Theoretical basics and modern status of radioactivity studies

Lecture 4: Few-body decays
Few-body theory
Classical few-body dynamics

More than 2
First of all three-body
Less than 6-7
Few-body basics: Jacobi variables
Few-body basics: Jacobi variables

4-body

\[ \text{He} = p + T = d + d = n + \text{He} \]

\[ \alpha\text{-particle} \]

\[ = p + T \quad = n + ^3\text{He} \quad = d + d \]
**Few-body basics: Jacobi variables**

- "Non-normalized" and normalized Jacobi variables in coordinate and momentum space
- Meaning of Jacobi vectors in coordinate and momentum space

\[
\begin{align*}
X &= r_1 - r_2 \\
Y &= \frac{m_1 r_2 + m_2 r_1}{m_1 + m_2} - r_3 \\
R &= \frac{m_1 r_1 + m_2 r_2 + m_3 r_3}{M}
\end{align*}
\]

\[
\begin{align*}
P_x &= \frac{m_1 p_1 - m_2 p_2}{m_1 + m_2} \\
P_y &= \frac{m_3 (p_1 + p_2) - (m_1 + m_2) p_3}{M} \\
P_R &= p_1 + p_2 + p_3
\end{align*}
\]

\[
M = m_1 + m_2 + m_3
\]

\[
A = A_1 + A_2 + A_3
\]

\[
\frac{D(r_1 r_2 r_3)}{D(r y x)} = \left(\frac{D(p_1 p_2 p_3)}{D(p_r p_y p_x)}\right)^{-1} = (A_1 A_2 A_3)^{-3/2}
\]
Few-body basics: Jacobi variables

- Special quadratic forms: plane wave and kinetic energy

\[ p_1 r_1 + p_2 r_2 + p_3 r_3 = P_x X + P_y Y + P_R R \]

- Behavior of these very important quadratic forms is conserved in Jacobi variables

\[ \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} + \frac{p_3^2}{2m_3} = \frac{m_1 + m_2}{2m_1 m_2} p_x^2 + \frac{M}{2(m_1 + m_2)m_3} p_y^2 + \frac{1}{2M} p_R^2 \]

\[ \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} + \frac{p_3^2}{2m_3} = \frac{p_r^2}{2m} + \frac{p_y^2}{2m} + \frac{p_x^2}{2m} \]

- In general case the kinetic energy have a strange mixed form

\[ T = \sum_{ij} k_i k_j / 2M_{ij} \]
\[ \rho = \sqrt{x^2 + y^2} ; \quad \theta_\rho = \arctan(x/y) \]
\[ \kappa = \sqrt{p_x^2 + p_y^2} = \sqrt{2mE} = \sqrt{2m(E_x + E_y)} \]
\[ \theta_\kappa = \arctan \left( \sqrt{E_x/E_y} \right) = \arctan \left( \frac{p_x}{p_y} \right) \]

\[ \Psi(X, Y) = \Psi(\rho, \Omega_\rho) = \frac{1}{\rho^{5/2}} \sum_{K\gamma} \chi_K\gamma(\rho) J_{K\gamma}(\Omega_\rho) \]

\[ J_{Kl_xl_y}^{JM}(\Omega) = \psi_{K}^{l_xl_y}(\theta) \left[ Y_{l_x} \otimes Y_{l_y} \right]_{JM} \]

\[ \psi_{K}^{l_xl_y}(\theta) = N_K^{l_xl_y}(\sin \theta)^{l_x}(\cos \theta)^{l_y} P_{K-(l_x, l_y)}^{l_x+1/2, l_y+1/2}(\cos 2\theta) \]

### 7.3.1 Some lowest harmonics

**Positive parity**

\[ \psi_0^{00}(\theta) = \frac{4}{\sqrt{\pi}} \]
\[ \psi_2^{00}(\theta) = \frac{8}{\sqrt{\pi}} \cos 2\theta \]
\[ \psi_2^{11}(\theta) = \frac{8}{\sqrt{3\pi}} \sin 2\theta \]
\[ \psi_2^{20}(\theta) = \frac{1}{\sqrt{5\pi}} \sin^2 \theta \]
\[ \psi_2^{02}(\theta) = \frac{1}{\sqrt{5\pi}} \cos^2 \theta \]

**Negative parity**

\[ \psi_1^{10}(\theta) = \frac{8}{\sqrt{2\pi}} \sin \theta \]
\[ \psi_1^{01}(\theta) = \frac{8}{\sqrt{2\pi}} \cos \theta \]
\[ \psi_3^{10}(\theta) = \frac{8}{\sqrt{6\pi}} (4 \cos 2\theta + 1) \sin \theta \]
\[ \psi_3^{12}(\theta) = \frac{32}{\sqrt{6\pi}} \cos^2 \theta \sin \theta \]
\[ \psi_3^{01}(\theta) = \frac{8}{\sqrt{6\pi}} (4 \cos 2\theta - 1) \cos \theta \]
HH method

