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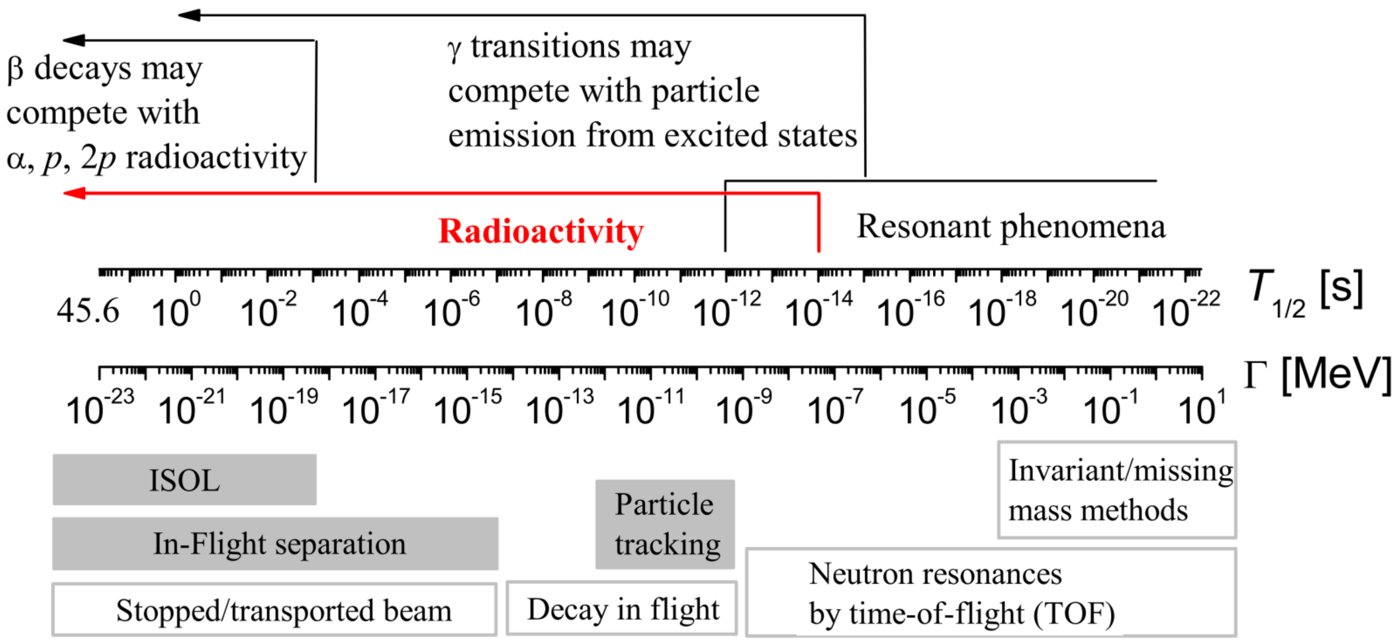
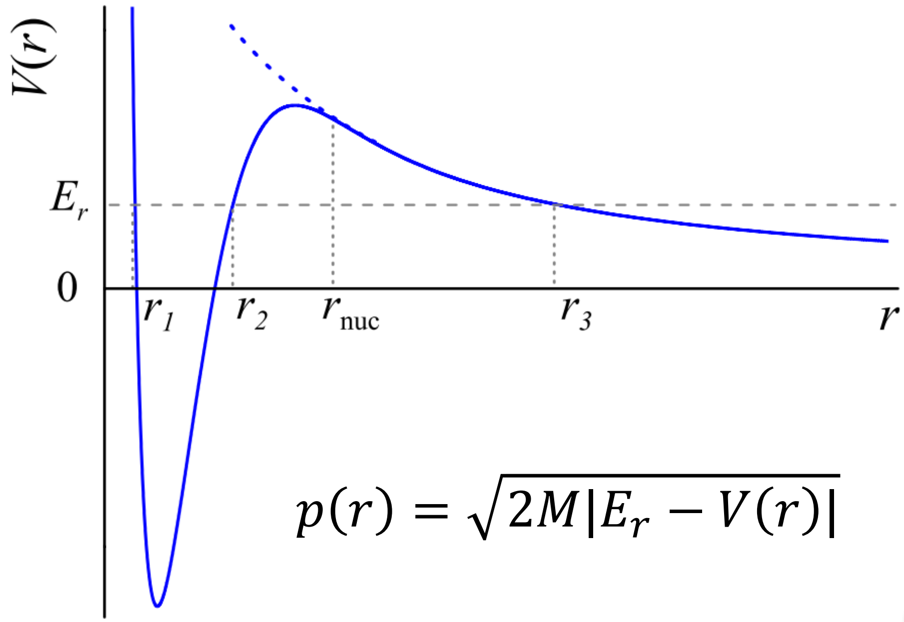
Theoretical basics and modern status of radioactivity studies

Lecture 3: Some theory

Long lifetimes



Particle decay. Gamow theory



$$p(r) = \sqrt{2M|E_r - V(r)|}$$

Quasiclassical WF

$$\Psi(r) \sim e^{iS(r)}$$

$$S(r) = \int_{r0}^{r} p(r')\, dr'$$

Preexponent **Exponent**

$$\Gamma = S_\alpha\ \omega\ exp\left[-2\int_{r2}^{r3} p(r)\, dr\right]$$

Dimensionless structure factor

Dimension energy "limiting width"

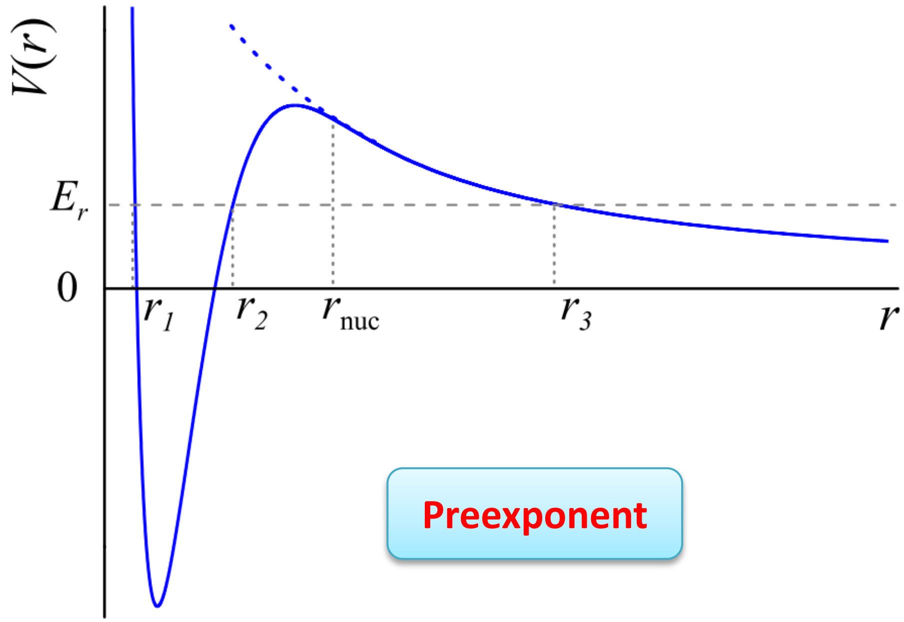
$$\int_0^{r3} p(r)\, dr = 2\eta \int_0^1 dx \sqrt{1 - 1/x} = \eta\pi$$

