Theoretical basics and modern status of radioactivity studies

Lecture 3: Some theory
Long lifetimes

$\beta$ decays may compete with $\alpha, p, 2p$ radioactivity

$\gamma$ transitions may compete with particle emission from excited states

Radioactivity

Resonant phenomena

$T_{1/2}$ [s]

$\Gamma$ [MeV]

ISOL

In-Flight separation

Stopped/transported beam

Particle tracking

Decay in flight

Invariant/missing mass methods

Neutron resonances by time-of-flight (TOF)
Particle decay, Gamow theory

\[ \Psi(r) \sim e^{iS(r)} \]

\[ S(r) = \int_{r_0}^{r} p(r') \, dr' \]

\[ p(r) = \sqrt{2M|E_r - V(r)|} \]

\[ \Gamma = S_\alpha \omega \exp\left[-2\int_{r_2}^{r_3} p(r) \, dr\right] \]

\[ \int_{0}^{r_3} p(r) \, dr = 2\eta \int_{0}^{1} dx \sqrt{1 - 1/x} = \eta\pi \]

\[ \eta = Z_1 Z_2 \alpha \sqrt{\frac{M}{2E}} = \frac{Z_1 Z_2 \alpha}{\nu} \]

Sommerfeld parameter

Dimensionless structure factor

Dimension energy “limiting width”
Particle decay. Gamow theory

\[ T = \int_{r_1}^{r_2} dt = \int_{r_1}^{r_2} \frac{1}{v} dr = \int_{r_1}^{r_2} \frac{M}{p(r)} dr \]

\[ \omega = \left[ 2 \int_{r_1}^{r_2} \frac{M}{p(r)} dr \right]^{-1} \]

\[ \Gamma = S_\alpha \omega \exp[-2\pi \eta] \]

\[ \ln T_{1/2} \sim a + b E^{-1/2} \]

Preexponent

Strange, but for majority of situations this is precise within few percent

Geiger-Nuttal law
\[ H_{fi} = (2\pi)\delta(E_f - E_i) \int \prod_{k=1}^{N} d^3r_k \Psi_f^*(r_1 \ldots r_N) \hat{H}_\beta(r_1 \ldots r_N) \Psi_i(r_1 \ldots r_N) \]

\[ \hat{H}_\beta(r_1, \ldots, r_N) = -\frac{G_V}{\sqrt{2}} \sum_{k=1}^{N} \left\{ L_0^*(r_k) + \lambda(L^*_\mu(r_k), \sigma_k) \right\} \tau_k^- \]

\[ L^*_\mu(r_k) = \overline{\Psi}_e(r_k) \gamma_\mu (1 + \gamma_5) \Psi_\nu(r_k) = \frac{\overline{u}_e \gamma_\mu (1 + \gamma_5) u_\nu}{\sqrt{2E_e2E_\nu V}} \exp(iqr_k) \quad q = p_e - p_\nu \]

\[ \sum_f |H_{fi}|^2 = \frac{1}{2J + 1} \frac{G_V^2 \lambda^2}{2} (2\pi)\delta(E_f - E_i) \Delta t \frac{6}{V^2} \left(1 - \frac{p_e p_\nu}{3E_e E_\nu}\right) \frac{2J' + 1}{3} \left| \sum_{k=1}^{N} \langle J' || \sigma_k || J \rangle \langle \tau_k^- \rangle \right|^2 \]

\[ dW = \frac{1}{2J + 1} \frac{G_V^2 \lambda^2}{2} (2\pi)\delta(E_e + E_\nu - Q) \frac{6}{V^2} \left(1 - \frac{p_e p_\nu}{3E_e E_\nu}\right) \times \]

\[ \times \frac{2J' + 1}{3} \left| \sum_{k=1}^{N} \langle J' || \sigma_k || J \rangle \langle \tau_k^- \rangle \right|^2 \frac{V^2 dp_e dp_\nu}{(2\pi)^6} \]