\[
\left[ \frac{d^2}{d\rho^2} - \frac{\mathcal{L}(\mathcal{L}+1)}{\rho^2} + 2M \{ E - V_{K\gamma,K\gamma}(\rho) \} \right] \chi_{K\gamma}(\rho) \\
= \sum_{K',\gamma'} 2MV_{K\gamma,K',\gamma'}(\rho) \chi_{K',\gamma'}(\rho),
\]

\[
V_{K\gamma,K',\gamma'}(\rho) = \int d\Omega_\rho \quad \mathcal{J}^\dagger_{K\gamma}(\Omega_\rho) \left[ \sum_{i>j} \hat{V}(r_{ij}) \right] \mathcal{J}_{K',\gamma'}(\Omega_\rho)
\]

\[ \mathcal{L} = K + 3/2 \]
Lippmann-Schwinger equations

\((\hat{H} - E)\Psi = 0\)
\((\hat{T} + V_0 + V_1 - E)\Psi = (\hat{H}_0 + V_1 - E)\Psi = 0\)
\((\hat{H}_0 - E)\Psi = -V_1 \Psi\)

\(\Psi = \Psi_{pw} - (\hat{H}_0 - E + i\epsilon)^{-1} V_1 \Psi\)

\(\Psi = \Psi_{pw} - \hat{G}_0 V_1 \Psi\)

Faddev equations

\[
\begin{align*}
\{ij\} + l & \rightarrow \{ij\} + l, \\
\rightarrow & \{il\} + j, \\
\rightarrow & \{jl\} + i, \\
\rightarrow & i + j + l,
\end{align*}
\]

\[
\begin{align*}
\alpha_{12}\Phi^{(12)} + \alpha_{23}\Phi^{(23)} + \alpha_{31}\Phi^{(31)},
\Psi = \Phi^{(0)} - \hat{G}_0 (\hat{V}_{12} + \hat{V}_{23} + \hat{V}_{31}) \Psi, \\
\Psi = \Psi^{(12)} - \hat{G}_{12} (\hat{V}_{23} + \hat{V}_{31}) \Psi, \\
\Psi = \Psi^{(23)} - \hat{G}_{23} (\hat{V}_{12} + \hat{V}_{31}) \Psi, \\
\Psi = \Psi^{(31)} - \hat{G}_{31} (\hat{V}_{12} + \hat{V}_{23}) \Psi.
\end{align*}
\]
Irreducible few-body dynamics

- Many nuclear models are based on utilization of the single-particle basis
- For certain situations (certain observables) single-particle basis is far from being adequate

\[ \Psi \neq \Psi(r_1) \Psi(r_2) \]

- Basis states based on collective coordinates. E.g. in hyperspherical harmonics method:

\[ \Psi = \psi(\rho) \mathcal{J}(\Omega) \]

\[ \rho^2 = \frac{A_1 A_2 A_3}{A} \left( \frac{r_{12}^2}{A_3} + \frac{r_{23}^2}{A_1} + \frac{r_{31}^2}{A_2} \right) \]

- Borromean halo nuclei: none of the subsystems are bound.
- Borromean rings logo: integrity and loyalty
Quantum mechanics and boundary conditions

### Two-body

- **Discrete**
  \[ \sim A \ e^{-kr/r} \]

- **Continuum**
  \[ \sim A(\theta) \ e^{ikr/r} \]

### Three-body

- **Discrete**
  \[ \sim A_3 \ e^{-\kappa \rho / \rho^{5/2}} + \sum_{i>j} A_{ij} \ e^{-k_{ij}r_{ij}/r_{ij}} \ e^{-k_k r_k / r_k} \]

- **Continuum**
  \[ \sim A_3 \ e^{ik \rho / \rho^{5/2}} + \sum_{i>j} A_{ij} \ e^{i k_{ij}r_{ij}/r_{ij}} \ e^{ik_k r_k / r_k} \]

- **ANC and scattering amplitudes**
- **Collective variable**: **hyperradius**

\[ \rho^2 = (M \ M_n)^{-1} \sum_{i>j} M_i \ M_j \ r_{ij}^2 \]

- **A_3** - is always not zero
- **A_{ij}** - some are typically equal to zero
- **True three-body** system - if all **A_{ij}** are equal to zero

Dynamics of the processes can not be reduced to the two-body dynamics and studies should be done using methods of the few-body theory
Energy conditions and few-body phenomena

Borromean 2n halo systems

Three-body
Core-n
n-n

Core-n-n

"Soft excitations"

Three-body
Core-N_1
Core-N_2
N_1 - N_2

Three-body virtual states

Three-body
Core-n
n-n

2p radioactivity

True three-body decay

Three-body
Core-p-p
Core-p
p-p

Democratic decays

Three-body
Core-p-p
Core-p
p-p

True 5n-body decay (4n radioactivity)

Five-body
(Core+3N)-N
(Core+2N)-2N
(Core+N)-3N
N_1 - N_2
Few-body dynamics at the driplines

Modern RIB research: move towards and beyond the driplines

Few-body dynamics at the driplines as consequence of corresponding clusterization

Exotic phenomena in vicinity of driplines:
Haloes (green)
True 2p/2n decays (red)
4p/4n emitters (blue)
NOT INVESTIGATED (gray)