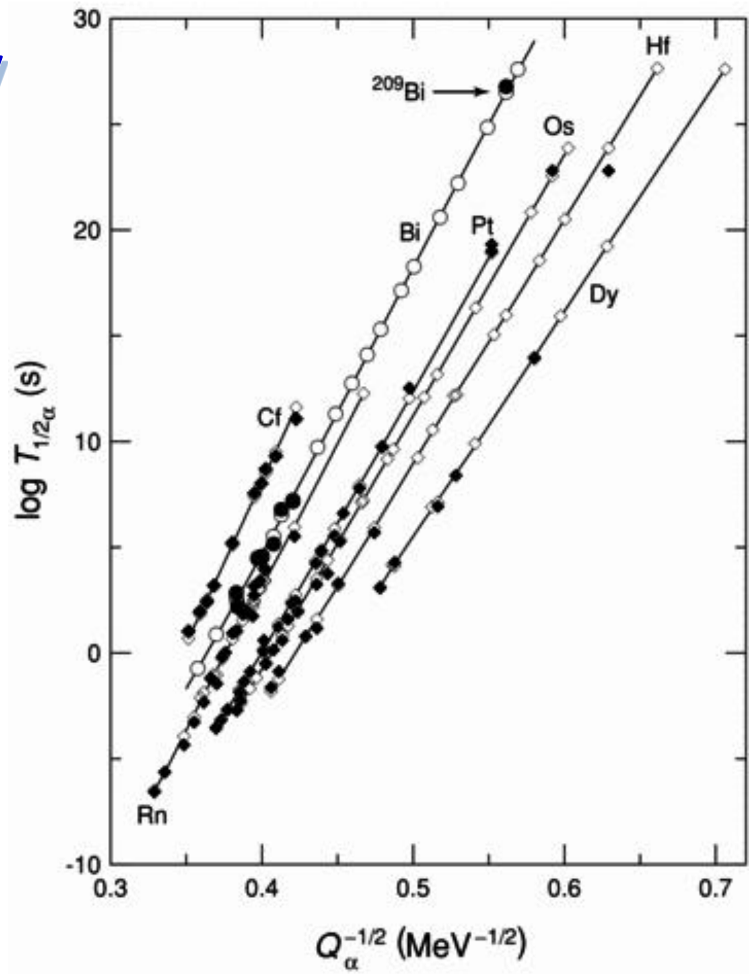
Sommerfeld parameter

$$\eta = Z_1 Z_2 \alpha \sqrt{\frac{M}{2E}} = \frac{Z_1 Z_2 \alpha}{v}$$

$$\Gamma = S_\alpha\ \omega\ exp[-2\pi\eta]$$

Particle decay. Gamow theory





Preexponent

$$T = \int_{r1}^{r2} dt = \int_{r1}^{r2} \frac{1}{v} dr = \int_{r1}^{r2} \frac{M}{p(r)} dr$$

$$\omega = \left[2 \int_{r1}^{r2} \frac{M}{p(r)} dr\right]^{-1}$$

$$\Gamma = S_\alpha \, \omega \, exp[-2\pi\eta]$$

$$\ln T_{1/2} \sim a + b\, E^{-1/2}$$

Strange, but for majority of situations this is precise within few percent

Geiger-Nuttal law

Beta decay. Gamow-Teller

$$H_{fi} = (2\pi)\delta\left(E_f - E_i\right) \int \prod_{k=1}^{N} d^3\mathbf{r}_k \Psi_f^*\left(\mathbf{r}_1 \ldots \mathbf{r}_N\right) \widehat{H}_\beta\left(\mathbf{r}_1 \ldots \mathbf{r}_N\right) \Psi_i\left(\mathbf{r}_1 \ldots \mathbf{r}_N\right)$$

$$\widehat{H}_\beta\left(\mathbf{r}_1, \ldots, \mathbf{r}_N\right) = -\frac{G_V}{\sqrt{2}} \sum_{k=1}^{N} \left\{L_0^*\left(\mathbf{r}_k\right) + \lambda\left(\mathbf{L}^*\left(\mathbf{r}_k\right), \boldsymbol{\sigma}_k\right)\right\} \tau_k^-$$

$$L_\mu^*\left(\mathbf{r}_k\right) = \overline{\Psi}_e\left(\mathbf{r}_k\right)\gamma_\mu\left(1+\gamma_5\right)\Psi_\nu\left(\mathbf{r}_k\right) = \frac{\overline{u}_e\gamma_\mu\left(1+\gamma_5\right)\overline{u}_\nu}{\sqrt{2E_e 2E_\nu}\,V} \exp\left(i\mathbf{q}\mathbf{r}_k\right) \qquad \mathbf{q} = \mathbf{p}_e - \mathbf{p}_\nu$$

$$\sum_f \overline{\left|H_{fi}\right|^2} = \frac{1}{2J+1}\frac{G_V^2\lambda^2}{2}(2\pi)\,\delta\left(E_f - E_i\right)\Delta t\,\frac{6}{V^2}\left(1 - \frac{\mathbf{p}_e\mathbf{p}_\nu}{3\,E_e E_\nu}\right)\frac{2J'+1}{3}\left|\sum_{k=1}^{N}\left\langle J'\left\|\sigma_k\right\|J\right\rangle\left\langle\tau_k^-\right\rangle\right|^2$$

$$dW = \frac{1}{2J+1}\frac{G_V^2\lambda^2}{2}(2\pi)\,\delta(E_e + E_\nu - Q)\,\frac{6}{V^2}\left(1 - \frac{\mathbf{p}_e\mathbf{p}_\nu}{3\,E_e E_\nu}\right)\times$$

$$\times\frac{2J'+1}{3}\left|\sum_{k=1}^{N}\left\langle J'\left\|\sigma_k\right\|J\right\rangle\left\langle\tau_k^-\right\rangle\right|^2 \frac{V^2\mathbf{dp}_e\mathbf{dp}_\nu}{(2\pi)^6}$$

$$dW = \frac{G_V^2\lambda^2}{2\pi^3}\cdot\left(\frac{2J'+1}{2J+1}\left|\sum_{k=1}^{N}\left\langle J'\left\|\sigma_k\right\|J\right\rangle\left\langle\tau_k^-\right\rangle\right|^2\right)\cdot\left(\delta\left(E_e + E_\nu - Q\right)p_e^2 dp_e p_\nu^2 dp_\nu\right)$$

Beta decay. Gamow-Teller

$$W = \frac{\ln 2}{t_{1/2}} = \frac{G_V^2 \lambda^2 m_e^5}{2\pi^3} \cdot B_{GT} \cdot f\left(A, Z, Q\right)$$

$\lambda = G_A / G_V = -1.268 \pm 0.002 \; (\lambda^2 = 1.608 \pm 0.004)$

$Q = E - m_e$ is the energy release in the reaction

$$f t_{1/2} = 2 \cdot \frac{\pi^3 \ln 2}{G_V^2 m_e^5} \cdot \frac{1}{\lambda^2 B_{GT}} = \frac{2 f t(0^+ \to 0^+)}{\lambda^2 B_{GT}}$$

$$f t(0^+ \to 0^+) = \frac{\pi^3 \ln 2}{G_V^2 m_e^5} = 3072.4 \pm 1.6 \text{ sec}$$

$$B_{GT} = \frac{2J' + 1}{2J + 1} \left| \sum_{k=1}^{N} \left\langle J' \left\| \sigma_k \right\| J \right\rangle \left\langle T' T_3' \left| \tau_k^- \right| T T_3 \right\rangle \right|^2$$

$$f\left(A, Z, Q\right) = \int\limits_1^{Q/m_e} \left(Q/m_e - \varepsilon\right)^2 \varepsilon \sqrt{\varepsilon^2 - 1}\, F\left(A, Z, Q\right) d\varepsilon$$