\[ dW = \frac{G_V^2 \lambda^2}{2\pi^3} \cdot \left( \frac{2J' + 1}{2J + 1} \left| \sum_{k=1}^{N} \langle J' || \sigma_k || J \rangle \langle \tau_k^- \rangle \right|^2 \right) \cdot (\delta(E_e + E_\nu - Q) p_e^2 dp_e p_\nu^2 dp_\nu) \]
Beta decay. Gamow-Teller

\[ W = \frac{\ln 2}{t_{1/2}} = \frac{G_V^2 \lambda^2 m_e^5}{2 \pi^3} \cdot B_{GT} \cdot f(A, Z, Q) \]

\[ ft_{1/2} = 2 \cdot \frac{\pi^3 \ln 2}{G_V^2 m_e^5} \cdot \frac{1}{\lambda^2 B_{GT}} = \frac{2 ft(0^+ \to 0^+)}{\lambda^2 B_{GT}} \]

\[ B_{GT} = \frac{2 J' + 1}{2 J + 1} \left| \sum_{k=1}^{N} \langle J' | \| \sigma_k \| | J \rangle \langle T'T_3' \mid \tau_k^- \mid TT_3 \rangle \right|^2 \]

\[ f(A, Z, Q) = \int_{1}^{\infty} \frac{(Q/m_e - \varepsilon)^2 \varepsilon \sqrt{\varepsilon^2 - 1} F(A, Z, Q) d\varepsilon}{F(A, Z, Q)} \]

\[ \lambda = G_A/G_V = -1.268 \pm 0.002 \ (\lambda^2 = 1.608 \pm 0.004) \]

\[ Q = E - m_e \] is the energy release in the reaction

\[ ft(0^+ \to 0^+) = \frac{\pi^3 \ln 2}{G_V^2 m_e^5} = 3072.4 \pm 1.6 \text{ sec} \]

\[ F(A, Z, Q) \]

\[ f \sim 1 \quad \text{for} \quad Q \sim 2.4 \ m_e \]

\[ \text{for} \ Q \gg m_e \ \rightarrow \ f \sim Q^5 \]

For \( Q \sim 10 \ m_e \) lifetime of the
tens of milliseconds is expected

For \( Q \sim 1.1 \ m_e \) lifetime of the
order of years is expected
Electromagnetic decay. Nuclear isomers

“Phase space” isomers

\[ \Gamma \sim E^3 \text{ for } E1 \text{ transitions} \]

\[ \Gamma \sim E^5 \text{ for } E2 \text{ transitions} \]

Angular momentum isomers

<table>
<thead>
<tr>
<th>E* (MeV)</th>
<th>J^\pi</th>
<th>T_{1/2}</th>
</tr>
</thead>
<tbody>
<tr>
<td>180^{Ta}</td>
<td>0</td>
<td>1^+</td>
</tr>
<tr>
<td>180^{mTa}</td>
<td>0.07</td>
<td>9^-</td>
</tr>
</tbody>
</table>

Shape isomers

\[ E_{rot} \sim J^2 \sim l(l+1) \]

If the system change shape it is likely that gamma lifetime abruptly increases.
Particle decay
Resonance phenomena
Nuclear dynamics vs. excitation energy

"Phase volume" dynamics – only initial state is important

Transition region – resonances are broad

Resonance phenomena

Radioactivity

Haloes

"Normal nuclei"
From formal dynamics point of view there is no clear borderline between resonance phenomena and radioactivity.