NOT SO EXOTIC: More or less every second isotope in vicinity of the driplines has features connected to few-body dynamics
Ternary fission

Not exactly few-body dynamics, but longest-known example of decay into three fragments
Ternary fission yields

Ternary alpha fission

Ternary heavy fragment fission

\[ \text{Ternary } \alpha \text{ fission} \]

\[ \text{Ternary heavy fragment fission} \]
Angular and energy distributions

\[ F_{\text{heavy}} > F_{\text{light}} \]

Classical effect

Light fragment

Heavy fragment

Quantum (?) effects

\( ^{252}\text{Cf} \)

- He4
  - \( E > 8 \text{ MeV} \)
  - \( fwhm = 11.2 \pm 0.1 \text{ MeV} \)
  - \( \langle E \rangle = 15.7 \pm 0.1 \text{ MeV} \)

- Yield [arb. units]
  - Energy [MeV]
  - \( \Theta_{\text{el}} \) [°]
Problem of nonclassical angular distribution for different ternary fragments
Two-proton radioactivity
Two-proton radioactivity: a qualitative view

Classical case: one particle emission is always possible

Quantum mechanical case: it could be that both particles should be emitted simultaneously

- No deeper bound orbitals.
- The common orbital for two protons exists only when both are “inside”.
- When one of them goes out, their common orbital do not exist any more and the second HAS to go out instantaneously.

Goldansky and Zeldovich, 1960
Pfutzner et al and Giovinazzo et al, 2002
Trivial approach

Proton decay

\[ \Gamma_2(E_r) \sim \exp \left[ -\frac{\pi(Z-1)\alpha\sqrt{M}}{\sqrt{E_r}} \right] \]

Two-proton decay. ONLY sharing of energy. OTHERWISE protons are uncorrelated

\[ \frac{d\Gamma_3(E_T)}{d\varepsilon} \sim \exp \left[ -\frac{2\pi(Z-2)\alpha\sqrt{M}}{\sqrt{E_T}} \left( \frac{1}{\sqrt{\varepsilon}} + \frac{1}{\sqrt{1-\varepsilon}} \right) \right], \]

\[ \Gamma_3(E_T) = \int_0^1 d\varepsilon \left[ d\Gamma_3(E_T)/d\varepsilon \right]. \]

Original estimate by Goldansky
Three-body cluster model

- Hyperspherical Harmonic method
- For narrow states
  \[ \Psi^{(+))(\rho, \Omega_{\rho}, t)} = \Psi^{(+))(\rho, \Omega_{\rho})} \exp[-iE_T t - (\Gamma / 2)t] \]
- Schoedinger Equation with complex energy
  \[ \left( \hat{H} - E_T + i\Gamma / 2 \right) \Psi^{(+))(\rho, \Omega_{\rho})} = 0 \]
- Actually solved equation
  \[ \left( \hat{H} - E_{\text{box}} \right) \Psi^{(+))(\rho, \Omega_{\rho})} = -i(\Gamma / 2)\Psi_{\text{box}}^{(+))(\rho, \Omega_{\rho})} \]
  where
  \[ \left( \hat{H} - E_{\text{box}} \right) \Psi_{\text{box}}^{(+))(\rho, \Omega_{\rho})} = 0 \]
- “Natural” definition of width

\[
\Gamma = \frac{j(\rho_{\max})}{N(\rho_{\text{box}})} = \frac{\text{Im} \int d\Omega_{\rho} \Psi^{(+)}(\rho_{\max}) \rho_{\max}^{5/2} \int \frac{d\rho}{\rho_{\text{box}}^{5/2}} \rho_{\text{box}}^{5/2} \Psi^{(+)}(\rho_{\max})}{M \int d\Omega_{\rho} \int_0^{\rho_{\text{box}}} d\rho \rho_{\text{box}}^{5} |\Psi^{(+)}|^2}
\]

Typical precision: stable solution for \( \Gamma/E_T > 10^{-30} \)


M. Pfutzner, L.V. Grigorenko, M. Karny, and K. Riisager, Rev. Mod. Phys. 84 (2012) 567
True 2p decay lifetime systematics

20 orders of the magnitude variation of the lifetime

Different experimental techniques are required: implantation, decay in flight, missing mass

In broad lifetime ranges the true 2p lifetime measurements are not accessible

Nice agreement overall. Problem with $^{12}$O and $^{16}$Ne lifetimes were recently resolved
20 orders of the magnitude variation of the lifetime

Different experimental techniques are required: implantation, decay in flight, missing mass

In broad lifetime ranges the true 2p lifetime measurements are not accessible

Nice agreement overall. Problem with $^{12}$O lifetime is recently resolved, problem with $^{16}$Ne lifetime to be resolved
Three-body correlations
Three-body correlations in decays and reactions

2-body decay: state is defined by 2 parameters - energy and width

3-body decays: 2-dimensional “internal” 3-body correlations

3-body continuum in reactions: there is a selected direction. 5-dimensional correlations: “internal” + “external”

"Internal" energy of 3-bodies
\[ \{k_x, k_y\} \implies E_T = E_x + E_y \]

"Internal" 3-body correlations
\[ \{k_x, k_y\} \implies \varepsilon = E_x / E_T \]
\[ \cos \theta_k = (k_x, k_y) / (k_x, k_y) \]

"External" 3-body correlations
\[ \{q, k_x, k_y\} \implies \{\alpha, \beta, \gamma\} \]

Which kind of useful information (if any) can be obtained from three-body correlations?

