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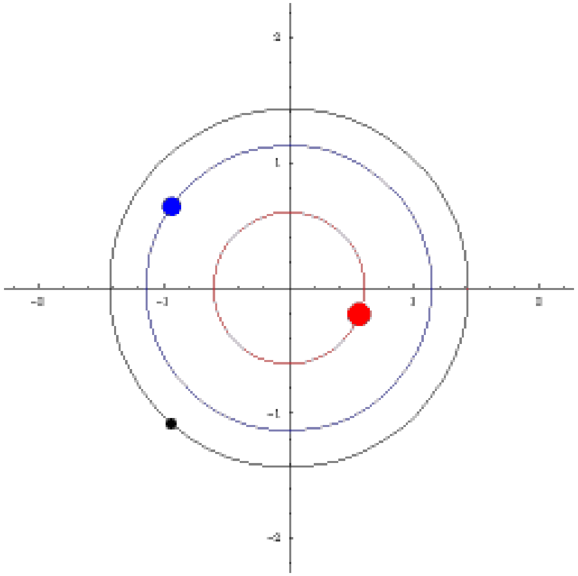
Theoretical basics and modern status of radioactivity studies

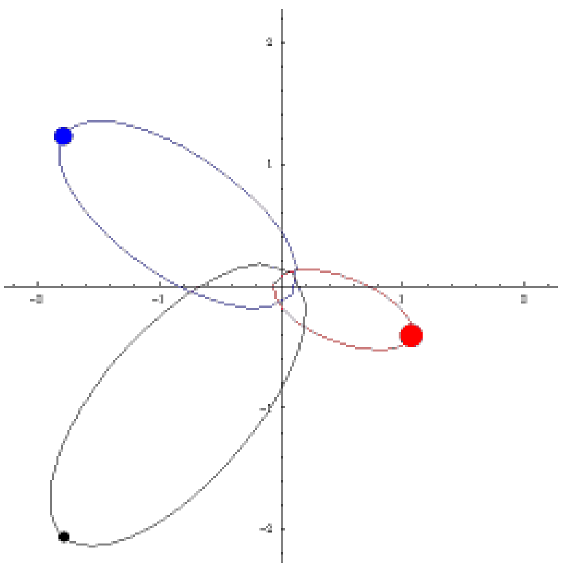
Lecture 4: Few-body decays

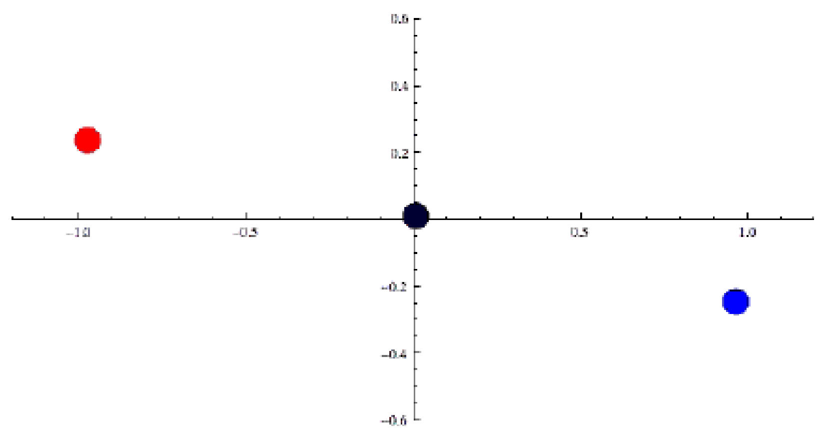
Few-body theory

Classical few-body dynamics

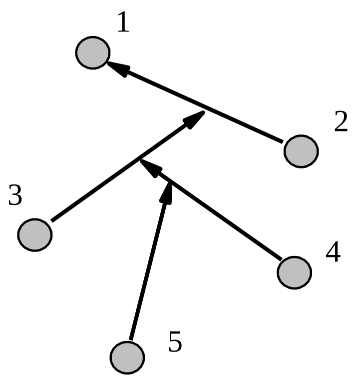


More than 2
First of all three-body
Less than 6-7

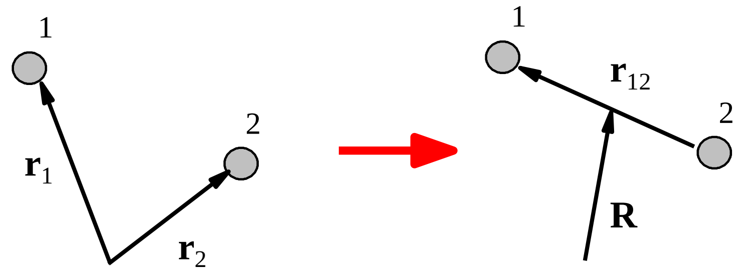




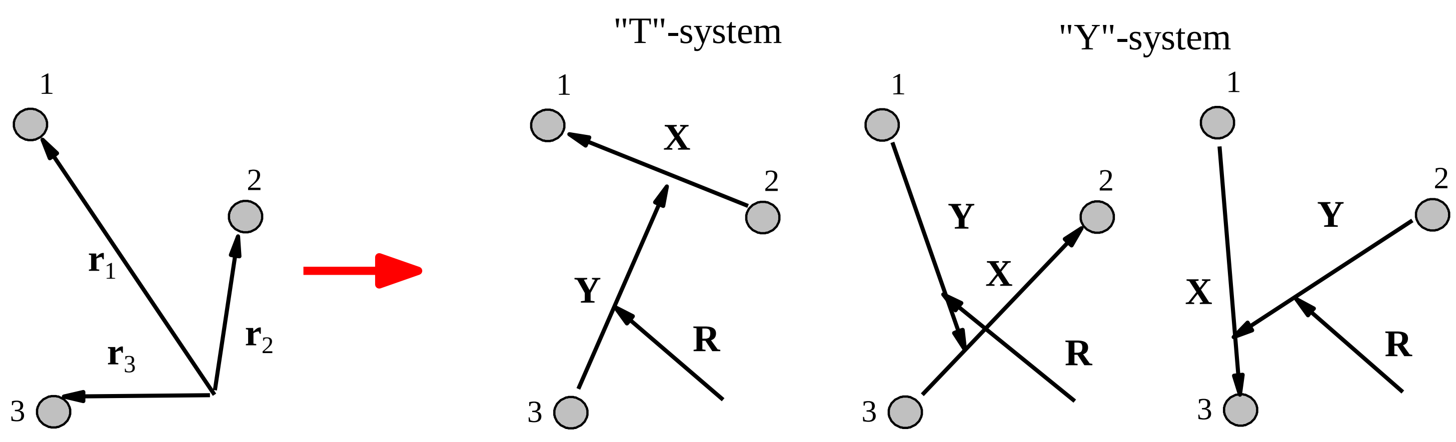
Few-body basics: Jacobi variables



2-body

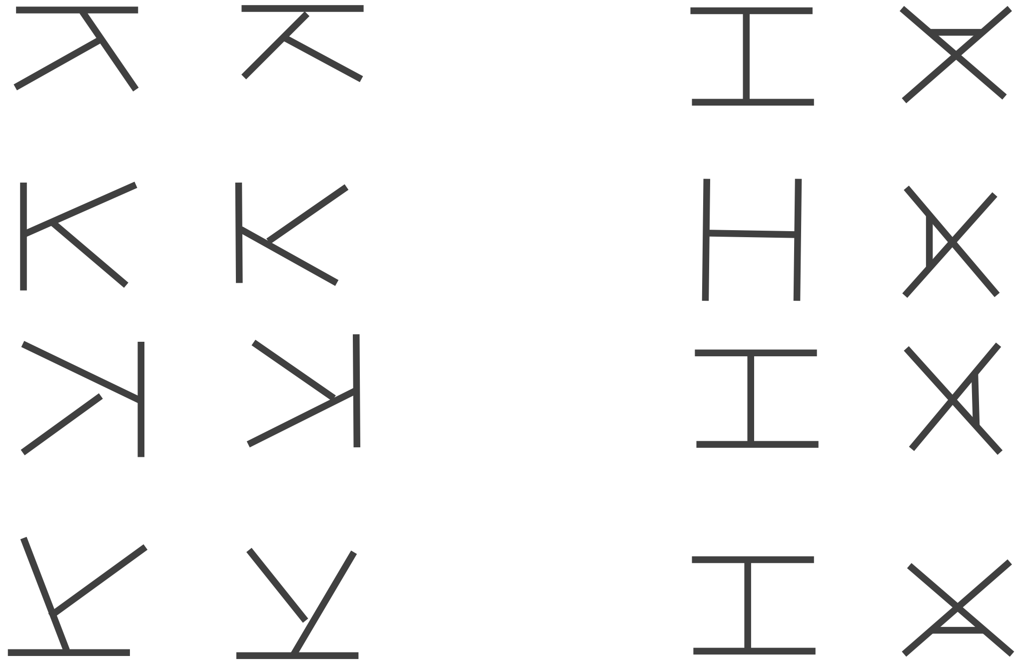


3-body

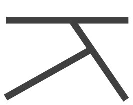
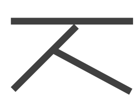
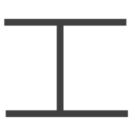