$F\left(A, Z, Q\right)$ is the Fermi function

***f* ~ 1 for *Q* ~ 2.4 *m*$_e$**

for *Q* >> *m*$_e$ -> *f* ~ *Q*^{5}

For *Q* ~ 10 *m*$_e$ lifetime of the tens of milliseconds is expeced

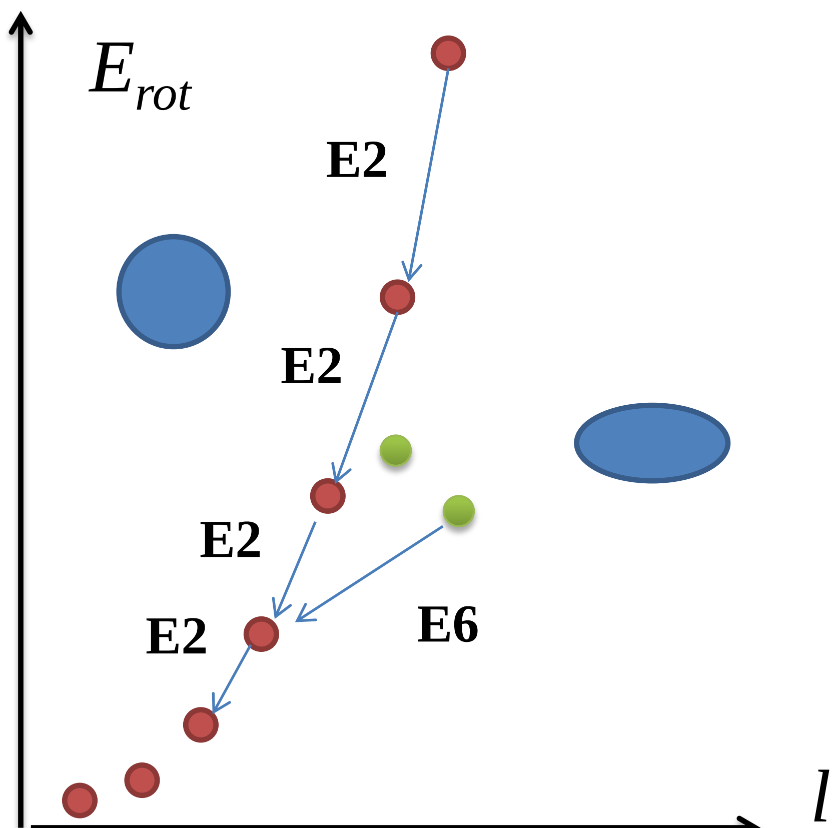
For *Q* ~ 1.1 *m*$_e$ lifetime of the order of years is expected

Electromagnetic decay. Nuclear isomers

"Phase space" isomers

$\Gamma \sim E^3$ for $E1$ transitions

$\Gamma \sim E^5$ for $E2$ transitions



Angular momentum isomers

	E* (MeV)	J^π	$T_{1/2}$	
^{180}Ta	0	1^+	8	hours
^{180m}Ta	0.07	9^-	$>10^{15}$	years