For direct reactions, the selected direction is momentum transfer vector

For direct reactions, the selected direction is momentum transfer vector.

\[ k' = k_1 + k_2 + k_3 \]

\[ k' = k + q \]
Three-body correlations. “Internal” correlations

- 2-dimensional “internal three-body correlations” or “energy-angular correlations”

\[ \varepsilon = \frac{E_x}{E_T} \quad \cos(\theta_k) = \frac{(k_x, k_y)}{k_x k_y} \]

- “T” and “Y” Jacobi systems reveal different dynamical aspects

- Three-body variables in coordinate and in momentum space.

Simple way to understand three-body correlations: “quasibinary kinematics” \( \varepsilon \to 0 \) and \( \varepsilon \to 1 \)
Common properties of correlations

- **Energy correlation in the core-p channel** well corresponds to original prediction of Goldansky: energies of the emitted protons tend to be equal.

- **Energy correlation in the p-p channel** in the s-d shell nuclei quantitatively depend on the structure.

- **Energy correlation in the p-p channel** in the p-f shell nuclei qualitatively depend on the structure.

How can we use the correlation information?
Between theory and experiment
Monte-Carlo codes

Observables in reactions:
- Nuclear structure +
- Reaction mechanism +
- Final state interaction

Experimental bias:
- Acceptance +
- Efficiency +
- Resolution +
- Physical backgrounds

- For studies of correlations full quantum-mechanical Monte Carlo simulations are required
- Decompose experimental particle correlation data over hyperspherical amplitudes in the momentum space. HH amplitudes automatically take into account PP, angular momenta in the subsystems and spin. Calculated or parameterized.
- Density matrix formalism:

\[
\frac{dW}{dq\,dE\,d\Omega_5} = \sum_{JM,J'M'} \rho_{JM}^{JM'}(q, E) A^\dagger_{J'M'}(E, \Omega_5) A_{JM}(E, \Omega_5)
\]

- Density matrix has especially simple form in the system of transferred momentum for direct reactions
- Three-body decay -> eightfold differential cross section

M.S. Golovkov et al., PRC 72 (2005) 064612.
L.V. Grigorenko et al., PRC 82 (2010) 014615.
T.A. Golubkova et al., PLB 762 (2016) 263.
How experiment distort correlations

$^{6}\text{Be} = \alpha + p + p$ populated in (p,n) charge-exchange reaction on $^6\text{Li}$

Energy correlations for 0$^+$ state in different angular ranges
Three-body correlations and nuclear structure
**45**Fe: the first found and the best studied

**Pfützner et al., EPJA 14 (2002) 279**
**Giovinazzo et al., 89 (2002) 102501**
**Dossat et al., PRC 72 (2005) 054315**

\[ Q_{2p} = 1.154 \text{ MeV} \]

**Miernik et al., PRL 99 (2007) 192501**

- Special design Optical TPC → nuclear physics “life video”
- Improved lifetime:

\[ \Gamma_{2p} = 1.3^{+0.22}_{-0.16} \times 10^{-19} \text{ MeV} \quad T_{1/2}(2p) = 3.5(5) \text{ ms} \]

- Complete momentum correlations provided

**L.Grigorenko et al., PLB 677 (2009) 30**
**L.Grigorenko et al., PRC 82 (2010) 014615**

**Brown 1991: energy – yes, lifetime – no**

**Grigorenko 2001: energy – no, lifetime – yes**
45Fe: internal correlations

Miernik et al., PRL 99 (2007) 192501
➢ Complete kinematics reconstructed
➢ Both lifetime and correlations provide $W(p^2) \sim 30\%$
High-precision studies of three-body correlations

Three-body decay model provides very precise and parameter-free description of correlations in the well-defined three-cluster nuclear systems. This is extensively checked experimentally.
6Be at MSU: correlations on resonance

Experiment:
R. Charity and coworkers, MSU \( ^7\text{Be}(^9\text{Be},X)^{6}\text{Be} \)

- High statistics (~10^6 events/state)
- High resolution
- Nice agreement with the previous (Texas A&M, Dubna) experimental data


![Graphs showing data distributions for different states and reactions (T and Y).](attachment:image.png)
Three-body Coulomb continuum problem
Approximate boundary conditions

In general case the boundary conditions of the three-body Coulomb problem are analytically unknown

- Boundary conditions are obtained by diagonalization of the three-body Coulomb interaction on the truncated hyperspherical basis.

