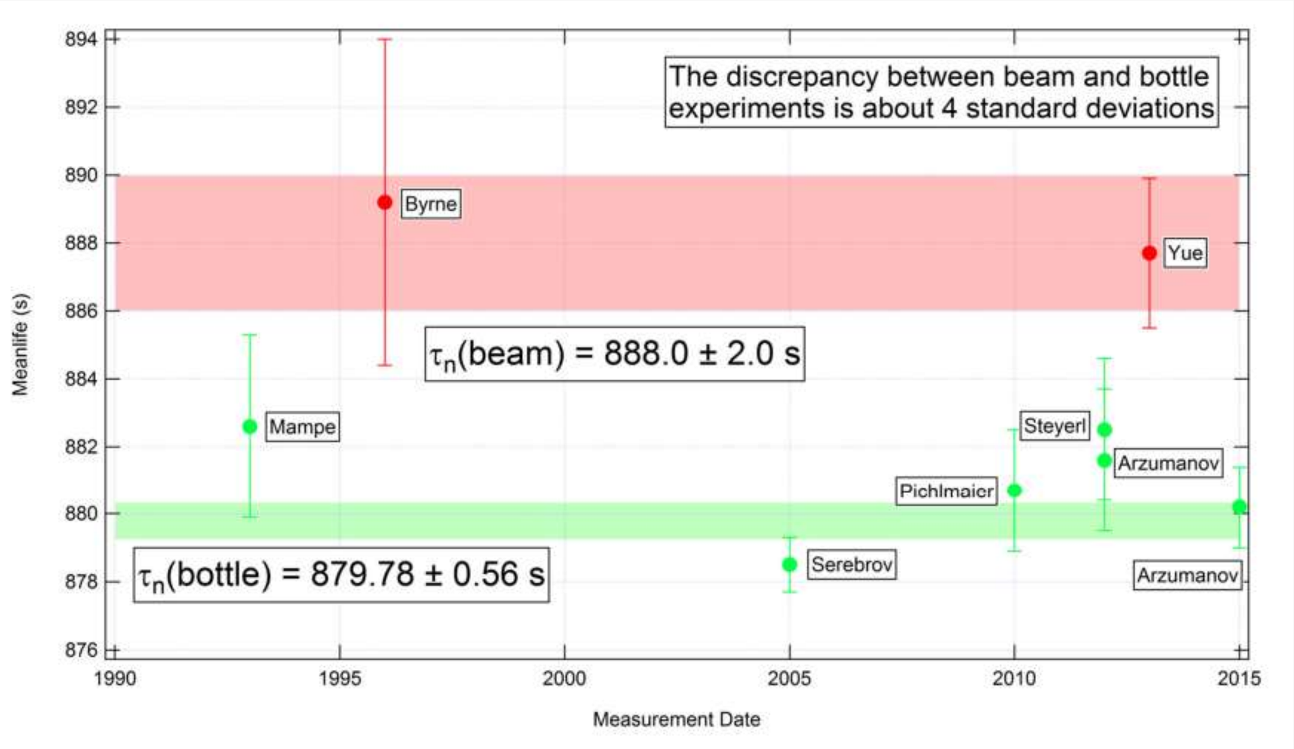
… and τ_n is deduced from the extrapolation of results to a virtual infinite trap

Last section: CVC, V_{ud} & CKM

V_{ud}, CKM: the neutron case $n \to p + e^- + \bar{\nu}_e$

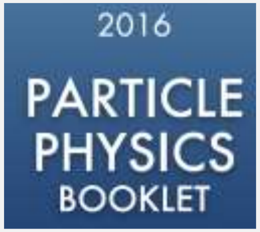
Why results remain limited ?

1. neutron manipulation is difficult
2. discrepancy on $T_{1/2}$ results depending on method used



between 2010 & 2012, the PDG value shifted by 5.6 s which corresponds to ~ 7 old standard deviations…

2016 PDG value:
880.2 ± 1.0 s



This is the END…?