Few-body basics: Jacobi variables

4-body



α-particle

 = p + T  = n + ^{3}He  = d + d

Few-body basics: Jacobi variables

- "Non-normalized" and normalized Jacobi variables in coordinate and momentum space
- Meaning of Jacobi vectors in coordinate and momentum space

$$\begin{cases} \mathbf{X} = & \mathbf{r}_1 - \mathbf{r}_2 \\ \mathbf{Y} = & \dfrac{m_1\mathbf{r}_2 + m_2\mathbf{r}_1}{m_1 + m_2} - \mathbf{r}_3 \\ \mathbf{R} = & \dfrac{m_1\mathbf{r}_1 + m_2\mathbf{r}_2 + m_3\mathbf{r}_3}{M} \end{cases}$$

$$\begin{cases} \mathbf{P}_x = & \dfrac{m_1\mathbf{p}_1 - m_2\mathbf{p}_2}{m_1 + m_2} \\ \mathbf{P}_y = & \dfrac{m_3(\mathbf{p}_1 + \mathbf{p}_2) - (m_1 + m_2)\mathbf{p}_3}{M} \\ \mathbf{P}_R = & \mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3 \end{cases}$$

$$\begin{cases} \mathbf{x} = & \sqrt{\dfrac{A_1A_2}{A_1 + A_2}}\,(\mathbf{r}_1 - \mathbf{r}_2) \\ \mathbf{y} = & \sqrt{\dfrac{(A_1 + A_2)A_3}{A}}\left(\dfrac{A_1\mathbf{r}_1 + A_2\mathbf{r}_2}{A_1 + A_2} - \mathbf{r}_3\right) \\ \mathbf{r} = & \dfrac{A_1\mathbf{r}_1 + A_2\mathbf{r}_2 + A_3\mathbf{r}_3}{\sqrt{A}} \end{cases}$$

$$M = m_1 + m_2 + m_3$$

$$A = A_1 + A_2 + A_3$$

$$\begin{cases} \mathbf{p}_x = & \sqrt{\dfrac{A_1 + A_2}{A_1A_2}}\,\dfrac{A_1\mathbf{p}_1 - A_2\mathbf{p}_2}{A_1 + A_2} \\ \mathbf{p}_y = & \sqrt{\dfrac{A}{(A_1 + A_2)A_3}}\,\dfrac{A_3(\mathbf{p}_1 + \mathbf{p}_2) - (A_1 + A_2)\mathbf{p}_3}{A} \\ \mathbf{p}_r = & \dfrac{\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3}{\sqrt{A}} \end{cases}$$

$$\frac{D\,(\mathbf{r}_1\mathbf{r}_2\mathbf{r}_3)}{D\,(\mathbf{RYX})} = \frac{D(\mathbf{p}_1\mathbf{p}_2\mathbf{p})}{D(\mathbf{P}_R\mathbf{P}_y\mathbf{P}_x)} = 1$$

$$\frac{D\,(\mathbf{r}_1\mathbf{r}_2\mathbf{r}_3)}{D(\,\mathbf{r}\,\mathbf{y}\,\mathbf{x}\,)} = \left(\frac{D(\mathbf{p}_1\mathbf{p}_2\mathbf{p}_3)}{D(\mathbf{p}_r\mathbf{p}_y\mathbf{p}_x)}\right)^{-1} = (A_1A_2A_3)^{-3/2}$$

Few-body basics: Jacobi variables

- Special quadratic forms: plane wave and kinetic energy

$$\mathbf{p}_1\mathbf{r}_1+\mathbf{p}_2\mathbf{r}_2+\mathbf{p}_3\mathbf{r}_3=\mathbf{P}_x\mathbf{X}+\mathbf{P}_y\mathbf{Y}+\mathbf{P}_R\mathbf{R} \qquad \mathbf{p}_1\mathbf{r}_1+\mathbf{p}_2\mathbf{r}_2+\mathbf{p}_3\mathbf{r}_3=\mathbf{p}_x\mathbf{x}+\mathbf{p}_y\mathbf{y}+\mathbf{p}_r\mathbf{r}$$

- Behavior of these very important quadratic forms is conserved in Jacobi variables

$$\frac{\mathbf{p}_1^2}{2m_1}+\frac{\mathbf{p}_2^2}{2m_2}+\frac{\mathbf{p}_3^2}{2m_3}=\frac{m_1+m_2}{2m_1m_2}\mathbf{P}_x^2+\frac{M}{2(m_1+m_2)m_3}\mathbf{P}_y^2+\frac{1}{2M}\mathbf{P}_R^2$$

$$\frac{\mathbf{p}_1^2}{2m_1}+\frac{\mathbf{p}_2^2}{2m_2}+\frac{\mathbf{p}_3^2}{2m_3}=\frac{\mathbf{p}_r^2}{2m}+\frac{\mathbf{p}_y^2}{2m}+\frac{\mathbf{p}_x^2}{2m}$$

- In general case the kinetic energy have a strange mixed form

$$T=\sum_{ij}\mathbf{k}_i\mathbf{k}_j/2M_{ij}$$

HH method

Hyperspherical variables

$$\rho = \sqrt{x^2 + y^2} \quad ; \qquad \theta_\rho = \arctan(x/y)$$

$$\varkappa = \sqrt{p_x^2 + p_y^2} = \sqrt{2mE} = \sqrt{2m(E_x + E_y)}$$

$$\theta_\varkappa = \arctan\left(\sqrt{E_x/E_y}\right) = \arctan\left(p_x/p_y\right)$$

$$\rho^2 = \frac{A_1 A_2 A_3}{A}\left(\frac{\mathbf{r}_{12}^2}{A_3} + \frac{\mathbf{r}_{23}^2}{A_1} + \frac{\mathbf{r}_{31}^2}{A_2}\right)$$

HH expansion for three-body WF: generalization of the spherical function expansiuon

$$\Psi(\mathbf{X},\mathbf{Y}) = \Psi(\rho,\Omega_\rho) = \frac{1}{\rho^{5/2}} \sum_{K\gamma} \chi_{K\gamma}(\rho)\mathcal{J}_{K\gamma}(\Omega_\rho)$$

$$\mathcal{J}_{Kl_xl_y}^{JM}(\Omega) = \psi_K^{l_xl_y}(\theta)\ \left[Y_{l_x} \otimes Y_{l_y}\right]_{JM}$$

$$\psi_K^{l_xl_y}(\theta) = N_K^{l_xl_y}(\sin\theta)^{l_x}(\cos\theta)^{l_y} P_{\frac{K-l_x-l_y}{2}}^{l_x+1/2,l_y+1/2}(\cos 2\theta)$$

7.3.1 Some lowest harmonics

Positive parity

$$\psi_0^{00}(\theta) = \frac{4}{\sqrt{\pi}}$$
$$\psi_2^{00}(\theta) = \frac{8}{\sqrt{\pi}}\cos 2\theta$$
$$\psi_2^{11}(\theta) = \frac{8}{\sqrt{3\pi}}\sin 2\theta$$
$$\psi_2^{20}(\theta) = \frac{16}{\sqrt{5\pi}}\sin^2\theta$$
$$\psi_2^{02}(\theta) = \frac{16}{\sqrt{5\pi}}\cos^2\theta$$

Negative parity

$$\psi_1^{10}(\theta) = \frac{8}{\sqrt{2\pi}}\sin\theta$$
$$\psi_1^{01}(\theta) = \frac{8}{\sqrt{2\pi}}\cos\theta$$
$$\psi_3^{10}(\theta) = \frac{8}{\sqrt{6\pi}}(4\cos 2\theta + 1)\sin\theta$$
$$\psi_3^{12}(\theta) = \frac{32}{\sqrt{6\pi}}\cos^2\theta\sin\theta$$
$$\psi_3^{01}(\theta) = \frac{8}{\sqrt{6\pi}}(4\cos 2\theta - 1)\cos\theta$$

HH method

$$\left[\frac{d^2}{d\rho^2} - \frac{\mathcal{L}(\mathcal{L}+1)}{\rho^2} + 2M\{E - V_{K\gamma,K\gamma}(\rho)\}\right]\chi_{K\gamma}(\rho)$$

$$= \sum_{K'\gamma'} 2MV_{K\gamma,K'\gamma'}(\rho)\chi_{K'\gamma'}(\rho),$$

$$V_{K\gamma,K'\gamma'}(\rho) = \int d\Omega_\rho \quad \mathcal{J}^\dagger_{K\gamma}(\Omega_\rho)\left[\sum_{i>j} \hat{V}(\mathbf{r}_{ij})\right]\mathcal{J}_{K'\gamma'}(\Omega_\rho)$$

HH partial equations: motion in effective "strongly deformed" field

$$\mathcal{L} = K + 3/2$$

Effective centrifugal barrier is always nonzero !