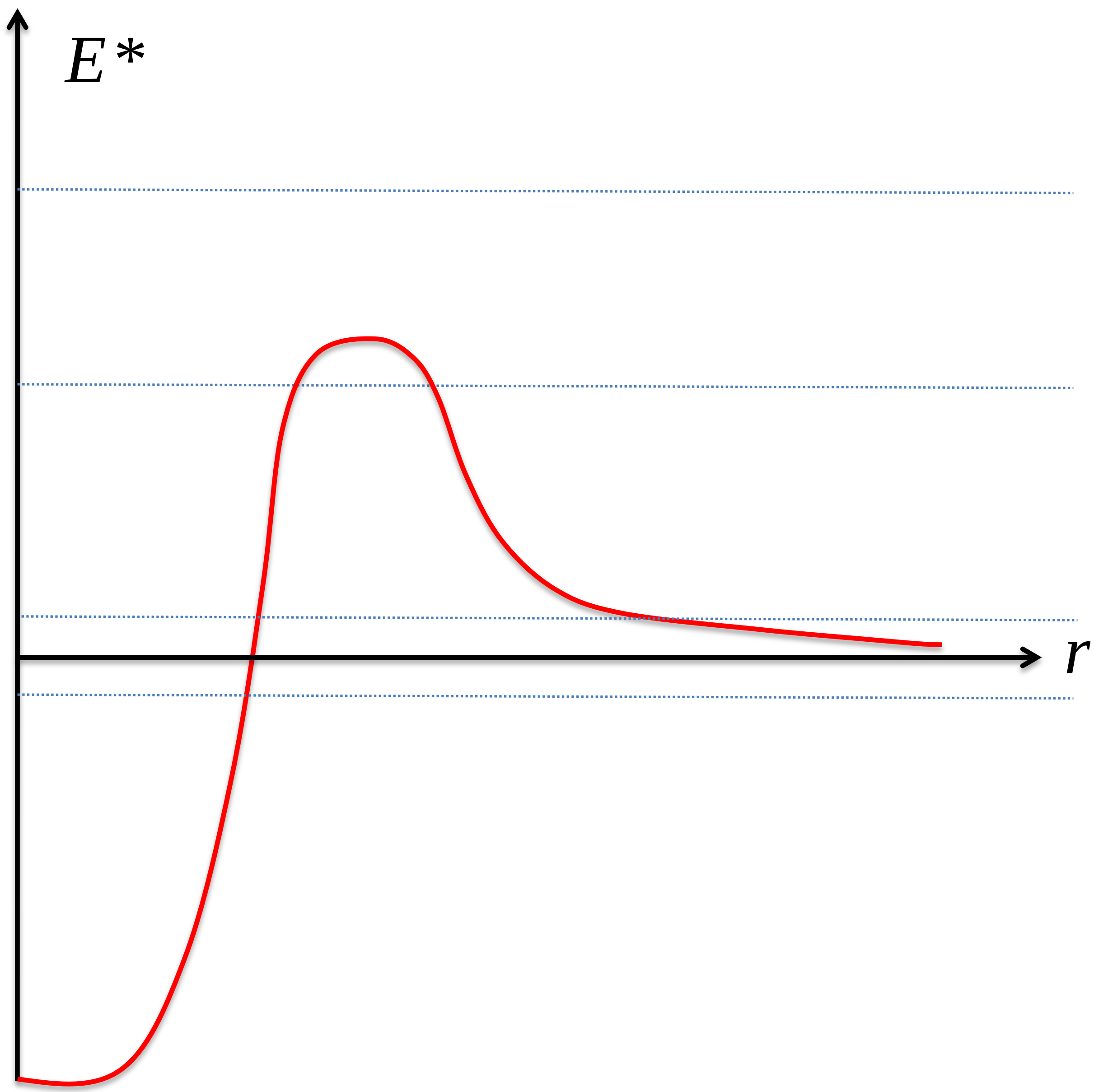
Shape isomers

$$E_{rot} \sim J^2 \sim l(l+1)$$

If the system change shape it is likely that gamma lifetime abruptly increases

Particle decay
Resonance phenomena

Nuclear dynamics vs. excitation energy



"Phase volume" dynamics – only initial state is important

Transition region – resonances are broad

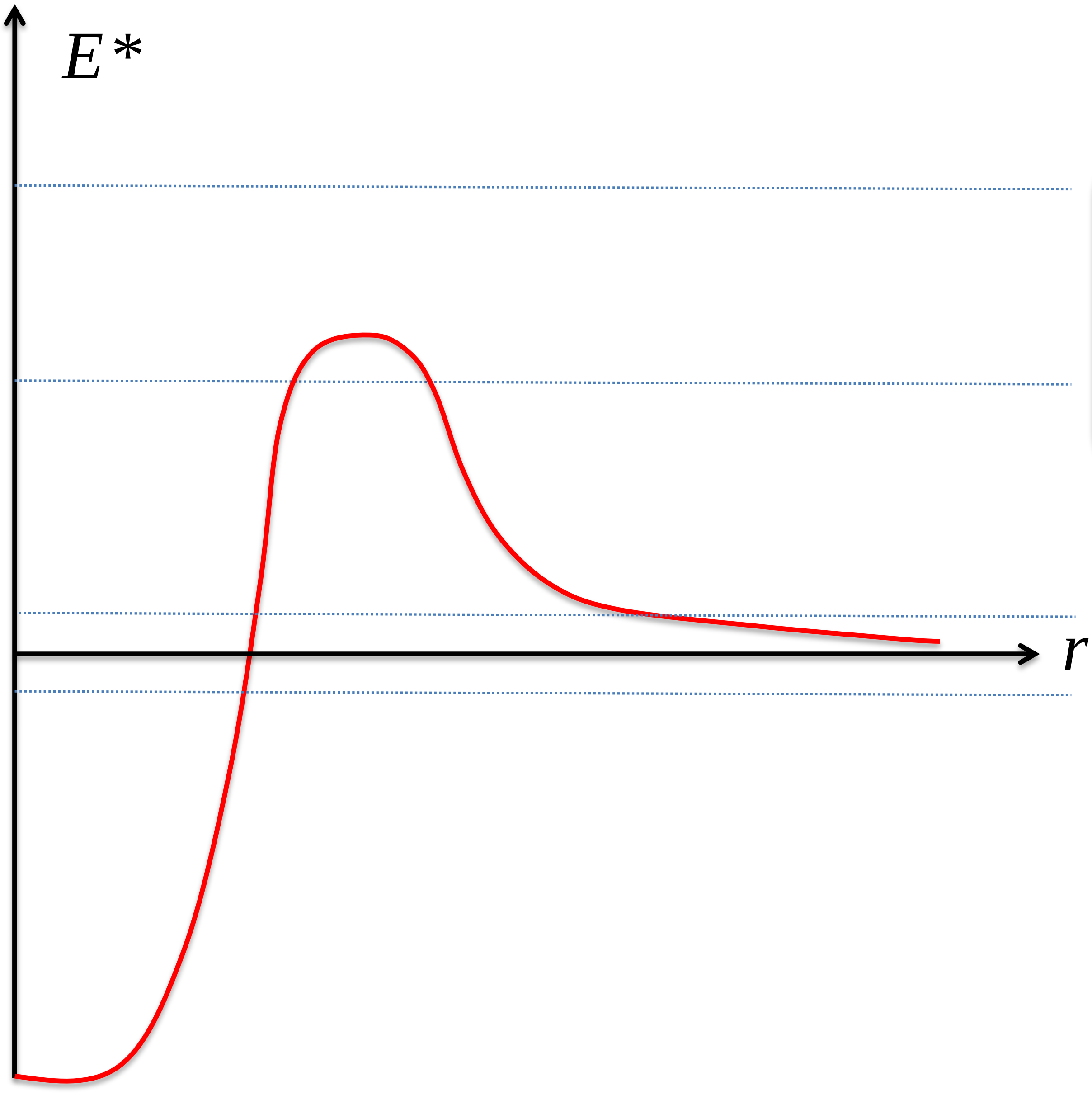
Resonance phenomena

Radioactivity

Haloes

"Normal nuclei"

Nuclear dynamics vs. excitation energy



From formal dynamics point of view there is no clear borderline between resonance phenomena and radioactivity

From formal structure point of view there is no clear borderline between stationary and quasistationary states (radioactivity)

Resonances in elastic scattering

Elastic scattering formulation is not comfortable for radioactivity studies

$$(\bar{H} - E)\Phi_{\mathbf{k}}(\mathbf{r}) = (T + \bar{V}^{\mathrm{nuc}} + V^{\mathrm{coul}} - E)\Phi_{\mathbf{k}}(\mathbf{r}) = 0,$$

$$\Phi_{\mathbf{k}}(\mathbf{r}) = 4\pi \sum\nolimits_l i^l (kr)^{-1} \varphi_l(kr) \sum\nolimits_m Y^*_{lm}(\hat{k}) Y_{lm}(\hat{r}),$$

$$\varphi_l(kr) = \frac{i}{2}[(G_l(kr) - i F_l(kr)) - \bar{S}_l(G_l(kr) + i F_l(kr))],$$

$$\varphi_l(kr) = \exp(i\bar{\delta}_l)[F_l(kr)\cos(\bar{\delta}_l) + G_l(kr)\sin(\bar{\delta}_l)].$$

At resonance energy E_r,

$$S_l(E_r) = e^{2i\delta_l(E_r)} = e^{2i\pi/2} = -1,$$

$$\varphi_l(kr) = i\, G_l(kr)$$

Phase shift $\delta_l(E) = 90^{\mathrm{o}}$

Elastic cross section has peak

$$\sigma = \frac{4\pi}{k^2} \sum_l (2l+1)\ \sin^2(\delta_l)$$

Internal normalization has peak

$$N_l(E,R) = \int_0^R dr |\psi_{E,l}(r)|^2$$

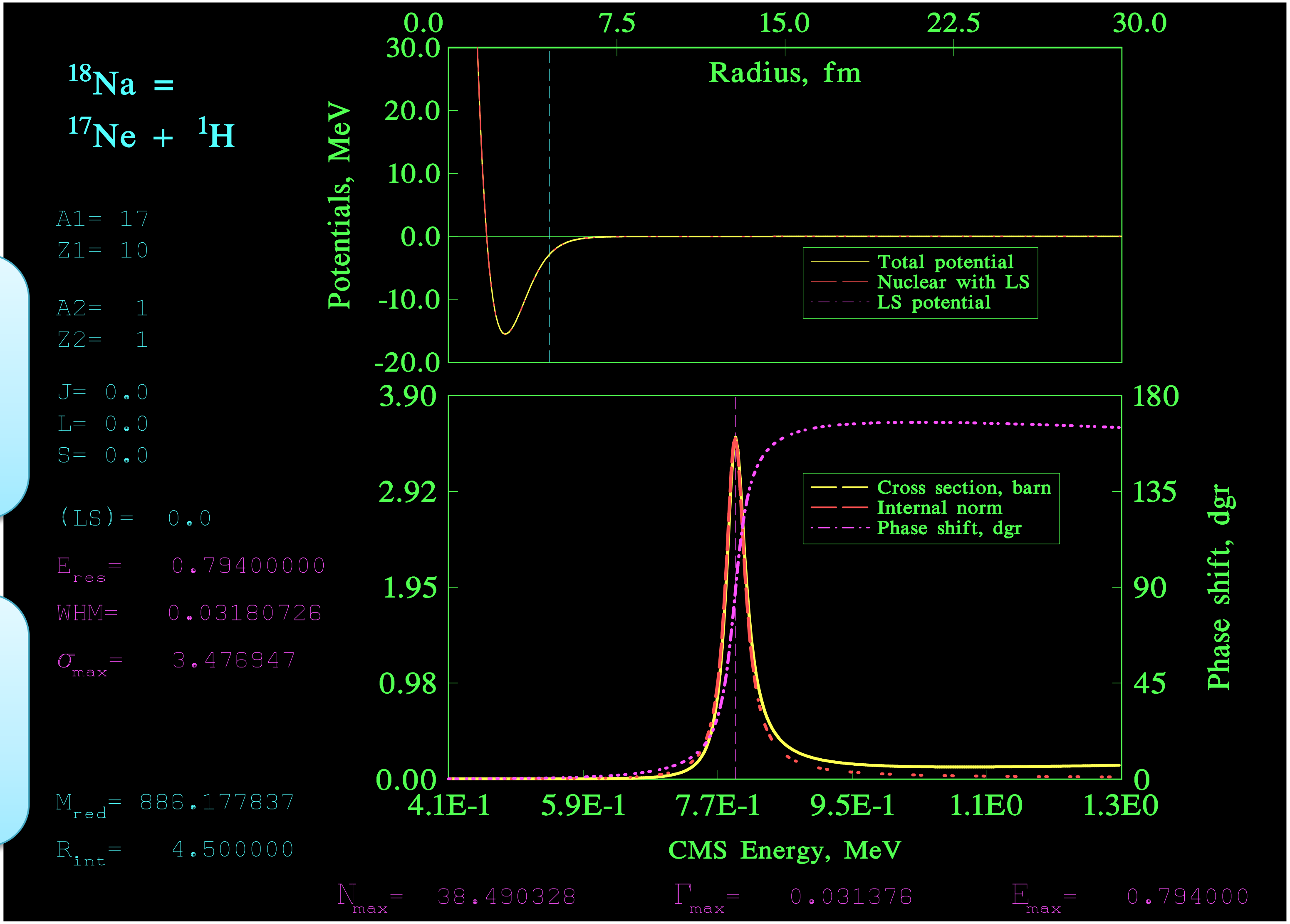
Resonance in elastic in elastic scattering