From formal structure point of view there is no clear borderline between stationary and quasistationary states (radioactivity).
Resonances in elastic scattering

\[ (\tilde{H} - E)\Phi_k(\mathbf{r}) = (T + \tilde{V}^{\text{nuc}} + V^{\text{coul}} - E)\Phi_k(\mathbf{r}) = 0, \]

\[ \Phi_k(\mathbf{r}) = 4\pi \sum_l i^l(kr)^{-1} \varphi_l(kr) \sum_m Y_{lm}^*(\hat{k})Y_{lm}(\hat{r}), \]

\[ \varphi_l(kr) = \frac{i}{2} [ (G_l(kr) - i F_l(kr)) - \bar{S}_l(G_l(kr) + i F_l(kr))], \]

\[ \varphi_l(kr) = \exp(i\bar{\delta}_l)[ F_l(kr) \cos(\delta_l) + G_l(kr) \sin(\delta_l)]. \]

At resonance energy \( E_r \),

\[ S_l(E_r) = e^{2i\delta_l(E_r)} = e^{2i\pi/2} = -1, \]

\[ \varphi_l(kr) = i G_l(kr) \]

Phase shift \( \delta_l(E) = 90^\circ \)

Elastic cross section has peak

\[ \sigma = \frac{4\pi}{k^2} \sum_l (2l + 1) \sin^2(\delta_l) \]

Internal normalization has peak

\[ N_l(E, R) = \int_0^R dr |\psi_{E,l}(r)|^2 \]
Resonance in elastic in elastic scattering

\[ ^{18}\text{Na} = ^{17}\text{Ne} + ^{1}\text{H} \]

- Lorentian profile peak in the internal normalization (red)
- Lorentian profile peak in the cross section (yellow)

\[ \begin{aligned}
A_1 &= 17 \\
Z_1 &= 10 \\
A_2 &= 1 \\
Z_2 &= 1 \\
J &= 0.0 \\
L &= 0.0 \\
S &= 0.0 \\
(\text{LS}) &= 0.0 \\
E_{\text{res}} &= 0.79400000 \\
\text{WHM} &= 0.03180726 \\
\sigma_{\text{max}} &= 3.476347 \\
M_{\text{red}} &= 886.177837 \\
R_{\text{int}} &= 4.500000 \\
\end{aligned} \]

“Normal resonance” under barrier

Phase shift pass 90° (magenta)
Joke 1

Peaks above the barrier

Two expressed peaks

WF is not passing 90° from below

There is WF concentration in the interior ONLY for the first peak

In reality – test for internal structure

\[ {}^{18}\text{F} = {}^{14}\text{N} + {}^{4}\text{He} \]

\( A_1 = 14 \)

\( Z_1 = 7 \)

\( A_2 = 4 \)

\( Z_2 = 2 \)

\( J = 0.0 \)

\( I = 0.0 \)

\( S = 2.0 \)

\( (LS) = -3.0 \)

\( E_{\text{res}} = 6.42000000 \)

\( \text{WHM} = 3.41675188 \)

\( \sigma_{\max} = 0.005282 \)

\( M_{\text{red}} = 2921.087711 \)

\( R_{\text{int}} = 4.500000 \)

\( F_{\max} = 0.022432 \)

\( \Gamma_{\max} = 7.289170 \)

\( E_{\max} = 6.820000 \)
Cross section dropdown to zero.

No scattering at all instead of very active scattering.

Total “transparency”
Joke 3

“Shallow water resonance”

WF concentration is provided by
(i) small velocity above the “step” and
(ii) Interference of the waves reflected from origin
and from the right part of the step

\[ ^2n = ^1n + ^1n \]

A1 = 1
Z1 = 0
A2 = 1
Z2 = 0
J = 0.0
L = 0.0
S = 0.0

\( (LS) = 0.0 \)

\( \sigma_{\text{max}} = 27.673779 \)

\( M_{\text{red}} = 469.782815 \)

\( E_{\text{1n}} = 7.000000 \)

\( N_{\text{max}} = 0.888479 \)

\( \Gamma_{\text{max}} = 3.062052 \)