Lippmann-Schwinger equations

$$(\widehat{H} - E)\Psi = 0$$

$$(\widehat{T} + V_0 + V_1 - E)\Psi = (\widehat{H}_0 + V_1 - E)\Psi = 0$$

$$(\widehat{H}_0 - E)\Psi = -V_1\Psi$$

$$\Psi = \Psi_{pw} - (\widehat{H}_0 - E + i\varepsilon)^{-1} V_1 \Psi \qquad \Psi = \Psi_{pw} - \widehat{G}_0 V_1 \Psi$$

Faddev equations

$$\begin{array}{l} \{ij\} + l \to \{ij\} + l, \\ \quad \to \{il\} + j, \\ \quad \to \{jl\} + i, \\ \quad \to i + j + l, \end{array}$$

$$\left.\begin{aligned} \Psi &= \Phi^{(0)} - \widehat{G}_0 (\widehat{V}_{12} + \widehat{V}_{23} + \widehat{V}_{31}) \Psi, \\ \Psi &= \Psi^{(12)} - \widehat{G}_{12} (\widehat{V}_{23} + \widehat{V}_{31}) \Psi, \\ \Psi &= \Psi^{(23)} - \widehat{G}_{23} (\widehat{V}_{12} + \widehat{V}_{31}) \Psi, \\ \Psi &= \Psi^{(31)} - \widehat{G}_{31} (\widehat{V}_{12} + \widehat{V}_{23}) \Psi. \end{aligned}\right\}$$

$$\alpha_{12}\Phi^{(12)} + \alpha_{23}\Phi^{(23)} + \alpha_{31}\Phi^{(31)},$$

$$\left.\begin{aligned} \Psi &= \alpha_{12}\Phi^{(12)} - \widehat{G}_{12} (\widehat{V}_{23} + V_{31}) \Psi, \\ \Psi &= \alpha_{23}\Phi^{(23)} - \widehat{G}_{23} (\widehat{V}_{12} + \widehat{V}_{31}) \Psi, \\ \Psi &= \alpha_{31}\Phi^{(31)} - \widehat{G}_{31} (\widehat{V}_{12} + \widehat{V}_{23}) \Psi. \end{aligned}\right\}$$

Irreducible few-body dynamics

- Many nuclear models are based on utilization of the single-particle basis
- For certain situations (certain observables) single-particle basis is far from being adequate

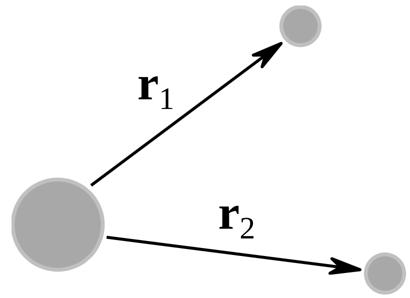
$$\Psi \neq \Psi(\mathbf{r}_1)\,\Psi(\mathbf{r}_2)$$

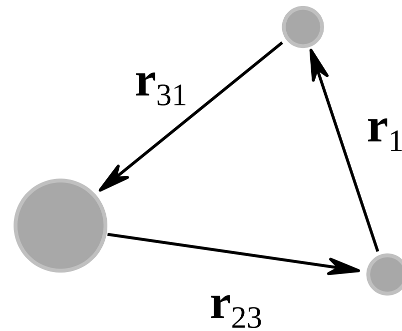
- Basis states based on collective coordinates. E.g. in hyperspherical harmonics method:

$$\Psi = \psi(\rho)\,\mathcal{J}(\Omega_5)$$

$$\rho^2 = \frac{A_1 A_2 A_3}{A}\left(\frac{\mathbf{r}_{12}^2}{A_3} + \frac{\mathbf{r}_{23}^2}{A_1} + \frac{\mathbf{r}_{31}^2}{A_2}\right)$$

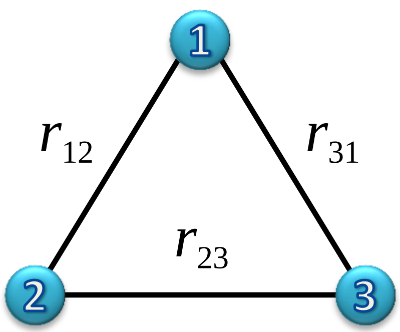
- Borromean halo nuclei: none of the subsystems are bound.
- Borromean rings logo: integrity and loyalty





Quantum mechanics and boundary conditions

	Two-body	Three-body
Discrete	$\sim A\, e^{-kr}/r$	$\sim A_3\, e^{-\kappa\rho}/\rho^{5/2} + \sum_{i>j} A_{ij}\, e^{-k_{ij}r_{ij}}/r_{ij}\; e^{-k_k r_k}/r_k$
Continuum	$\sim A(\theta)\, e^{ikr}/r$	$\sim A_3\, e^{i\kappa\rho}/\rho^{5/2} + \sum_{i>j} A_{ij}\, e^{i\,k_{ij}r_{ij}}/r_{ij}\; e^{ik_k r_k}/r_k$



- **ANC** and **scattering amplitudes**
- Collective variable: **hyperradius**

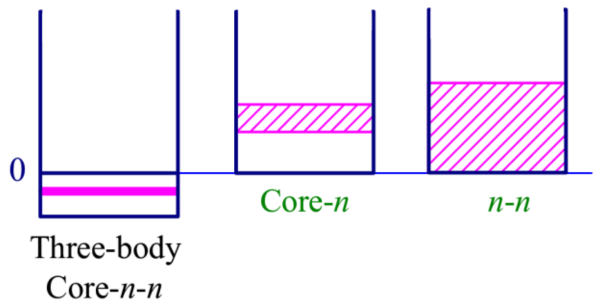
$$\rho^2 = (M\,M_n)^{-1}\sum_{i>j} M_i\, M_j\, r_{ij}^{\,2}$$

- A_3 - is always not zero
- A_{ij} - some are typically equal to zero
- **True three-body** system - if all A_{ij} are equal to zero

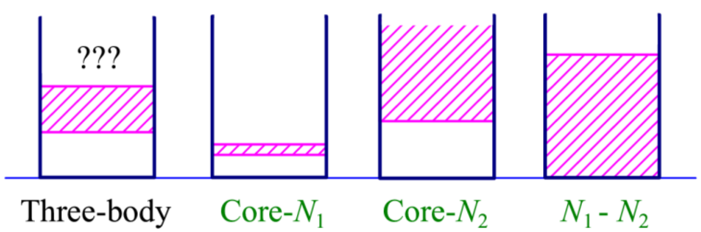
Dynamics of the processes can not be reduced to the two-body dynamics and studies should be done using methods of the few-body theory

Energy conditions and few-body phenomena

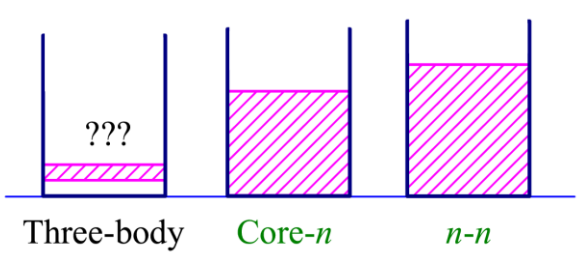
Borromean 2n halo systems



"Soft excitations"

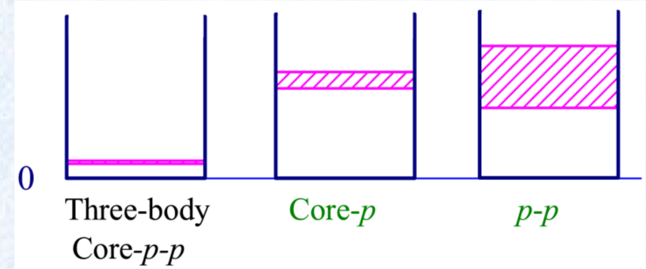


Three-body virtual states

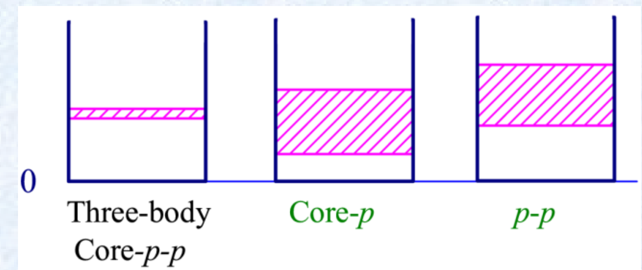


True three-body decay

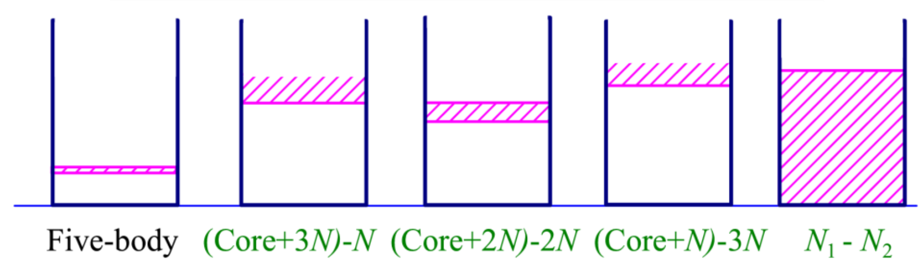
2p radioactivity



Democratic decays



True 5n-body decay (4n radioactivity)