"Normal resonance" under barrier

Phase shift pass 90° (magenta)

Lorentian profile peak in the internal normalization (red)

Lorentian profile peak in the cross section (yellow)



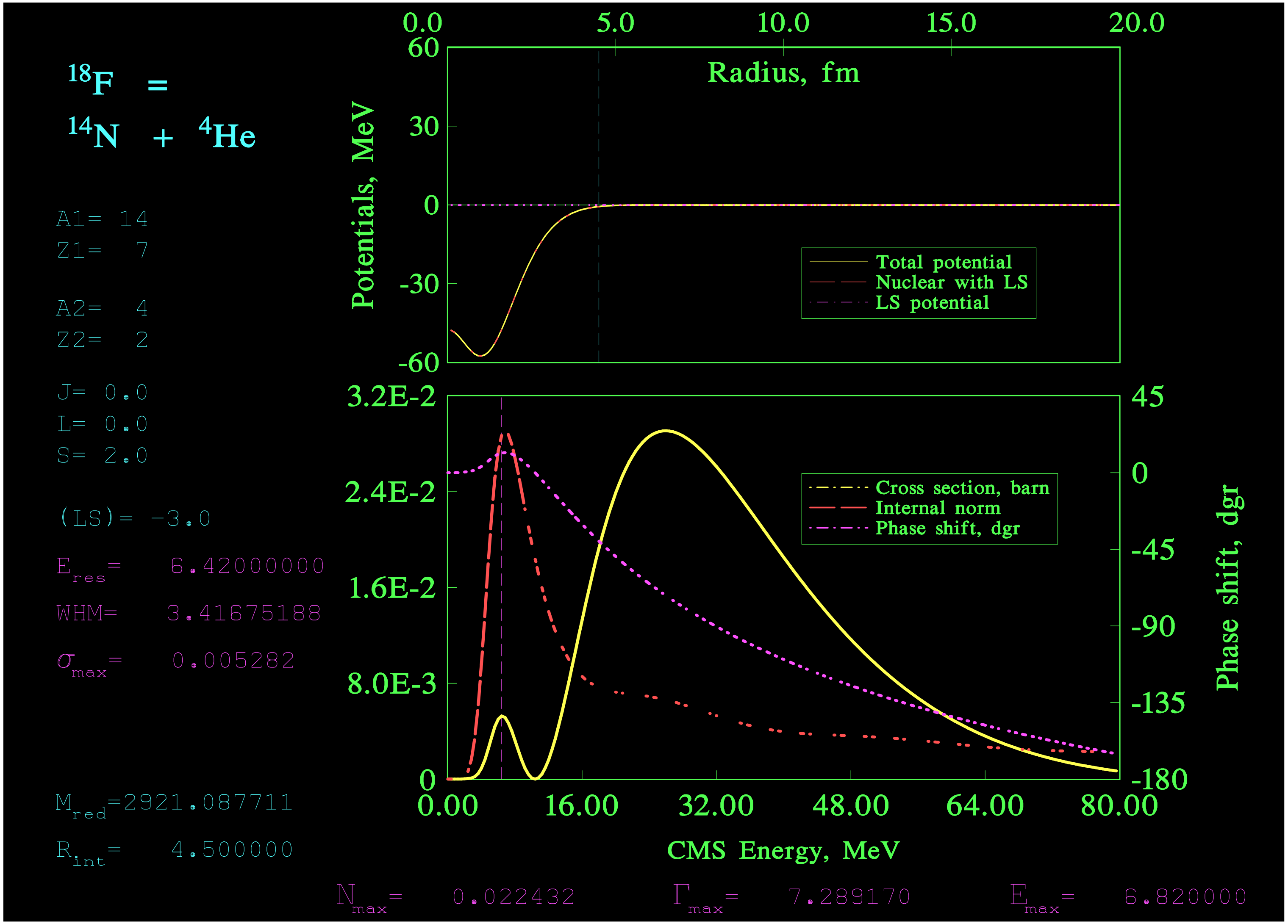
Joke 1

Peaks above barrier

Two expressed peaks

WF is not passing 90° from below

There is WF concentration in the interior ONLY for the first peak



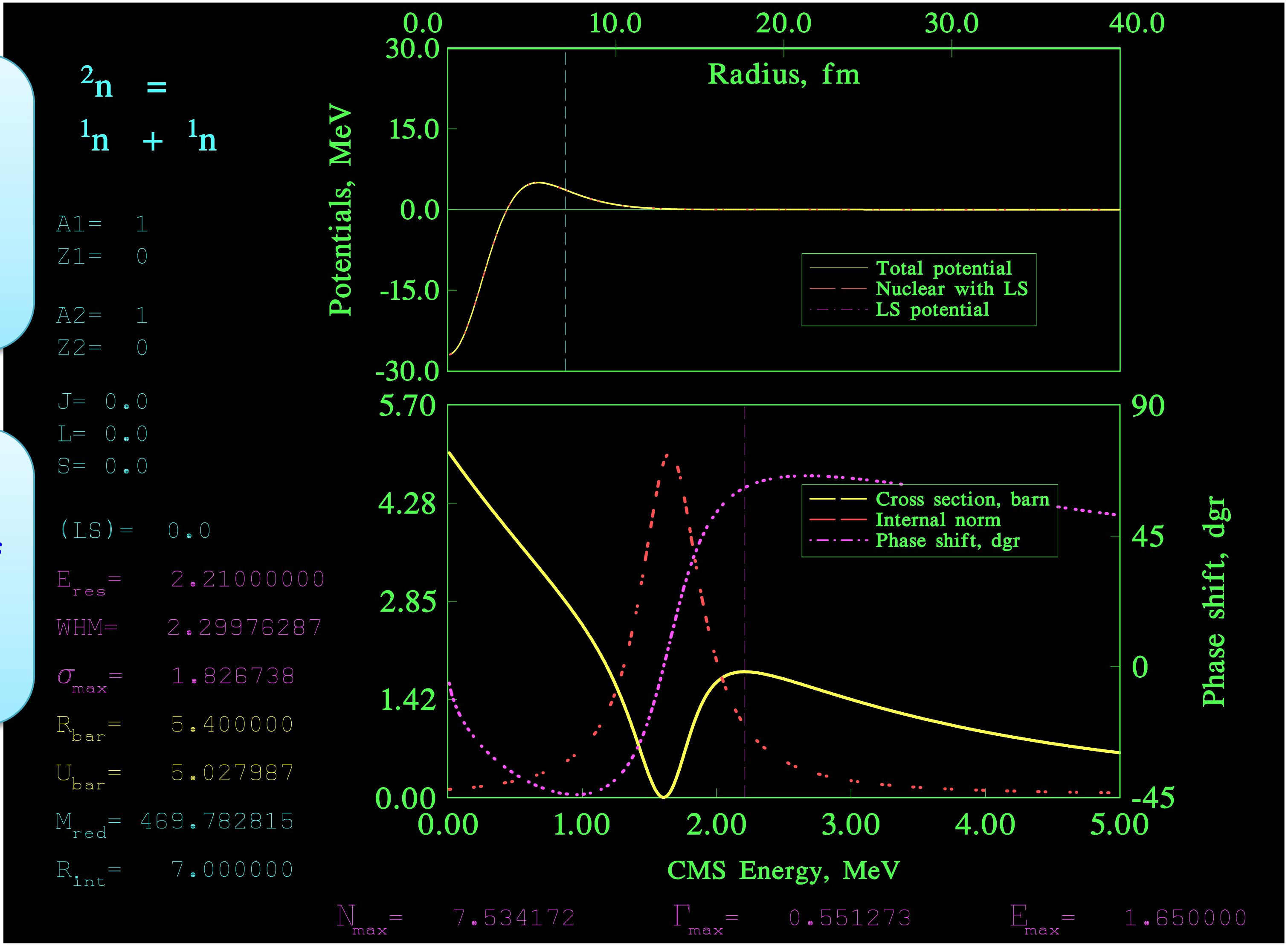
In reality – test for internal structure

Joke 2

Total "transparency"

Cross section dropdown to zero.