\( E_{\text{max}} = 1.18000 \)
**R-matrix formalism**

Elastic scattering formulation is not comfortable for radioactivity studies

Gamow approach has problems (i) only probabilities no amplitudes (ii) width behave wrong on top of the barrier

\[ \Gamma_{p_i}(E) = 2\gamma^2 P_l(E, R, Z_1Z_2). \]

\[ P_l(E, R, Z_1Z_2) = \frac{kR}{F_l^2(\eta, kR) + G_l^2(\eta, kR)}. \]

\[ \gamma_{WL}^2 = \frac{1}{2MR^2} \theta^2, \quad \theta^2 = \frac{\left|\psi_l(kR)\right|^2}{\int_0^1 dx \left|\psi_l(kxR)\right|^2}. \]

**Dimension energy limiting width – Wigner limit**

No Coulomb, \( l = 0 \)

\[ \Gamma \sim 1/T \quad \text{inverse flight time through nuclear interior} \]

\[ \Gamma \sim \exp[-2\pi\eta] \quad \text{- the same as for Gamow approach} \]

\[ F_l(kr) \approx 0, \quad C_l(kr)^{l+1}, \quad G_l(kr) \approx 0, \quad \frac{(kr)^{-l}}{(2l + 1)C_l}, \]

\[ C_l = \frac{2^l}{(2l + 1)!} \left[ (1^2 + \eta^2)(2^2 + \eta^2) \ldots (l^2 + \eta^2) \right]^{1/2} C_0 \]

\[ C_0 = \left[ \frac{2\pi\eta e^{-2\pi\eta}}{1 - e^{-2\pi\eta}} \right]^{1/2}. \]

\[ \eta = Z_1Z_2 \alpha \sqrt{\frac{M}{2E}} = \frac{Z_1Z_2\alpha}{\nu} \]
R-matrix phenomenology

\[ \delta(E) = \arctan \left( \frac{\Gamma(E)}{2(E_r - E)} \right) \]

For broad states the energy-dependent corrections to reduced width are provided in terms of “level shift function” \( S_l(E) \) (Lane and Tomas, 1958):

\[ \gamma^2 \rightarrow \gamma^2 / \left[ 1 + \gamma^2 dS_l(E)/dE \right] , \]

\[ S_l(E, R, Z_1 Z_2) = k R \frac{F_l(kR)F_l'(kR) + G_l(kR)G_l'(kR)}{F_l^2(kR) + G_l^2(kR)} . \]

In applications of the R-matrix theory the dimensionless reduced width is identified as phenomenological spectroscopic factor

\[ \theta^2 \rightarrow S = \frac{A!}{A_1!A_2!} \int_0^R dr \left| \langle A | A_1, A_2 \rangle \right|^2 \]

In contrast with normalizations of WFs spectroscopic factors are overlaps and their norm could be larger than 1.
Bohr’s compound nucleus theory

Kinematical limit. QM cross section for spinless particles cannot be larger

\[ \sigma_{\alpha\beta}(E) = \frac{\pi}{(k_\alpha)^2} \frac{(2J + 1)}{(2J_\alpha_1 + 1)(2J_\alpha_2 + 1)} \frac{\Gamma_{\alpha}(E_\alpha)\Gamma_{\beta}(E_\beta)}{(E - E_R)^2 - \Gamma^2/4} \]

Statistical factor – QM mantra: sum over the final states, average over initial

Ingoing and outgoing channels

Lorentian (Breit-Wigner) profile peak in the cross section

\[ \Gamma = \sum_i \Gamma_i(E_R) \]

Total resonance width – sum of partial widths ON RESONANCE

Compound state resonance is like a pool with attached pipes. Each pipe is a decay channel. Pool can be filled via one selected pipe, but the water is coming out via all opened pipes.