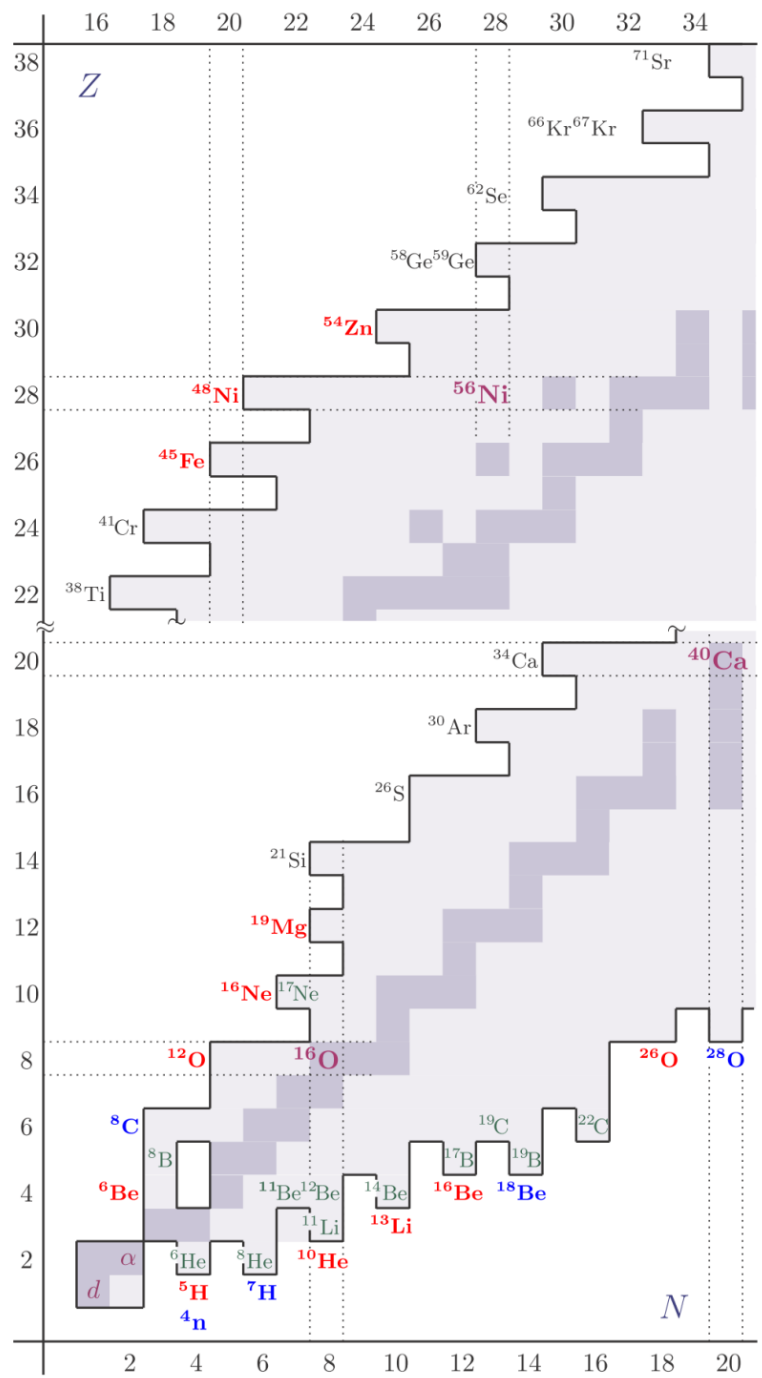
Few-body dynamics at the driplines

Modern RIB research: move towards and beyond the driplines

Few-body dynamics at the driplines as consequence of corresponding clusterization

Exotic phenomena in vicinity of driplines:
Haloes (green)
True 2p/2n decays (red)
4p/4n emitters (blue)
NOT INVESTIGATED (gray)

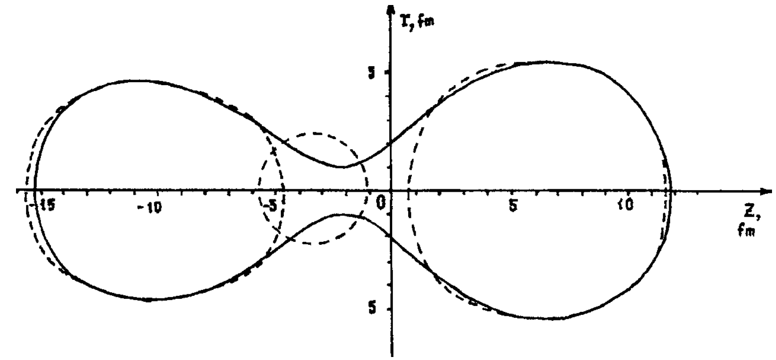
NOT SO EXOTIC: More or less every second isotope in vicinity of the driplines has features connected to few-body dynamics



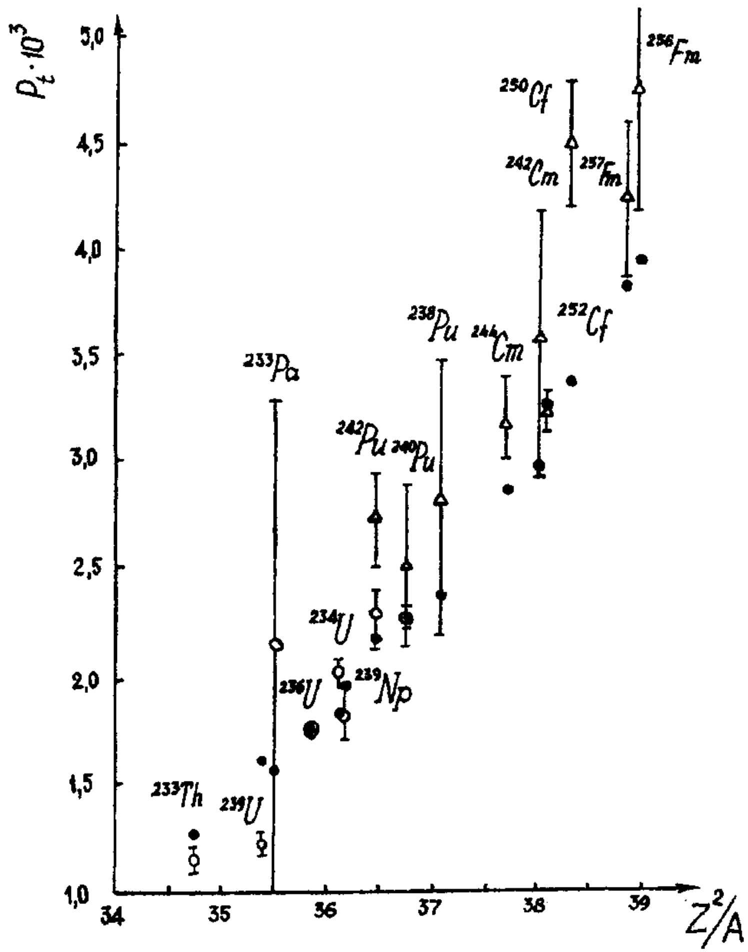
Ternary fission

Not exactly few-body dynamics, but longest-known example of decay into three fragments