L. Grigorenko et al., PRC 64 (2001) 054002.

\[ V_{K\gamma, K\gamma'} \sim \frac{\alpha_{K\gamma, K\gamma'}}{\rho} + \delta_{KK'} \delta_{\gamma\gamma'} \frac{(K + 3/2)(K + 5/2)}{\rho^2} + \frac{\beta_{K\gamma, K\gamma'}}{\rho^{N_{K\gamma, K\gamma'} \geq 3}} \]

\[ \tilde{V}_{K\gamma, K\gamma'} \sim \delta_{KK'} \delta_{\gamma\gamma'} \frac{\tilde{\alpha}_{K\gamma}}{\rho} + \frac{\lambda_{K\gamma, K\gamma'}}{\rho^2} + \frac{\tilde{\beta}_{K\gamma, K\gamma'}}{\rho^3} \]

\[ \tilde{\Psi}_{K\gamma}(\rho) \sim H_{\tilde{\alpha}_{K\gamma}, \lambda_{K\gamma, K\gamma'}}(\kappa \rho) \rightarrow \Psi_{K\gamma, K\gamma'}(\rho) \]

- Procedure is exact on the truncated HH basis
- The above procedure was first proposed by Merkuriev. It provide very stable results for the true three-body decays.

Two-body decay WF asymptotic
\[ \Psi^{(+)}(r) \sim H^{(+)}(r) \sim \exp[+ikr] \]

Three-body Coulomb problems is one of “eternal problems” of theoretical and mathematical physics

Diagram:
- \( ^{48}\text{Ni}, E_T = 2.09 \text{ MeV} \)
- Plane wave
- Scale of panel 3
- Diagonal Coulomb
- Estimated numerical accuracy -- 0.3%
- Diagonialized Coulomb

Graph:
- \( \Gamma (\text{MeV} \times 10^{-13}) \)
- \( \rho_{\text{max}} (\text{fm}) \)
Classical extrapolation

- Approximate boundary conditions do not work good enough for momentum distributions.
- Improvement of the momentum distributions by classical trajectory extrapolation.

\[ M_x \ddot{X} = \frac{\alpha Z_1 Z_2 X}{X^3} - \frac{\alpha Z_2 Z_3 c_1 r_{23}}{r_{23}^3} + \frac{\alpha Z_2 Z_1 c_2 r_{31}}{r_{31}^3}, \]

\[ M_y \ddot{Y} = \frac{\alpha Z_2 Z_3 r_{23}}{r_{23}^3} + \frac{\alpha Z_3 Z_1 r_{31}}{r_{31}^3}. \]

- Trajectories 1000, 1400, 2200, 4000, and \( 10^5 \) fm
- Arrive to borderline with atomic phenomena

Electron density -> “plasma” screening

\( ^{45}\text{Fe}, \ E_T = 1.154 \text{ MeV} \)
How far we need to go?

Three radial scales of the calculations for three-body Coulomb problem

Energy of state: 20-30 fm

Width of state: 200-300 fm

Three-body correlations: $10^4$-$10^5$ fm
Long-range character of 3-body Coulomb by example of $^{45}$Fe

- Start point for extrapolation: typical range of 1000 fm in $\rho$ value
- End point for extrapolation: typical range of 100000 fm in $\rho$ value
- Complicated treatment of experimental effects

$^{45}$Fe, $E_T = 1.154$ MeV

Consistence, with data but no solid evidence

Long-range character of three-body Coulomb by example of $^{16}\text{Ne}$

- New level of experimental precision. MSU 2013: $^{16}\text{Ne}$ populated in n knockout from $^{17}\text{Ne}$

  K. Brown et al., PRL 113 (2014) 232501

- The energy distribution in “Y” Jacobi system only reproduced for extreme range of calculation

$^{16}\text{Ne}$ g.s., $E_T = 1.476$ MeV

![Graph showing energy distribution with theoretical and experimental data]
Three-body decay mechanisms
Energy conditions and few-body phenomena

**2p radioactivity/true 2p decay**

\[ E_T = -S_{2p} > 0 \]
\[ S_{2p} < 0 \]

\[ E_r \]
\[ A \rightarrow (A-1) + N \rightarrow (A-2) + 2N \]

**Democratic decays**

\[ E_r \]
\[ A \rightarrow (A-1) + p \rightarrow (A-2) + 2p \]
\[ S_{2p} > 0 \]
\[ S_{2p} < 0 \]

**Sequential decay**

\[ E_T \]
\[ E_r \]
\[ S_{2p} < 0 \]
\[ A \rightarrow (A-1) + N \rightarrow (A-2) + 2N \]

**Light 2p and majority of 2n emitters**
6Be at MSU: correlations on resonance

Experiment:
R. Charity and coworkers, MSU 7Be(9Be,X)6Be


- High statistics (~10^6 events/state)
- High resolution
- Nice agreement with the previous (Texas A&M, Dubna) experimental data
6Be at MSU: energy evolution of correlations

Note: the higher decay energy – the more developed is low-energy p-p correlation ("diproton")

Note: above 2$^+$ the $\varepsilon$ distribution is practically insensitive to decay energy

Note: when two-body states enters the decay window the intensity at expected peak position is suppressed

Note: sequential decay patterns appears only for $E_T > 2E_r + \Gamma$
Sequential decay pattern in energy distribution is formed at 6-9 meV.