\( \alpha \) \hspace{2cm} \( \beta \) \hspace{2cm} \( \gamma \) \\
\( \delta \hspace{2cm} \) etc…
**Decay states with complex energy**

\[ \Psi^{(+)\text{r}}(r_i, t) = \exp[iEt - \Gamma t/2] \Psi^{(+)\text{r}}(r_i) \]

\[ (H - \tilde{E}_r)\Psi^{(+)\text{r}}_{lm}(\mathbf{r}) = (T + V - \tilde{E}_r)\Psi^{(+)\text{r}}_{lm}(\mathbf{r}) = 0 \]

\[ \tilde{E}_r = \tilde{k}_r^2/(2M) = E_r - i\Gamma/2, \quad \tilde{k}_r \approx k_r - i\Gamma/(2v_r) \]

Applying Green's procedure to the complex energy WF

\[ \psi^{(+\text{t})\dagger}[(H - \tilde{E}_r)\psi^{(+)\text{r}}] - [(H - \tilde{E}_r)\psi^{(+)\text{r}}]^{(+)\dagger} = 0, \]

we get for the partial components at pole energy \( \tilde{E}_r \)

\[ i\Gamma \psi^{(+)\text{r}}_l \psi^{(+)\text{r}} = \frac{1}{2M} \left[ \psi^{(+)\text{r}}_l \frac{d^2 \psi^{(+)\text{r}}_l}{dr^2} - \frac{d^2 \psi^{(+)\text{r}}_l}{dr^2} \psi^{(+)\text{r}}_l \right]. \]

\[ \Gamma = \frac{\left[ \psi^{(+)\text{r}}_l \frac{d}{dr} \psi^{(+)\text{r}}_l - \left( \frac{d}{dr} \psi^{(+)\text{r}}_l \right)^* \psi^{(+)\text{r}}_l \right]}{2Mi \int_0^R |\psi^{(+)\text{r}}_l|^2 dr} \bigg|_{r=R} = \frac{j_l}{N_l}, \quad (3) \]

which corresponds to a definition of the width as a decay probability (reciprocal of the lifetime):

\[ N = N_0 \exp \left[ -t/\tau \right] = N_0 \exp \left[ -\Gamma t \right]. \]
Asymptotic of the decay WF

\[ \Psi^{(+)}(r_i, t) = \exp[iEt - \Gamma t/2] \Psi^{(+)}(r_i) \]

\[ (H - \tilde{E}_r) \Psi^{(+)}_{lm}(r) = (T + V - \tilde{E}_r) \Psi^{(+)}_{lm}(r) = 0 \]

\[ \tilde{E}_r = \tilde{k}_r^2/(2M) = E_r - i\Gamma/2, \quad \tilde{k}_r \approx k_r - i\Gamma/(2v_r) \]

\[ \psi^{(+)}_i(\tilde{k}_r r) \overset{r \geq R}{=} H^{(+)}_i(\tilde{k}_r r) = G_l(\tilde{k}_r r) + i F_l(\tilde{k}_r r). \]

\[ \psi^{(+)}_i(\tilde{k}_r r) \overset{r \geq R}{\approx} \exp[-i\tilde{k}_r r] \approx \exp[+ik_r r] \exp[+\Gamma r/(2v_r)] \]

Decay WF with complex energy shows unphysical exponential growth at large energy

Reason – simple Ansatz above do not work in all time domain

For decay of Radium with \( Q \sim 10 \text{ MeV} \) and \( T_{1/2} \sim 5000 \text{ years} \) the integral of the WF in the outer part become comparable with inner part for \( r > 100 \) light years