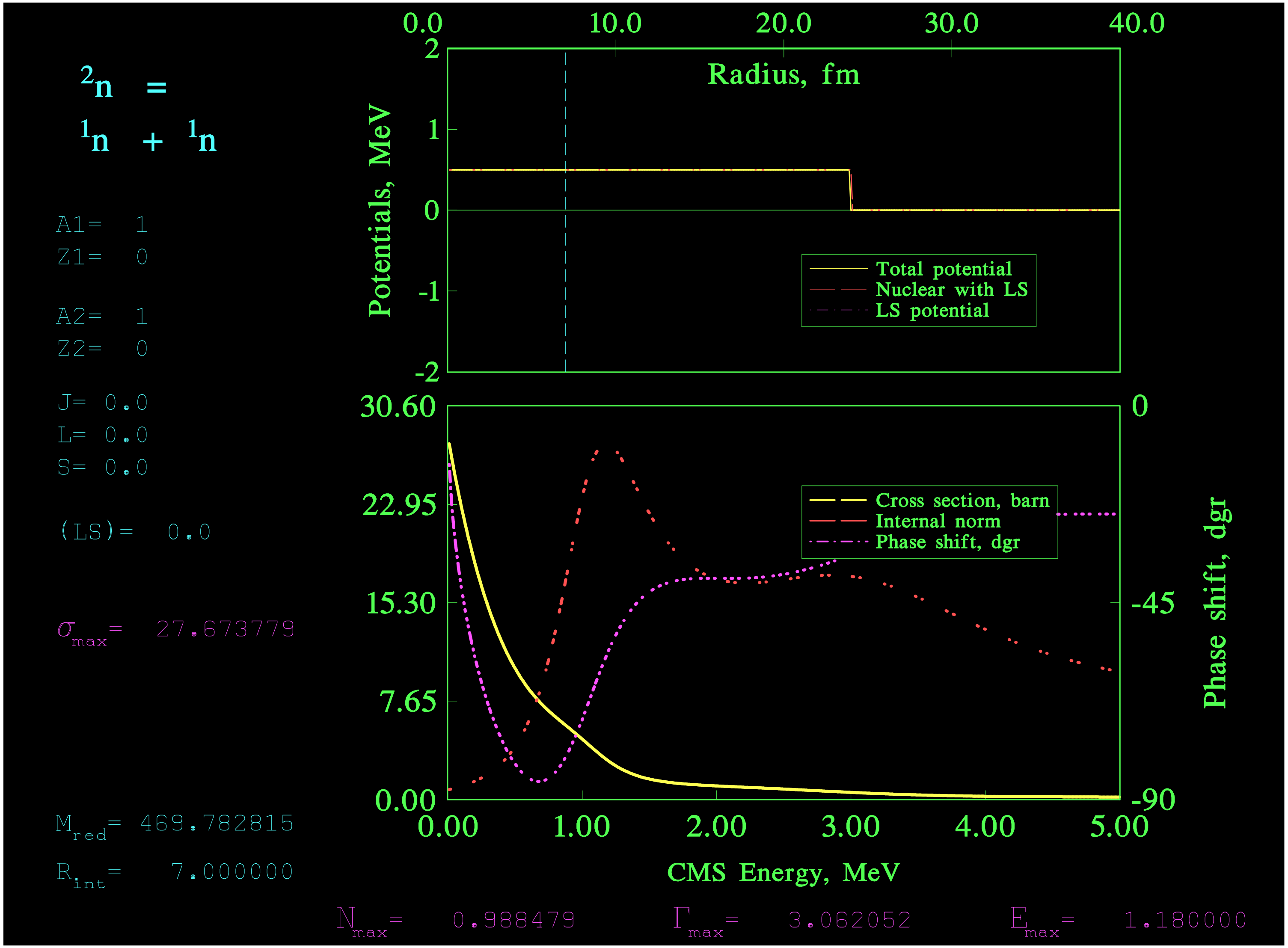
No scattering at all instead of very active scattering



Joke 3

“Shallow water resonance”

WF concentration is provided by
(i) small velocity above the “step” and
(ii) Interference of the waves reflected from origin and from the right part of the step



R-matrix formalism

Preexponent

Exponent

$$\Gamma_{p_i}(E) = 2\gamma^2 P_{l_i}(E, R, Z_1 Z_2).$$

$$P_l(E, R, Z_1 Z_2) = \frac{kR}{F_l^2(\eta, kR) + G_l^2(\eta, kR)}.$$

$$\gamma_{WL}^2 = \frac{1}{2MR^2}\theta^2, \quad \theta^2 = \frac{|\psi_l(kR)|^2}{\int_0^1 dx\, |\psi_l(kxR)|^2}.$$

Dimension energy limiting width – Wigner limit

Dimensionless structure factor

No Coulomb, *l = 0*

Γ ~ 1/T inverse flight time through nuclear interior

Elastic scattering formulation is not comfortable for radioactivity studies

Gamow approach has problems (i) only probabilities no amplitudes (ii) width behave wrong on top of the barrier

Strong Coulomb, *l = 0*

$$F_l(kr) \overset{kr\to 0}{\sim} C_l (kr)^{l+1}, \quad G_l(kr) \overset{kr\to 0}{\sim} \frac{(kr)^{-l}}{(2l+1)C_l},$$

$$C_l = \frac{2^l}{(2l+1)!}\left[(1^2+\eta^2)(2^2+\eta^2)\ \ldots\ (l^2+\eta^2)\right]^{1/2} C_0$$

$$C_0 = \left[\frac{2\pi\eta e^{-2\pi\eta}}{1-e^{-2\pi\eta}}\right]^{1/2}.$$

$$\eta = Z_1 Z_2 \alpha \sqrt{\frac{M}{2E}} = \frac{Z_1 Z_2 \alpha}{v}$$

Γ ~ exp[−2πη] - the same as for Gamow approach

R-matrix phenomenology

Description of elastic/inelastic scattering

$$\delta_l(E) = \arctan\left[\frac{\Gamma(E)}{2(E_r - E)}\right]$$

Effects of broad levels

For broad states the energy-dependent corrections to reduced width are provided in terms of "level shift function" $S_l(E)$ (Lane and Tomas, 1958):

$$\begin{aligned} \gamma^2 &\rightarrow \gamma^2/\left[1+\gamma^2 dS_l(E)/dE\right], \\ S_l(E,R,Z_1Z_2) &= kR\frac{F_l(kR)F_l'(kR)+G_l(kR)G_l'(kR)}{F_l^2(kR)+G_l^2(kR)}. \end{aligned}$$

Effects of structure

In applications of the R-matrix theory the dimensionless reduced width is identified as phenomenological spectroscopic factor

$$\theta^2 \rightarrow S = \frac{A!}{A_1!A_2!}\int_0^R dr\,\left|\langle A|A_1,A_2\rangle\right|^2$$

In contrast with normalizations of WFs spectroscopic factors are overlaps and their norm could be larger than 1