Is the decay sequential? NO.

Angular correlation shows complex behavior

Energy angular correlations elucidate the actual situation: mixture of p-p and a-p FSIs

Highly detailed data allows deep insights in the decay dynamics
Interplay between sequential and prompt two-proton decay from the first excited state of $^{16}\text{Ne}$

K. Brown et al., PRC 113 (2015) 034329

Data vs full three-body vs sequential
"Tethered decay mechanism" for the $^{16}\text{Ne} \; 2^+$ state

\[
\theta_\rho \sim \pi/4 \quad \rightarrow \quad X \sim Y \sim \rho/\sqrt{2}.
\]

\[
\theta_\rho \rightarrow \theta_\chi, \quad E_x = E_T \sin^2(\theta_\chi), \quad E_y = E_T \cos^2(\theta_\chi).
\]
Simplified approaches to three-body decays

One do not necessarily need extremely complicated computational codes to get decent idea about certain aspects of three-body decays. Important results could be obtained based on the models with simplified three-body Hamiltonians leading to compact analytical expressions.
Simplified approach to 2p decay: direct decay model

Green’s function for the three-body Coulomb problem is unknown in a general analytical form.
Approximations

- Proton-proton interaction is neglected (reasonable in heavy-core systems).
- One of core-proton potentials assumed to depend on Jacobi Y coordinate (becomes precise as core mass goes to infinity).

Analytical Green’s function
Resonance is provided by three-body potential depending on ρ.
“Correction procedure”

\[ \bar{G}_E^{(+)} (XX', YY') = \frac{1}{2\pi i} \int_{-\infty}^{\infty} d\varepsilon \bar{G}_E^{(+)} (XX') \bar{G}_E^{(+)} (YY') \]

\[ \Psi_{\text{corr}}^{(+)} (X, Y) = \int dX' dY' \bar{G}_E^{(+)} (XX', YY') [\vec{V} - V] \Psi_{\text{appr}}^{(+)} (X', Y') \]

\[ j_{\text{corr}} = \frac{1}{M_x} \sum_x R^2_x \frac{1}{M} \text{Im} \left[ X^2 d\Omega \int dY \Psi_{\text{corr}}^{(+)*} \frac{1}{v_x(\varepsilon)v_y(\varepsilon)} \Psi_{\text{corr}}^{(+)} \right] \]

\[ \Gamma_{\text{corr}} = \frac{j_{\text{corr}}}{N_{\text{corr}}} = \frac{8}{\pi} \int d\varepsilon \frac{1}{v_x(\varepsilon)v_y(\varepsilon)} |A(\varepsilon)|^2 \]

\[ A(\varepsilon) = \int_0^\infty dX dY \varphi_{l_x}(k_x X) \varphi_{l_y}(k_y Y) [\vec{V} - V] \tilde{\psi}_l(X, Y) \]

\[ \Gamma_{\text{dir}} (E_T) = \frac{E_T \langle V_3 \rangle^2}{2\pi} \int_0^1 d\varepsilon \frac{\Gamma_{p1}(\varepsilon E_T)}{(\varepsilon E_T - E_{p1})^2 + \Gamma_{p1}(\varepsilon E_T)^2/4} \times \frac{\Gamma_{p2}((1-\varepsilon)^2 E_T)}{((1-\varepsilon)E_T - E_{p2})^2 + \Gamma_{p2}((1-\varepsilon)E_T)^2/4} \]
Diproton model: no, thank you

- Proposed as one of a possible approximations in pioneering paper on 2p decay
  Goldansky, NPA 19 (1960) 482.
- Factorization of the degrees of freedom in “T” Jacobi system. Exact Green’s
  function exist for the system.
- Derivation of simple expression for preexponent is not possible.
- Used properly this model underestimate width typically 2 orders of the
  magnitude.
- Application of this formula is pure phenomenology without theoretical basis.
Correlations in the simplified models

- Three-body model reproduce experimental distributions in details.
- Direct decay model provide some distributions correctly and some not.
- Diproton model does not provide any momentum distributions correctly.
Precision check of the three-body calculations with simplified Hamiltonians

- For direct decay model

- For diproton model

L.V. Grigorenko, et al., PRC 82, 014615 (2010)
Transitional dynamics and “phase transition” diagram for the three-body decay

Studies of decays which dynamics lies in between well defined reaction mechanisms may lead to important results as in the case of such transitional dynamics observables have strong sensitivity to parameters typical for phase transition situations
Mechanisms of 2p decay defined by separation energies

Three major decay mechanisms: True 2p, Democratic 2p, Sequential 2p

Three principal parameters:
\[ E_T, E_r, \Gamma_r \]