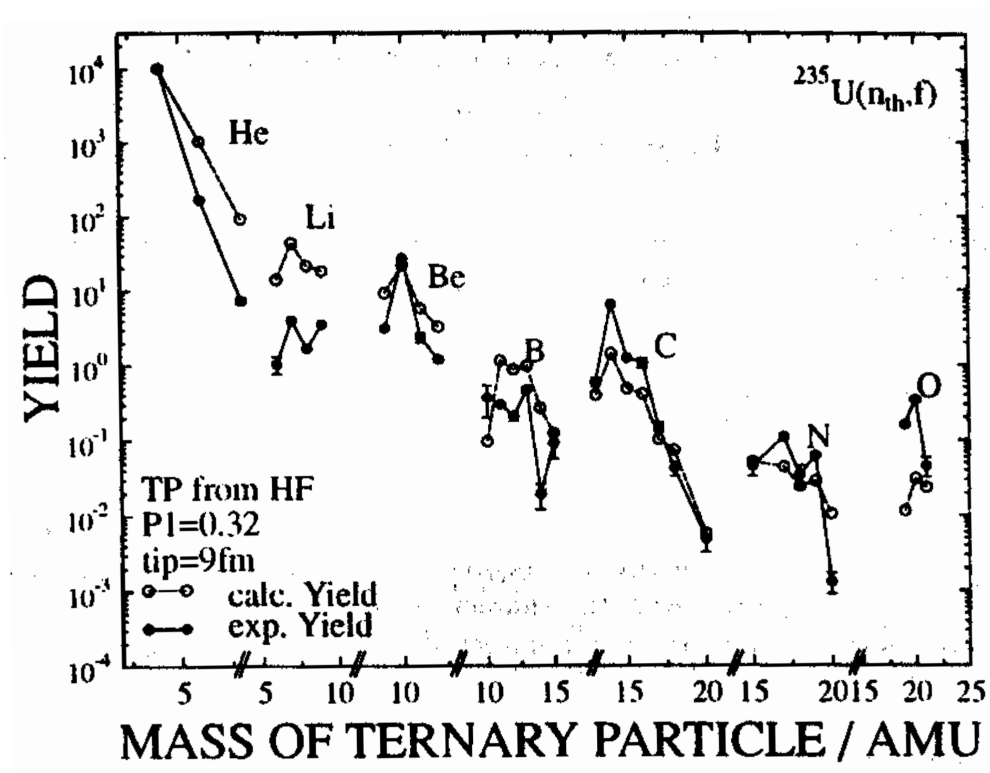
Ternary fission yeilds



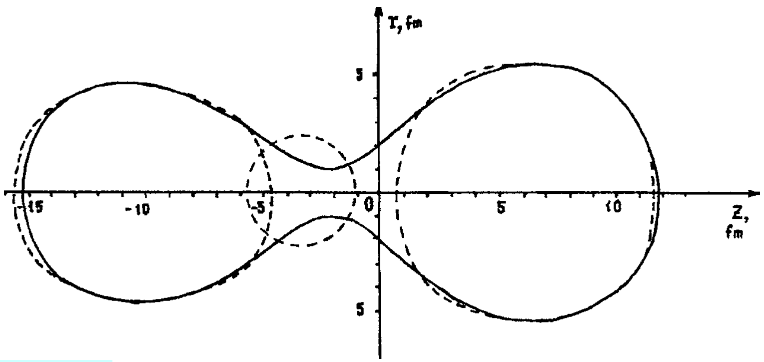
Ternary alpha fission

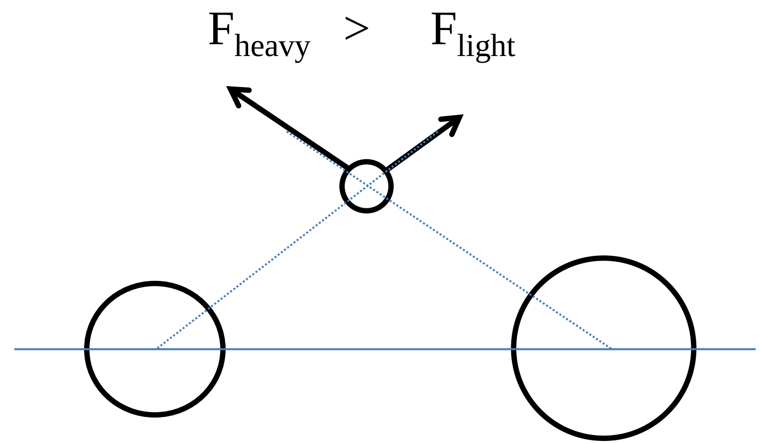


Ternary heavy fragment fission



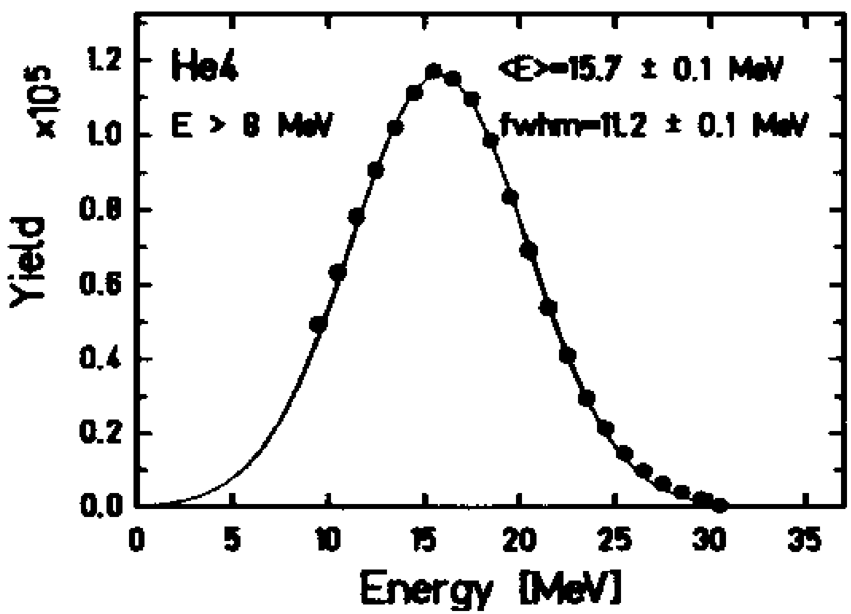
Angular and energy distributions





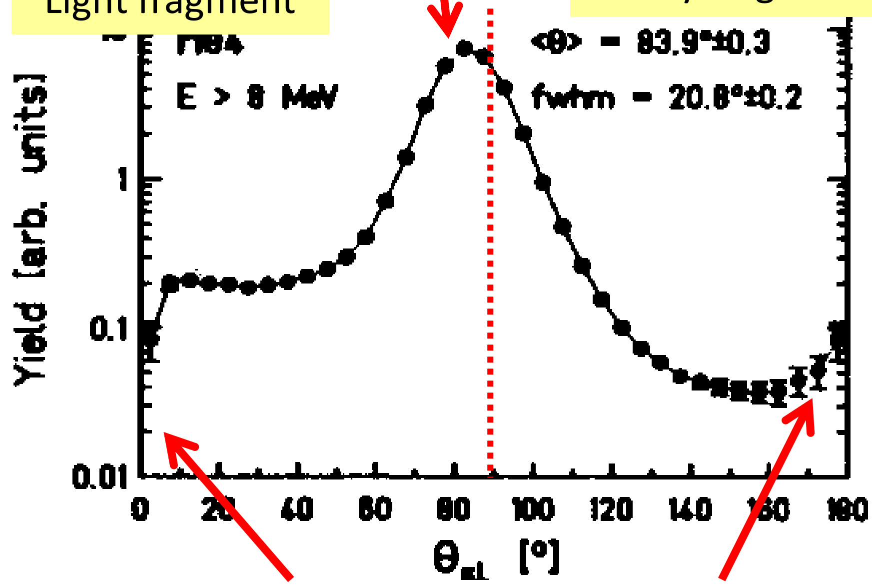
^{252}Cf

Classical effect



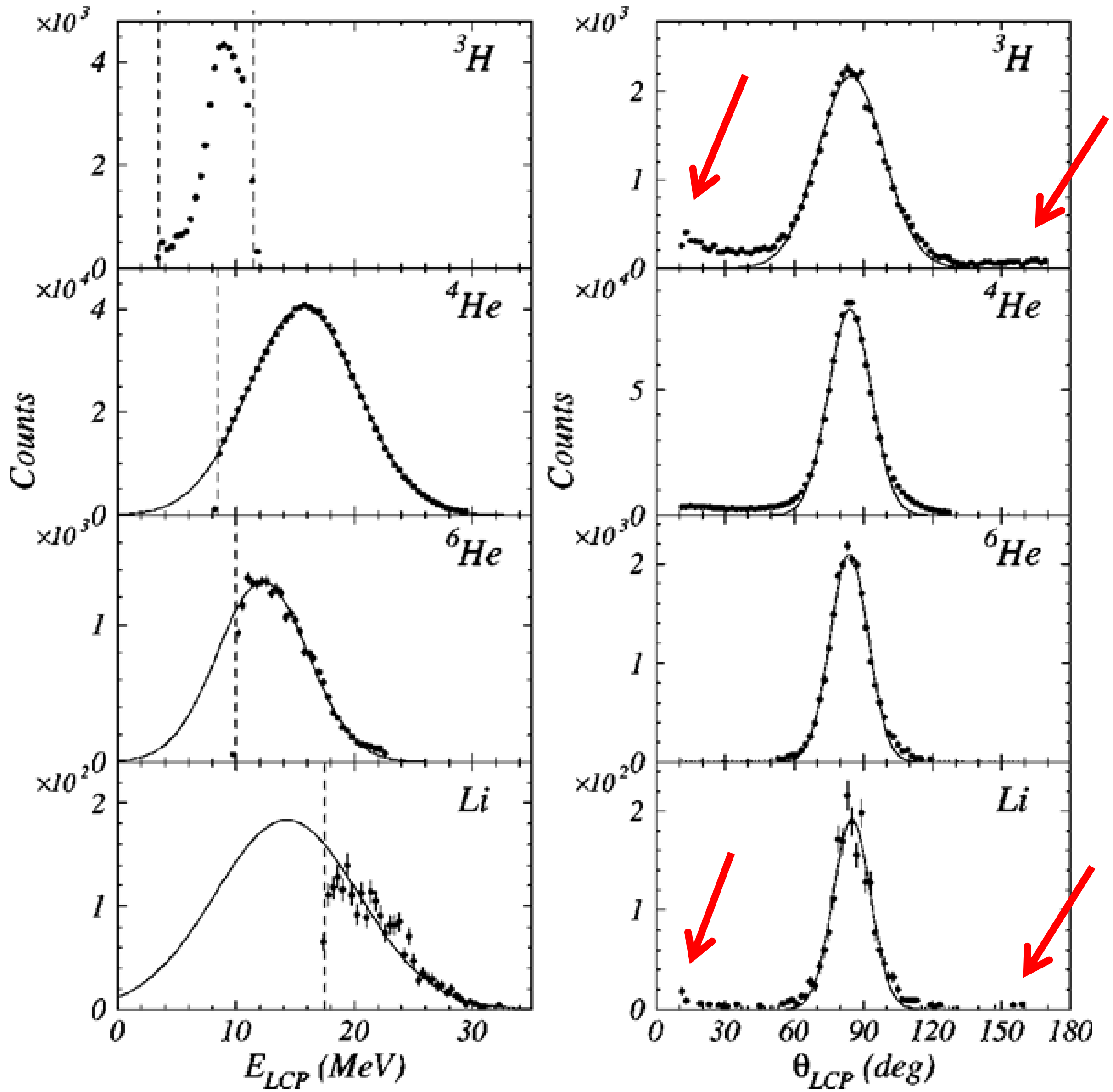
Light fragment

Heavy fragment



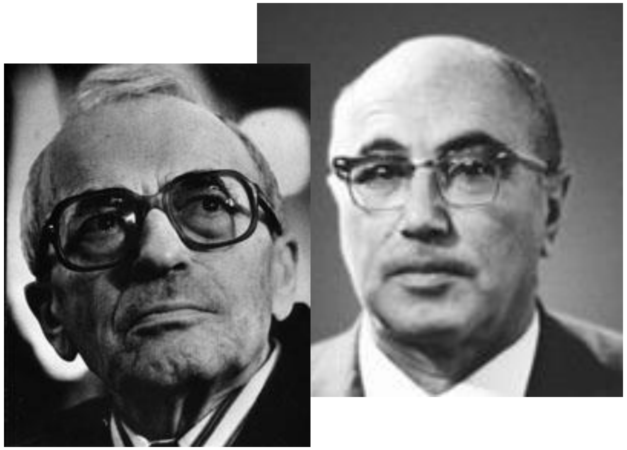
Quantum (?) effects

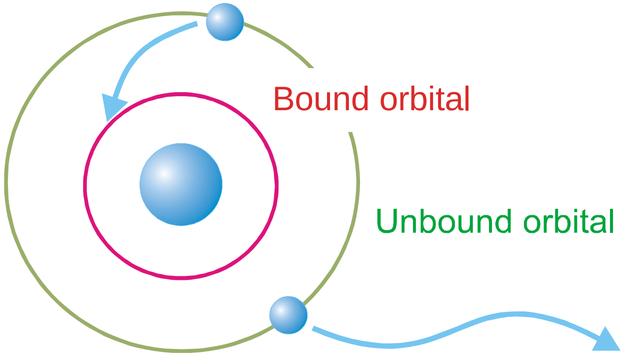
Problem of nonclassical angular distribution for different ternary fragments