Bohr's compound nucleus theory

Kinematical limit. QM cross section for spinless particles cannot be larger

Statistical factor – QM mantra: sum over the final states, average over initial

Ingoing and outgoing channels

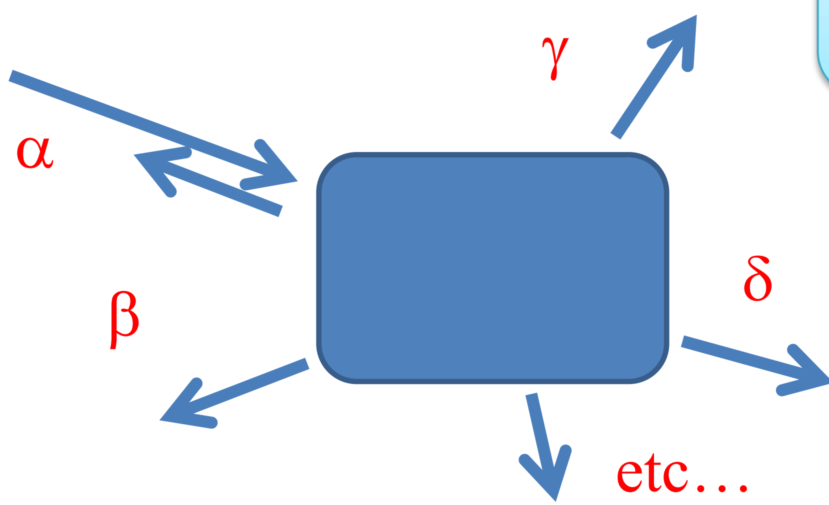
Lorentian (Breit-Wigner) profile peak in the cross section

$$\sigma_{\alpha\beta}(E) = \frac{\pi}{(k_\alpha)^2}\,\frac{(2J+1)}{(2J_{\alpha 1}+1)(2J_{\alpha 2}+1)}\,\frac{\Gamma_\alpha(E_\alpha)\Gamma_\beta(E_\beta)}{(E-E_R)^2-\Gamma^2/4}$$

$$\Gamma = \sum_i \Gamma_i(E_R)$$

Total resonance width – sum of partial widths ON RESONANCE

Compound state resonance is like a pool with attached pipes. Each pipe is a decay channel. Pool can be filled via one selected pipe, but the water is coming out via all opened pipes.



Decay states with complex energy

$$\Psi^{(+)}(r_i, t) = \exp[iEt - \Gamma t/2]\, \Psi^{(+)}(r_i)$$

$$(H - \tilde{E}_r)\Psi_{lm}^{(+)}(\mathbf{r}) = (T + V - \tilde{E}_r)\Psi_{lm}^{(+)}(\mathbf{r}) = 0$$

$$\tilde{E}_r = \tilde{k}_r^2/(2M) = E_r - i\Gamma/2, \quad \tilde{k}_r \approx k_r - i\Gamma/(2v_r)$$

Applying Green's procedure to the complex energy WF

$$\Psi^{(+)\dagger}[(H - \tilde{E}_r)\Psi^{(+)}] - [(H - \tilde{E}_r)\Psi^{(+)}]^\dagger \Psi^{(+)} = 0,$$

we get for the partial components at pole energy $\tilde{E}_r$

$$i\Gamma \psi_l^{(+)*}\psi_l^{(+)} = \frac{1}{2M}\left[\psi_l^{(+)*}\frac{d^2\psi_l^{(+)}}{dr^2} - \frac{d^2\psi_l^{(+)*}}{dr^2}\psi_l^{(+)}\right].$$

$$\Gamma = \frac{\left[\psi_l^{(+)*}\left(\frac{d}{dr}\psi_l^{(+)}\right) - \left(\frac{d}{dr}\psi_l^{(+)*}\right)\psi_l^{(+)}\right]\Big|_{r=R}}{2Mi\int_0^R \left|\psi_l^{(+)}\right|^2 dr} = \frac{j_l}{N_l}, \quad (3)$$

which corresponds to a definition of the width as a decay probability (reciprocal of the lifetime):

$$N = N_0 \exp[-t/\tau] = N_0 \exp[-\Gamma t].$$

Time dependent WF with probability exponentially decreasing with time

Complex energy Hamiltonian

Green's procedure for complex conjugate

"Natural" definition of width: for WF with pure outgoing asymptotic width is outgoing flux devided by normalization ("number of particles") in the internal region

Asymptotic of the decay WF

$$\Psi^{(+)}(r_i,t) = \exp[iEt - \Gamma t/2]\,\Psi^{(+)}(r_i)$$

$$(H - \tilde{E}_r)\Psi^{(+)}_{lm}(\mathbf{r}) = (T + V - \tilde{E}_r)\Psi^{(+)}_{lm}(\mathbf{r}) = 0$$

$$\tilde{E}_r = \tilde{k}_r^2/(2M) = E_r - i\Gamma/2, \quad \tilde{k}_r \approx k_r - i\Gamma/(2v_r)$$

$$\psi_l^{(+)}(\tilde{k}_r r) \overset{r>R}{=} H_l^{(+)}(\tilde{k}_r r) = G_l(\tilde{k}_r r) + i\,F_l(\tilde{k}_r r).$$

$$\psi_l^{(+)}(\tilde{k}_r r) \overset{r>R}{\sim} \exp[+i\tilde{k}_r r] \approx \exp[+ik_r r]\exp[+\Gamma r/(2v_r)]$$

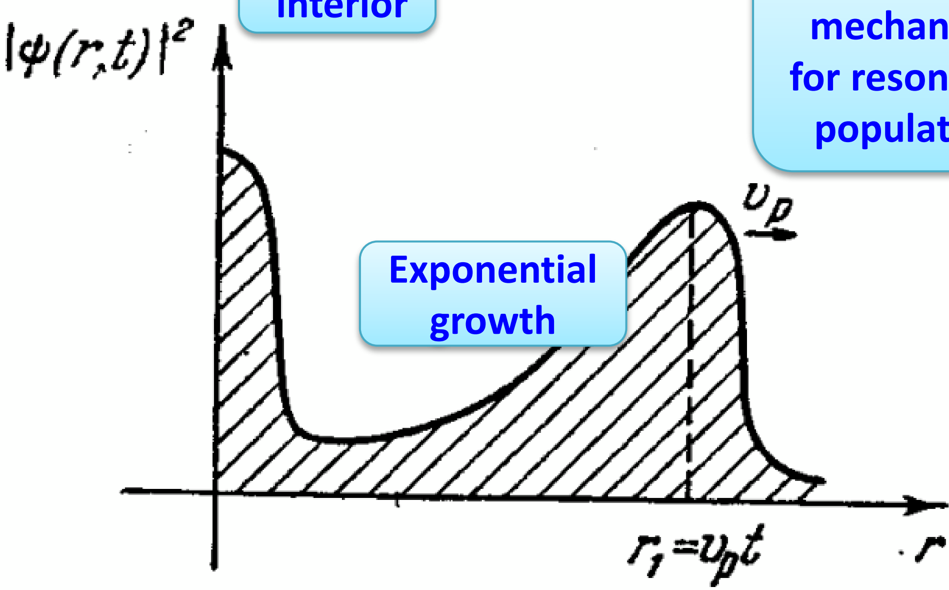
Decay WF with complex energy shows unphysical exponetial growth at large energy

Reason – simple Ansatz above do not work in all time domain

Nuclear interior

Reaction mechanism for resonance population

Exponential growth



For decay of Radium with Q ~ 10 MeV and $T_{1/2}$~ 5000 years the integral of the WF in the outer part become comparable with inner part for r > 100 light years

Computation to astronomical radial scale are needed to see decay WF in all its complexity

Quasistationary state

$$(H - E)\Psi_{\mathbf{k}}(\mathbf{r}) = (T + V^{\mathrm{nuc}} + V^{\mathrm{coul}} - E)\Psi_{\mathbf{k}}(\mathbf{r}) = 0,$$

$$\Psi_{\mathbf{k}}(\mathbf{r}) = 4\pi \sum_l i^l (kr)^{-1} \psi_l(kr) \sum_m Y^*_{lm}(\hat{k}) Y_{lm}(\hat{r}),$$

$$\psi_l(kr) = \frac{i}{2}[(G_l(kr) - i\,F_l(kr)) - S_l(G_l(kr) + i\,F_l(kr))].$$

$$S_l(E_r) = e^{2i\delta_l(E_r)} = e^{2i\pi/2} = -1$$

on resonance

$$\psi_l(k_r r) \overset{r>R}{=} i\,G_l(k_r r)$$

$$\tilde{\psi}_l(k_r r) = \frac{(-i)\psi_l(k_r r)}{\left(\int_0^R |\psi_l(k_r x)|^2 dx\right)^{1/2}} = -i\,\frac{\psi_l(k_r r)}{{N_l}^{1/2}}$$

Quasistaionary WF is normalized in internal region

$$\varphi_l(kr) = \sqrt{\frac{\pi v}{2}} \frac{\sqrt{\Gamma_r(E)}}{E_r - E - i\Gamma_r(E)/2}\,\hat{\psi}_l(k_a, r)$$

Near resonance the radial and energy degrees of freedom are factorized

WF in this form combines properties of bound and scattering WFs and thus demonstrates how transition from discrete to continuous spectrum happens

Integral formula for width

Formulation
K. Harada and E. A. Rauscher, 1968.
S. G. Kadmenskii and V. E. Kalechits, 1970.