There SHOULD EXIST transition region between them
Mechanisms of $2p$ decay

Democratic $2p \leftrightarrow$ Sequential $2p$

Energy correlations between core and one proton

Lifetime systematics

$a)$

$E_r$ : $\Gamma(E_r) \sim (0.2 \div 0.3)E_r$

$E_r \sim 0.5E_T$

Sequential

$b)$

$E_{2\gamma} = 1.54$ MeV

$E_{2\gamma} \sim 0.8E_T$

$c)$

$E_T$ (MeV) $\Gamma$ (MeV)$^*$

$E_{2\gamma} = 1.3$ MeV
Transition decay mechanism beyond the dripline

Systematics of proton and two-proton separation energies

Even number of protons. Upper and lower s-d shell

Systematic change of dynamics on the way to the dripline and beyond: “normal” -> Borromean -> true 2p -> transitional -> sequential 2p

All three mechanisms of 2p emission as well as transition situation change each other on the move away from the dripline
Transition decay dynamics in simplified semianalytical models

\[ \Gamma_{j_1j_2}(E_T) = \frac{E_T \langle V_3 \rangle^2}{2\pi} \int_0^1 d\varepsilon \frac{\Gamma_{j_1}(\varepsilon E_T)}{(\varepsilon E_T - E_{j_1})^2 + \Gamma_{j_1}(\varepsilon E_T)^2/4} \times \frac{\Gamma_{j_2}((1 - \varepsilon)E_T)}{((1 - \varepsilon)E_T - E_{j_2})^2 + \Gamma_{j_2}((1 - \varepsilon)E_T)^2/4} \]

Stems from simplified three-body Hamiltonian

Basic feature - strong dependence on two resonances in the subsystems

Recent upgrade to “improved direct decay model”

T.A. Golubkova et al., PLB 762 (2016) 263

Correct angular momentum coupling, amplitude symetries, NN FSI correction, etc.

\[ \frac{d\Gamma(E_{3r})}{d\Omega_\star} = \sum_{LS} \frac{E_{3r}}{2\pi(2L + 1)} \sum_{ML} \left| \sum_\gamma A_{S\gamma}^{LM_L}(\Omega_\star) \right|^2 , \]

\[ A_{S\gamma}^{LM_L}(\Omega_\star) = C_{\gamma}^{JLS} V_{\gamma}^{J} [l_1 \otimes l_2]_{LM_L} A_{j_1 l_1}(E_1) A_{j_2 l_2}(E_2) , \]

\[ [l_1 \otimes l_2]_{LM_L} = \sum_{m_1 m_2} C_{l_1 m_1 l_2 m_2}^{LM_L} Y_{l_1 m_1}(\hat{r}_1) Y_{l_2 m_2}(\hat{r}_1) . \]

\[ A_{S\gamma}^{LM_L}(\Omega_\star) \rightarrow \frac{C_{\gamma}^{JLS} V_{\gamma}^{J} A_{S}^{(pp)}(E_x)}{E_{r_1} + E_{r_2} - E_T - i [\Gamma_1(E_{r_1}) + \Gamma_2(E_{r_2})]/2} \times \hat{O}_S \left( [l_{x_1}^{Y_1} \otimes l_{y_1}^{Y_1}]_{LM_L} A_{j_x l_x}(E_{X_1}) \sqrt{\Gamma_1(E_{Y_1})} \right. \]

\[ + \left. [l_{x_2}^{Y_2} \otimes l_{y_2}^{Y_2}]_{LM_L} A_{j_y l_y}(E_{X_2}) \sqrt{\Gamma_2(E_{Y_2})} \right) . \]

\[ A_{jl}(E) = \frac{\sqrt{\Gamma_r(E)}}{E_r - E - i\Gamma_r(E)/2} + A_{jl}^{(p)}(E) \]
General view of transitional dynamics

Energy correlations between core and one proton

"30Ar"

Transition

True 2p

Sequential 2p

$E_r: \Gamma(E_r) \sim (0.2 \div 0.3) E_r$

$E_r \sim 0.5 E_T$

$E_r \sim 0.8 E_T$

Democratic

Sequential

True 2p

Sequential 2p

$\Gamma_r = 60$ keV

$\Gamma_r = 170$ keV

$\Gamma_r = 330$ keV

$\Gamma_r = 500$ keV

$E = \frac{E(\text{core-p})}{E_T}$
Two-proton events:

1. Fragmentation in the target
   \[ A \rightarrow A+1 \]
2. Exponential "tail" due to decay in the flight
   \[ \frac{A-2}{Z-2}X \rightarrow \frac{A}{Z}X \]

Basic idea is old, technological realization is modern and sophisticated

New method of radioactivity studies:
Investigation of radioactive decays with particle emission by product tracking

Simultaneously (i) lifetime in the ps range (ii) spectroscopy of the short-lived states

Extremely thick secondary targets up to ~25 mm

Efficient work with poor “cocktail” beams

Recent experiment in this technique
A. Lis et al., PRC 91 (2014) 064309.

Simultaneous results from tracking system (decay spectroscopy) and OTPC (β-delayed decays) – two experiments for the price of one
Energy is “easy” to measure, width could be very complicated. From $T_{1/2} \sim 1$ ps to $\Gamma \sim 100 - 200$ keV there is a “blind spot” no accessible for direct measurements.

$^{30}$Ar was found to have transition decay dynamics.

Strong dependence of the experimental signal on the g.s. properties of core+p subsystem – $^{29}$Cl.

Stringent limits for $^{29}$Cl g.s. width.