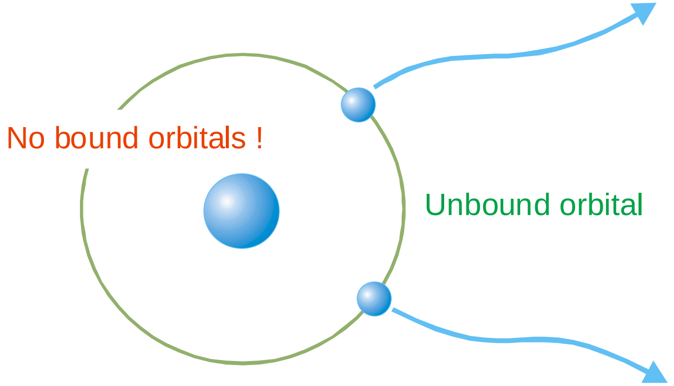


Two-proton radioactivity

Two-proton radioactivity: a qualitative view







Goldansky and Zeldovich, 1960

Pfutzner et al and Giovinazzo et al, 2002

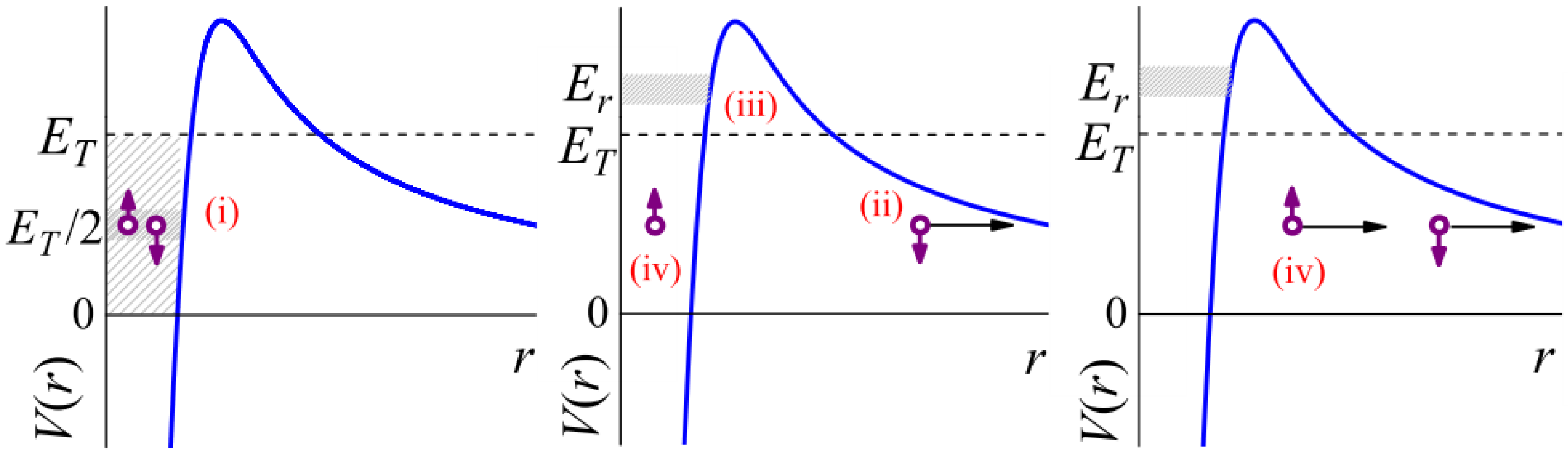
Classical case:
one particle emission is always possible

Quantum mechanical case:
it could be that both particles should be emitted simultaneously

Exclusive Quantum-Mechanical phenomenon

- **No deeper bound orbitals.**
- **The common orbital for two protons exists only when both are “inside”.**
- **When one of them goes out, their common orbital do not exist any more and the second HAS to go out instantaneously**

Trivial approach



Proton decay

$$\Gamma_2(E_r) \sim \exp\left[-\frac{\pi(Z-1)\alpha\sqrt{M}}{\sqrt{E_r}}\right]$$

Two-proton decay. ONLY sharing of energy. OTHERWISE protons are uncorrelated

$$\frac{d\Gamma_3(E_T)}{d\varepsilon} \sim \exp\left[-\frac{2\pi(Z-2)\alpha\sqrt{M}}{\sqrt{E_T}}\left(\frac{1}{\sqrt{\varepsilon}}+\frac{1}{\sqrt{1-\varepsilon}}\right)\right],$$

$$\Gamma_3(E_T) = \int_0^1 d\varepsilon\, [d\Gamma_3(E_T)/d\varepsilon] \,.$$

Original estimate by Goldansky

Three-body cluster model

Typical precision: stable solution for $\Gamma/E_T > 10^{-30}$

- Hyperspherical Harmonic method
- For narrow states

$$\Psi^{(+)}(\rho,\Omega_\rho,t) = \Psi^{(+)}(\rho,\Omega_\rho)\exp[-iE_T t - (\Gamma/2)t]$$

- Schoedinger Equation with complex energy

$$\left(\hat{H} - E_T + i\Gamma/2\right)\Psi^{(+)}(\rho,\Omega_\rho) = 0$$

- Actually solved equation

$$\left(\hat{H} - E_{box}\right)\Psi^{(+)}(\rho,\Omega_\rho) = -i(\Gamma/2)\Psi_{box}(\rho,\Omega_\rho)$$

where

$$\left(\hat{H} - E_{box}\right)\Psi_{box}(\rho,\Omega_\rho) = 0$$

- "Natural" definition of width

$$\Gamma = \frac{j(\rho_{\max})}{N(\rho_{box})} = \frac{\mathrm{Im}\int d\Omega_\rho \Psi^{(+)\dagger}\rho^{5/2}\frac{d}{d\rho}\rho^{5/2}\Psi^{(+)}\Big|_{\rho_{\max}}}{M\int d\Omega_\rho \int_0^{\rho_{box}} d\rho\, \rho^5 \left|\Psi^{(+)}\right|^2}$$

- L. V. Grigorenko, R. C. Johnson, I. G. Mukha, I. J. Thompson, and M. V. Zhukov, PRL **85** (2000) 22.
- L. V. Grigorenko, R. C. Johnson, I. G. Mukha, I. J. Thompson, and M. V. Zhukov, PRC **64** (2001) 054002.
- L. V. Grigorenko, I. G. Mukha, I. J. Thompson, and M. V. Zhukov, PRL **88** (2002) 042502.
- L. V. Grigorenko, I. G. Mukha, M. V. Zhukov, NPA 713 (2003) 372.
- L. V. Grigorenko, I. G. Mukha, M. V. Zhukov, NPA **714** (2003) 425.
- L. V. Grigorenko, M. V. Zhukov, PRC **68** (2003) 054005.