Computation to astronomical radial scale are needed to see decay WF in all its complexity
Quasistationary state

\[(H - E)\Psi_k(r) = (T + V^{nuc} + V^{coul} - E)\Psi_k(r) = 0,\]

\[\Psi_k(r) = 4\pi \sum_l i^l(kr)^{-1} \psi_l(kr) \sum_m Y_{lm}^*(\hat{k})Y_{lm}(\hat{r}),\]

\[\psi_l(kr) = \frac{i}{2}[(G_l(kr) - i F_l(kr)) - S_l(G_l(kr) + i F_l(kr))].\]

\[S_l(E_r) = e^{2i\delta_l(E_r)} = e^{2i\pi/2} = -1\]

\[\psi_l(k_r r) \quad r \geq R \quad = i G_l(k_r r)\]

\[\tilde{\psi}_l(k_r r) = \frac{(-i)\psi_l(k_r r)}{\left( \int_0^R |\psi_l(k_r x)|^2 dx \right)^{1/2}} = -i \frac{\psi_l(k_r r)}{N_l^{1/2}}\]

\[\varphi_l(kr) = \sqrt{\frac{\pi \nu}{2}} \frac{\sqrt{\Gamma_r(E)}}{E_r - E - i\Gamma_r(E)/2} \tilde{\psi}_l(k_a, r)\]

WF in this form combines properties of bound and scattering WFs and thus demonstrates how transition from discrete to continuous spectrum happens.

Near resonance the radial and energy degrees of freedom are factorized.

Quasistationary WF is normalized in internal region.

On resonance.
Integral formula for width

\( (H - E)\Psi_k(r) = (T + V_{\text{nuc}} + V_{\text{coul}} - E)\Psi_k(r) = 0, \)
\( \Psi_k(r) = 4\pi \sum_l i^l(kr)^{-1} \psi_l(kr) \sum_m Y_{lm}(\hat{k})Y_{lm}(\hat{r}), \)
\( \psi_l(kr) = \frac{i}{2} [(G_l(kr) - iF_l(kr)) - S_l(G_l(kr) + iF_l(kr))]. \)

Real Hamiltonian

\( S_l(E_r) = e^{2i\delta_l(E_r)} = e^{2i\pi/2} = -1 \)
\( \psi_l(k_r r) \stackrel{r \geq R}{=} i G_l(k_r r) \)

Wronskian after partial integration

\( 2M \int_0^R \varphi_l^*(V - \bar{V}) \psi_l dr = 2MiN_l^{1/2} \int_0^R \varphi_l^*(V - \bar{V}) \tilde{\psi}_l dr \)
\( = \exp(-i\delta_l) \cos(\delta_l)k_r W(F_l(k_r R) G_l(k_r R)) \)

Auxilliary Hamiltonian

\( (\bar{H} - E)\Phi_k(r) = (T + \bar{V}_{\text{nuc}} + V_{\text{coul}} - E)\Phi_k(r) = 0, \)
\( \Phi_k(r) = 4\pi \sum_l i^l(kr)^{-1} \varphi_l(kr) \sum_m Y_{lm}(\hat{k})Y_{lm}(\hat{r}), \)
\( \varphi_l(kr) = \frac{i}{2} [(G_l(kr) - iF_l(kr)) - \bar{S}_l(G_l(kr) + iF_l(kr))], \)
\( \varphi_l(kr) = \exp(i\delta_l)[F_l(kr) \cos(\delta_l) + G_l(kr) \sin(\delta_l)]. \)

Formulation


Green’s procedure

\( \Phi_k(r)^\dagger[(H - E)\Psi_k(r)] - [(\bar{H} - E)\Phi_k(r)]^\dagger\Psi_k(r) = 0, \)
\( \varphi_l^*(V - \bar{V})\psi = \frac{1}{2M} \left[ \varphi_l^* \left( \frac{d^2}{dr^2} \psi_l \right) - \left( \frac{d^2}{dr^2} \varphi_l^* \right) \psi_l \right]. \)
Integral formula for width

\[ 2M i N_l^{1/2} \int_0^R \varphi_l^*(V - \bar{V}) \tilde{\psi}_l \, dr = \exp(-i \delta_l) \cos(\delta_l) k_r \]