Real Hamiltonian

$$(H - E)\Psi_{\mathbf{k}}(\mathbf{r}) = (T + V^{\text{nuc}} + V^{\text{coul}} - E)\Psi_{\mathbf{k}}(\mathbf{r}) = 0,$$

$$\Psi_{\mathbf{k}}(\mathbf{r}) = 4\pi \sum\nolimits_l i^l (kr)^{-1}\psi_l(kr) \sum\nolimits_m Y^*_{lm}(\hat{k})Y_{lm}(\hat{r}),$$

$$\psi_l(kr) = \frac{i}{2}[(G_l(kr) - iF_l(kr)) - S_l(G_l(kr) + iF_l(kr))].$$

Auxilliary Hamiltonian

$$(\bar{H} - E)\Phi_{\mathbf{k}}(\mathbf{r}) = (T + \bar{V}^{\text{nuc}} + V^{\text{coul}} - E)\Phi_{\mathbf{k}}(\mathbf{r}) = 0,$$

$$\Phi_{\mathbf{k}}(\mathbf{r}) = 4\pi \sum\nolimits_l i^l (kr)^{-1}\varphi_l(kr) \sum\nolimits_m Y^*_{lm}(\hat{k})Y_{lm}(\hat{r}),$$

$$\varphi_l(kr) = \frac{i}{2}[(G_l(kr) - iF_l(kr)) - \bar{S}_l(G_l(kr) + iF_l(kr))],$$

$$\varphi_l(kr) = \exp(i\bar{\delta}_l)[F_l(kr)\cos(\bar{\delta}_l) + G_l(kr)\sin(\bar{\delta}_l)].$$

$$S_l(E_r) = e^{2i\delta_l(E_r)} = e^{2i\pi/2} = -1$$

$$\psi_l(k_r r) \overset{r \geq R}{=} iG_l(k_r r)$$

$$\tilde{\psi}_l(k_r r) = \frac{(-i)\psi_l(k_r r)}{\left(\int_0^R |\psi_l(k_r x)|^2 dx\right)^{1/2}} = -i\frac{\psi_l(k_r r)}{N_l{}^{1/2}}$$

Green's procedure

$$\Phi_{\mathbf{k}}(\mathbf{r})^\dagger[(H - E)\Psi_{\mathbf{k}}(\mathbf{r})] - [(\bar{H} - E)\Phi_{\mathbf{k}}(\mathbf{r})]^\dagger\Psi_{\mathbf{k}}(\mathbf{r}) = 0,$$

$$\varphi_l^*(V - \bar{V})\psi_l = \frac{1}{2M}\left[\varphi_l^*\left(\frac{d^2}{dr^2}\psi_l\right) - \left(\frac{d^2}{dr^2}\varphi_l^*\right)\psi_l\right].$$

Wronskian after partial integration

$$2M\int_0^R \varphi_l^*(V - \bar{V})\psi_l dr = 2MiN_l{}^{1/2}\int_0^R \varphi_l^*(V - \bar{V})\tilde{\psi}_l dr$$

$$= \exp(-i\bar{\delta}_l)\cos(\bar{\delta}_l)k_r W(F_l(k_r R)\; G_l(k_r R))$$

Integral formula for width

$$2MiN_l^{1/2}\int_0^R \varphi_l^*(V-\bar{V})\tilde{\psi}_l dr = \exp(-i\bar{\delta}_l)\cos(\bar{\delta}_l)k_r$$

Square both sides

Here flux is velocity and width is flux devided by internal normalization

$$\Gamma = \frac{v_r}{\int_0^R \left|\psi_l^{(+)}\right|^2 dr} \approx \frac{v_r}{\int_0^R |\psi_l|^2 dr} = \frac{v_r}{\left|N_l^{1/2}\right|^2},$$

$$\Gamma = \frac{4}{v_r\cos^2(\bar{\delta}_l)}\left|\int_0^R \varphi_l^*(V-\bar{V})\tilde{\psi}_l dr\right|^2.$$

If we take point like Coulomb potential for auxiliary Hamiltonian especially simple expression is obtained

$$\Gamma = \frac{4}{v}\left|\int_0^{r_{\max}} dr \quad F_l(\eta,kr)\left[V(r)-\frac{v\,\eta}{r}\right]\varphi_l(r)\right|^2$$

Analogous for expression for T matrix - <plane wave|potential|real WF>

Useful technique. Works when integral is converged on the upper bound. This is guaranteed if asymptotic behavior of real and auxilliary Hamiltonian are the same.

Time delay

Normalization for the scattering WF Eq. (1) inside sphere of radius $R > r_{\rm nuc}$ is (Wigner, 1955)

$$N_l(E,R) = \int_0^R dr |\psi_{E,l}(r)|^2$$
$$= \frac{1}{\pi}\left\{R + \frac{d\delta_l(E)}{dk} - \frac{1}{2k}\sin\left[2kR + 2\delta_l(E)\right]\right\}. \qquad (19)$$

It can be shown that the scattering process can be interpreted in terms of the *time delay* (Baz', 1967)

$$T_l(E,R) = 2\pi N_l(E,R)/v. \qquad (20)$$

$$T_l(E,R) \approx \frac{\Gamma(E)/4}{(E_r - E)^2 + \Gamma(E)^2/4}\left[1 + \frac{E_r - E}{E}(2\pi\eta - 1)\right].$$

For radioactivity scale widths Γ

ON resonance
$T \sim 1/\Gamma$
time is exponentially large

OF resonance
$T \sim \Gamma/(E-E_r)^2$
time is exponentially small

Decay states with real energy

nuclear reaction $A + B \rightarrow F + R$

In AB channel we have BOTH in and outgoing waves

$$\Psi = \Psi_{AB} + \Psi_{FR}^{(+)}$$

In FR channel we have ONLY outgoing waves

$$\Psi_{AB} = \psi_A \psi_B \psi_{AB}, \qquad \Psi_{FR} = \psi_{FR}^{(+)} \psi_R$$

$$\begin{cases} \left(\hat{H}_{AB} - E_{AB}\right) \psi_{AB} = \langle \psi_A \psi_B | \hat{V} | \Psi_{FR} \rangle, \\ \left(\hat{H}_{FR} - E_{FR}\right) \psi_{FR}^{(+)} = \langle \psi_R | \hat{V} | \Psi_{AB} \rangle. \end{cases}$$

$$\Psi_{AB} >> \Psi_{FR}^{(+)}$$

$$\left(\hat{H}_{FR} - E_{FR}\right) \psi_{FR}^{(+)} = \Phi$$

For weak channel coupling we can use static "source" function in inelastic channel

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Example – sudden removal approximation

$$\Phi(q, r_i) = \int d^3 r \, e^{iqr} \, \Psi(r, r_i)$$

Remove particle r from WF $\Psi(r, r_i)$

Instead of vector r we get vector q of transferred momentum in the source

Different facets of resonance phenomenon

Generic idea	Lorentian (Breit-Wigner) profile peak in the cross section
Decays	Lifetime; Exponentially growing WF long-range tail
Elastic scattering	Phase shift pass 90°; Delay time
Reactions	WF concentration in the interior
S-matrix	Pole
Quasistationary WF	Separation of energy and radial degrees of freedom

Resonance is not necessarily peak

Peak is not necessarily resonance

Resonance phenomena

vs

Excitation modes