M. Pfutzner, L.V. Grigorenko, M. Karny, and K. Riisager, Rev. Mod. Phys. **84** (2012) 567

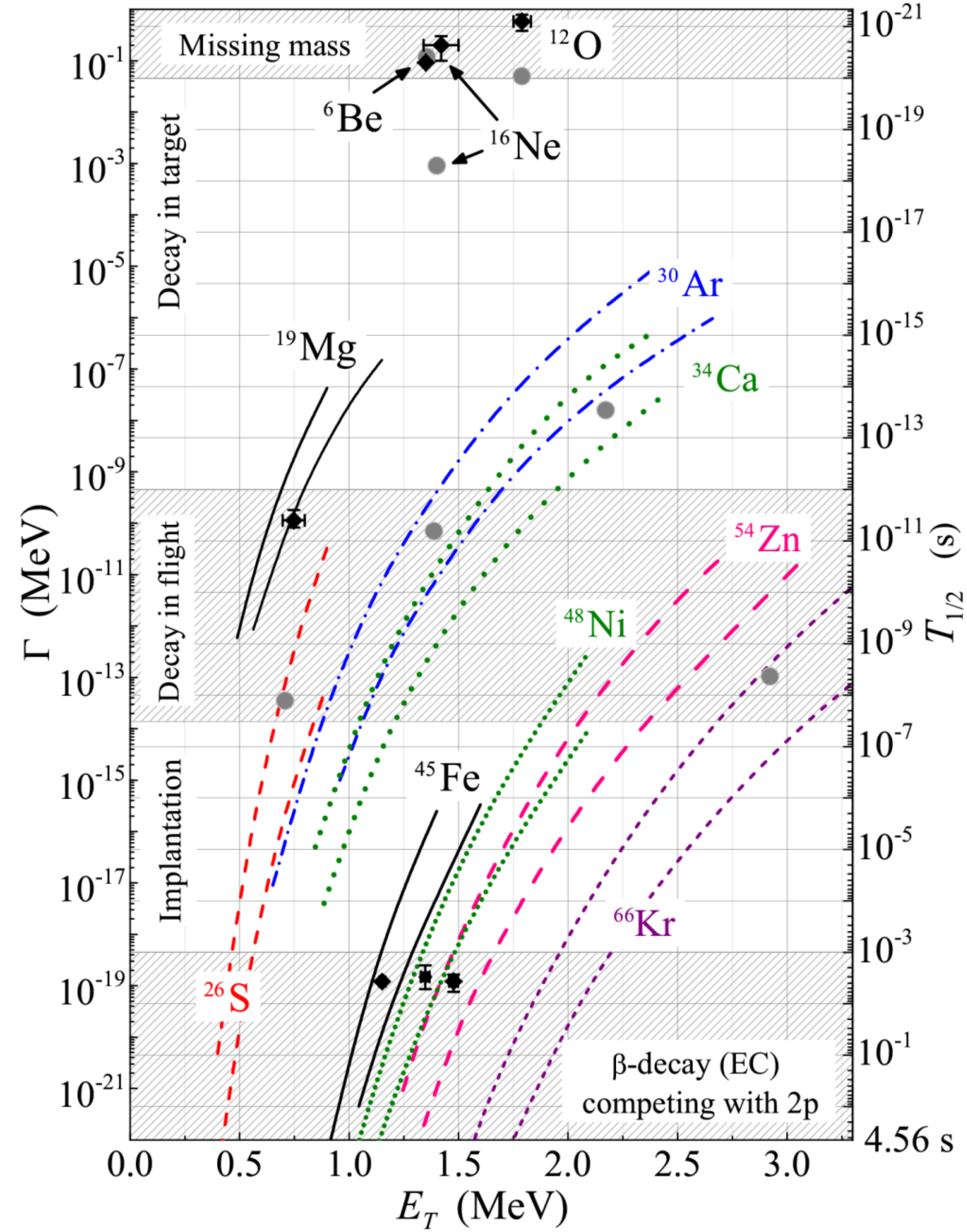
True 2p decay lifetime systematics

20 orders of the magnitude variation of the lifetime

Different experimental techniques are required: implantation, decay in flight, missing mass

In broad lifetime ranges the true 2p lifetime measurements are not accessible

Nice agreement overall. Problem with ^{12}O and ^{16}Ne lifetimes were recently resolved



True 2p decay lifetime systematics

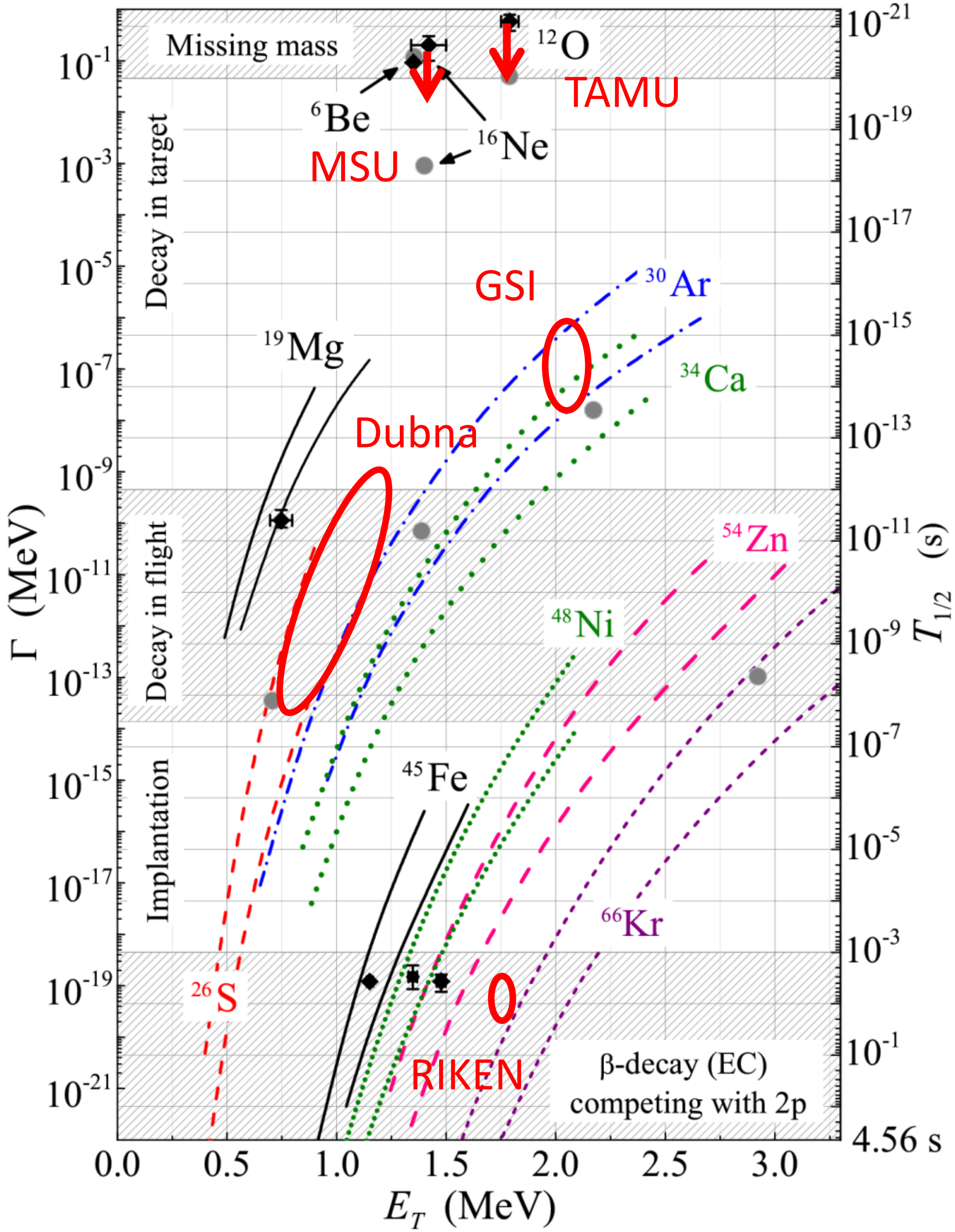
Recent findings

20 orders of the magnitude variation of the lifetime

Different experimental techniques are required: implantation, decay in flight, missing mass

In broad lifetime ranges the true 2p lifetime measurements are not accessible

Nice agreement overall. Problem with ^{12}O lifetime is recently resolved, problem with ^{16}Ne lifetime to be resolved



Three-body correlations

Three-body correlations in decays and reactions

2-body decay: state is defined by 2 parameters - energy and width

3-body decays: 2-dimensional “internal” 3-body correlations

3-body continuum in reactions: there is a selected direction. 5-dimensional correlations: “internal” + “external”

For <u>direct reactions</u> the selected direction is momentum transfer vector

Which kind of useful information (if any) can be obtained from three-body correlations?