Square both sides

Here flux is velocity and width is flux divided by internal normalization

\[ \Gamma = \frac{\nu_r}{\int_0^R |\psi_l^{(+)}|^2 \, dr} \approx \frac{\nu_r}{\int_0^R |\psi_l|^2 \, dr} = \frac{\nu_r}{|N_l^{1/2}|^2}, \]

\[ \Gamma = \frac{4}{\nu_r \cos^2(\delta_l)} \left| \int_0^R \varphi_l^*(V - \bar{V}) \tilde{\psi}_l \, dr \right|^2. \]

If we take point like Coulomb potential for auxiliary Hamiltonian especially simple expression is obtained

\[ \Gamma = \frac{4}{\nu} \left| \int_0^{r_{\text{max}}} \, dr \ F_l(\eta, kr) \left[ V(r) - \frac{\nu \eta}{r} \right] \varphi_l(r) \right|^2 \]

Analogous for expression for T matrix - \langle\text{plane wave}|\text{potential}|\text{real WF}\rangle

Useful technique. Works when integral is converged on the upper bound. This is guaranteed if asymptotic behavior of real and auxiliary Hamiltonian are the same.
Time delay

Normalization for the scattering WF Eq. (1) inside sphere of radius \( R > r_{\text{nuc}} \) is (Wigner, 1955)

\[
N_l(E, R) = \int_0^R dr |\psi_{E,l}(r)|^2
\]

\[
= \frac{1}{\pi} \left\{ R + \frac{d\delta_l(E)}{dk} - \frac{1}{2k} \sin [2kR + 2\delta_l(E)] \right\}. \quad (19)
\]

It can be shown that the scattering process can be interpreted in terms of the time delay (Baz’’, 1967)

\[
T_l(E, R) = \frac{2\pi N_l(E, R)}{v}. \quad (20)
\]

\[
T_l(E, R) \approx \frac{\Gamma(E)/4}{(E_r - E)^2 + \Gamma(E)^2/4} \left[ 1 + \frac{E_r - E}{E} (2\pi \eta - 1) \right].
\]

For radioactivity scale widths \( \Gamma \)

ON resonance

\( T \sim 1/\Gamma \)

time is exponentially large

OF resonance

\( T \sim \Gamma/ (E-E_r)^2 \)

time is exponentially small
Decay states with real energy

nuclear reaction \( A + B \rightarrow F + R \)

\[
\Psi = \Psi_{AB} + \Psi_{FR}^{(+)}
\]

\[
\Psi_{AB} = \psi_A \psi_B \psi_{AB}, \quad \Psi_{FR} = \psi_{FR}^{(+)} \psi_R
\]

\[
\begin{cases}
(\hat{H}_{AB} - E_{AB}) \psi_{AB} = \langle \psi_A \psi_B | \hat{V} | \Psi_{FR} \rangle,
\end{cases}
\]

\[
(\hat{H}_{FR} - E_{FR}) \psi_{FR}^{(+)} = \langle \psi_R | \hat{V} | \Psi_{AB} \rangle.
\]

\[
\Psi_{AB} \gg \Psi_{FR}^{(+)}
\]

\[
(\hat{H}_{FR} - E_{FR}) \psi_{FR}^{(+)} = \Phi
\]

**Example – sudden removal approximation**

\[
\Phi(q, r_i) = \int d^3r \ e^{iqr} \Psi(r, r_i)
\]

Remove particle \( r \) from WF \( \Psi(r, r_i) \)

Instead of vector \( r \) we get vector \( q \) of transferred momentum in the source
Different facets of resonance phenomenon

- Generic idea
- Lorentian (Breit-Wigner) profile peak in the cross section
- Elastic scattering
- Lifetime
- Lorentian profile peak in the cross section
- Phase shift pass 90°
- Delay time
- Reactions
- Exponentially growing WF long-range tail
- WF concentration in the interior
- Separation of energy and radial degrees of freedom
- S-matrix
- Pole
- Quasistationary WF

- Resonance is not necessarily peak
- Peak is not necessarily resonance
- Resonance phenomena
  - vs
  - Excitation modes