T.A. Golubkova et al., PLB 762 (2016) 263.
Prospects to observe transition dynamics in $^{15}$Ne

F. Wamers et al., PRL 112 (2014) 132502

$^{16}$Ne studies and $^{15}$Ne discovery, GSI

V. Goldberg et al., PLB 692 (2010) 307

$^{14}$F, TEXAS A&M

Levels in $^{14}$F.

<table>
<thead>
<tr>
<th>$E_R$ (MeV)</th>
<th>$E_\gamma$ (MeV)</th>
<th>$J^\pi$</th>
<th>$\Gamma$ (keV)</th>
<th>$\Gamma/\Gamma_{2p}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.56 ± 0.04</td>
<td>0.00</td>
<td>2$^-$</td>
<td>910 ± 100</td>
<td>0.85</td>
</tr>
<tr>
<td>2.1 ± 0.17</td>
<td>0.54</td>
<td>1$^-$</td>
<td>~1000</td>
<td>0.6</td>
</tr>
<tr>
<td>3.05 ± 0.060</td>
<td>1.49</td>
<td>3$^-$</td>
<td>210 ± 40</td>
<td>0.55</td>
</tr>
<tr>
<td>4.35 ± 0.10</td>
<td>2.79</td>
<td>4$^-$</td>
<td>550 ± 100</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Proposal: to study energy evolution of three-body correlations across the energy of broad ($\Gamma\sim0.6$ MeV) g.s. of $^{15}$Ne to extract $^{14}$F width
Problem of $^{67}\text{Kr}$

$^{67}\text{Kr}$
Decay energy 1690(17)
Decay width 7.3(3.0) ms

$^{67}\text{Kr}$ width is larger than any prediction of the three-body model. Possible reason: not true 2p, but transitional dynamics of 2p decay.

Proposal: to study correlations as the indicative signature of decay mechanism


Multi-neutron radioactivity

It could be that it is more probable to find very long-living (radioactivity timescale) four-neutron emitters than two-neutron.
For “true” N-body systems which dynamics is well described by finite set of hyperspherical equations the effective centrifugal barriers

\[
\left[ \frac{d^2}{d\rho^2} - \frac{\mathcal{L}(\mathcal{L} + 1)}{\rho^2} + 2M \left\{ E_T - V_{K\gamma,K\gamma}(\rho) \right\} \right] \chi_{K\gamma}(\rho) \\
= \sum_{K'\gamma'} 2MV_{K\gamma,K'\gamma'}(\rho)\chi_{K'\gamma'}(\rho) + f_{K\gamma}(\rho),
\]

\[
V_{K\gamma,K'\gamma'}(\rho) = \int d\Omega_\rho \mathcal{J}^{\dagger}_{K\gamma}(\Omega_\rho) \left[ \sum_{i>j} \hat{V}(r_{ij}) \right] \mathcal{J}_{K'\gamma'}(\Omega_\rho)
\]

\[
\mathcal{L} = K + (3A - 6)/2
\]
What about neutron radioactivity?

- Estimates: one-neutron radioactivity is highly unlikely.
- There are additional effective few-body “centrifugal” barriers making few-body emission relatively slower.
- Long-living Two-neutron decay states are reasonably probable.

L.V. Grigorenko, I.G. Mukha, C. Scheidenberger, and M.V. Zhukov, PRC 84 (2011) 021303(R)

26O Recent studies of 2n decay in:
- Lunderberg et al., PRL 108, 142503 (2012)
- C. Caesar et al., PRC 88, 034313 (2013)
- Z. Kohley et al., PRL 110, 152501 (2013)
- Y. Kondo et al., PRL 116, 102503 (2016)
2n radioactivity in $^{26}$O?

**Z. Kohley et al., PRL 110, 152501 (2013)**

$T_{1/2} = 4.5$ ps: 2n radioactivity discovered

Y. Kondo et al., PRL 116, 102503 (2016)

$E_T = 22^{+14}_{-11}$ keV: 2n radioactivity unlikely

Importance of fine three-body effects

2p radioactivity:
- Core recoil – negligible
- NN FSI – factor 200-500

2n radioactivity:
- Core recoil – factor 5-10
- NN FSI – factor 2000-10000

L.V. Grigorenko, I.G. Mukha, M.V. Zhukov,
PRL 111 (2013) 042501

26 $^8_8$O$_{18}$

true 2n
decay

Exp. RIKEN

Exp. MSU

$[s^2]$ $[d^2]$ $[f^2]$ $[p^2]$
**Four-neutron radioactivity search prospects**

- **$n$ radioactivity** realistic only in f- and higher waves. Not achieved regions of the dripline.

- **$2n$ radioactivity.** Simple estimate spoiled by paring interaction leading to $d^2 \rightarrow s^2$ “diffusion”. Realistic energy $< 1$ keV.

- **$4n$ radioactivity.** Minimal effective barrier is high. Also minimal quantum configuration is Pauli-prohibited. Minimal is $[s^2p^2]$ and $L = 13/2$.

- **$4n$ radioactivity.** Realistic energy window is 100-200 keV.