"Internal" energy of 3-bodies

$\{k_x, k_y\} \rightarrow E_T = E_x + E_y$

"Internal" 3-body correlations

$\{\mathbf{k}_x, \mathbf{k}_y\} \rightarrow$

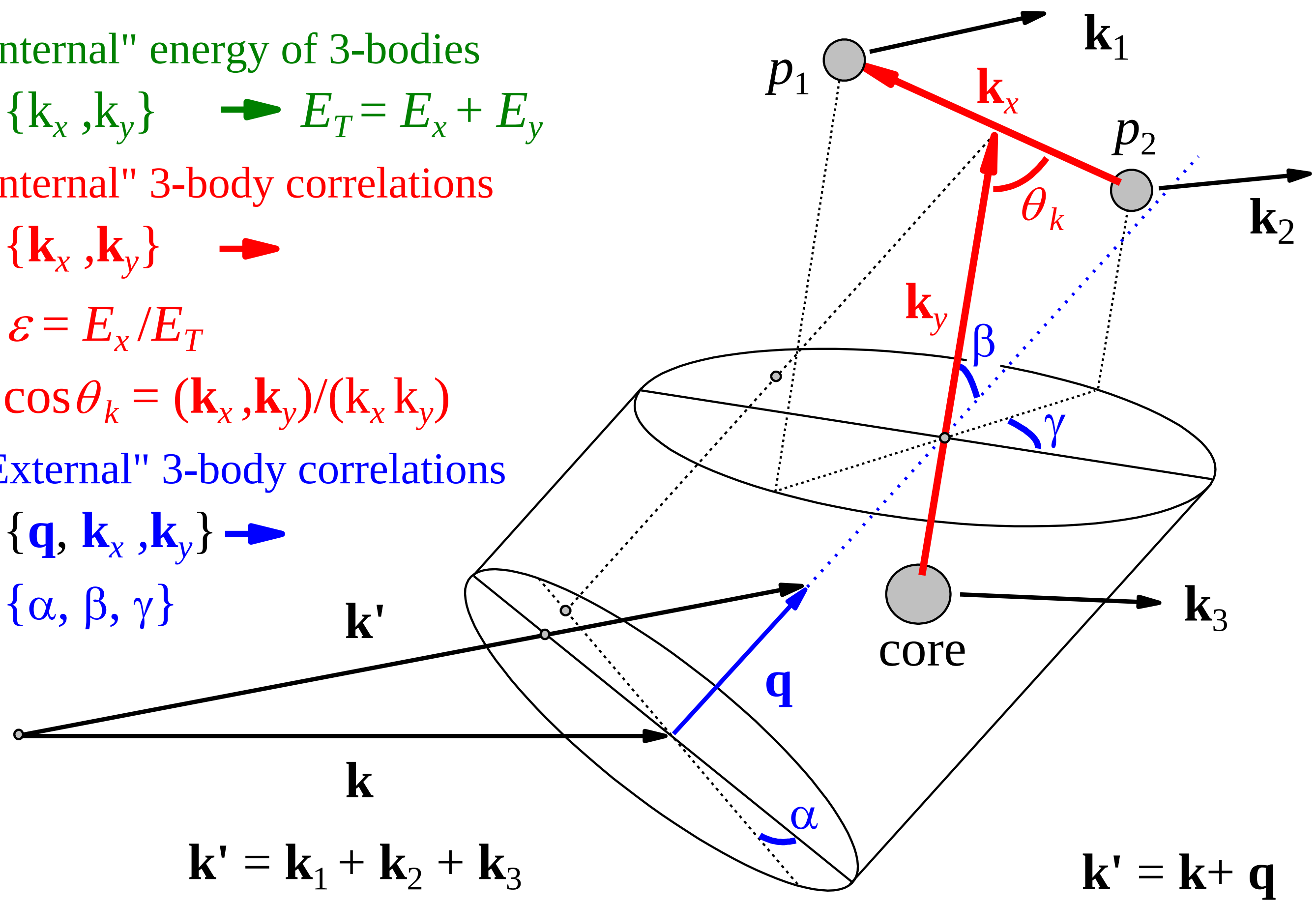
$\varepsilon = E_x / E_T$

$\cos\theta_k = (\mathbf{k}_x, \mathbf{k}_y)/(k_x k_y)$

"External" 3-body correlations

$\{\mathbf{q}, \mathbf{k}_x, \mathbf{k}_y\} \rightarrow$

$\{\alpha, \beta, \gamma\}$

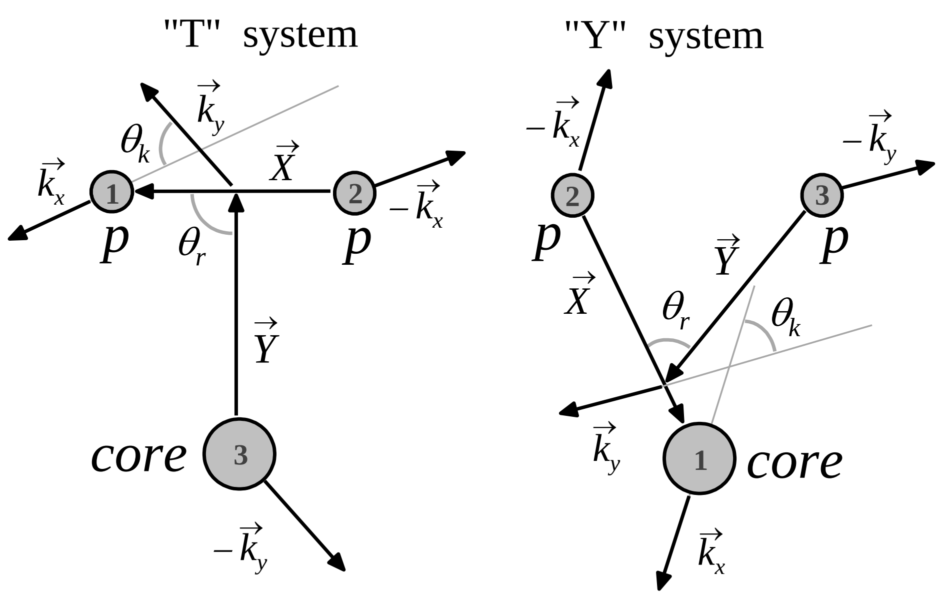


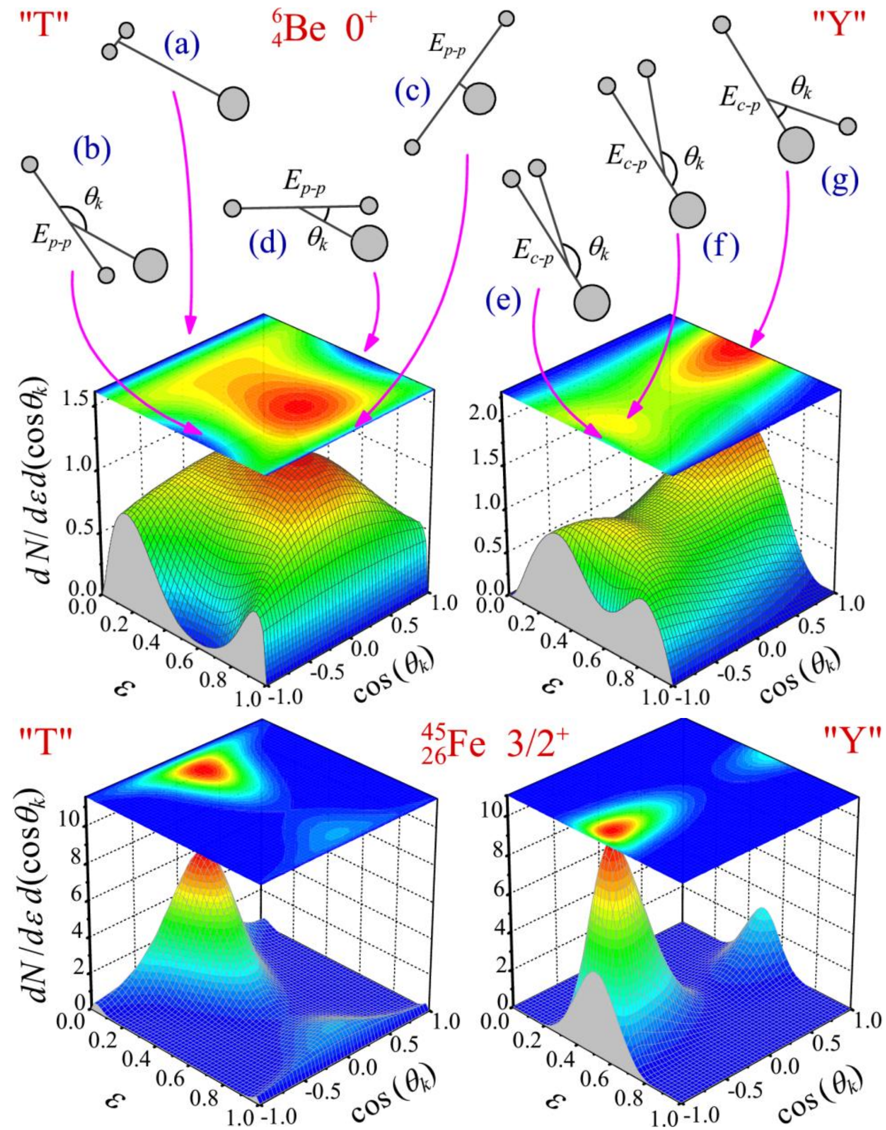
Three-body correlations. "Internal" correlations

- 2-dimensional "internal three-body correlations" or "energy-angular correlations"

$$\varepsilon = E_x / E_T \qquad \cos(\theta_k) = (\mathbf{k}_x, \mathbf{k}_y)/k_x k_y$$

- "T" and "Y" Jacobi systems reveal different dynamical aspects
- Three-body variables in coordinate and in momentum space.



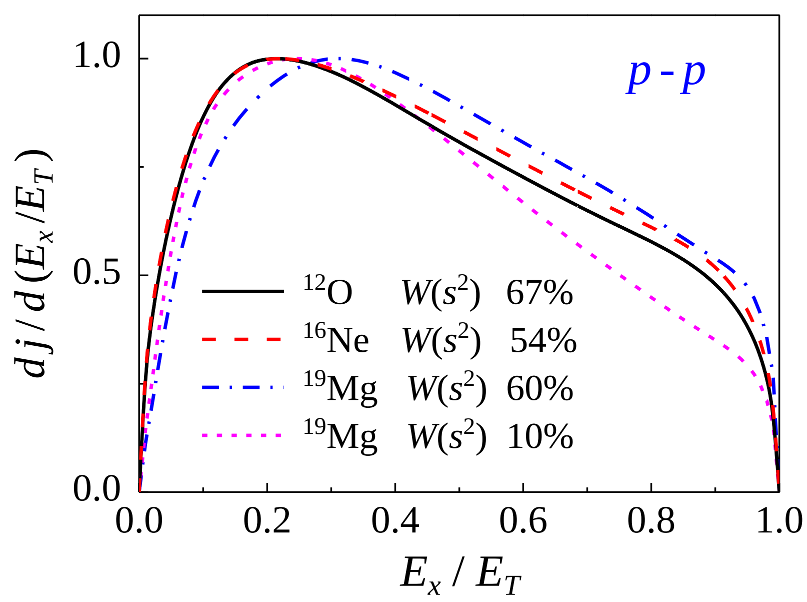


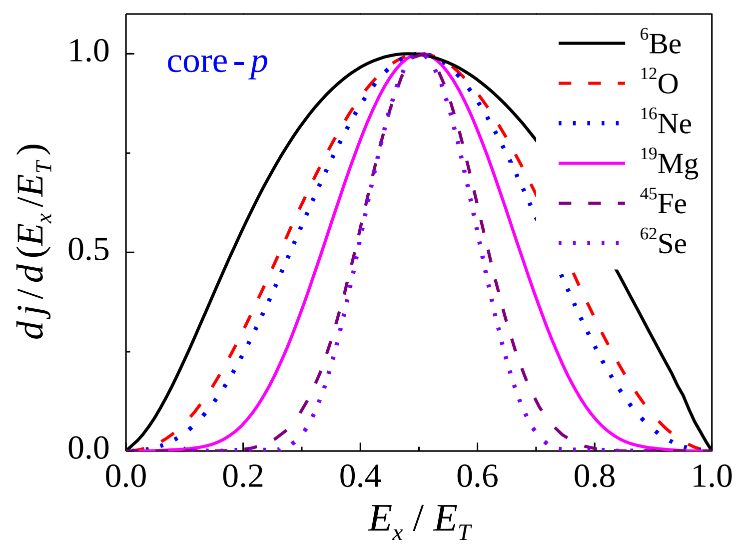
Simple way to understand three-body correlations: "quasibinary kinematics" ε -> 0 and ε -> 1

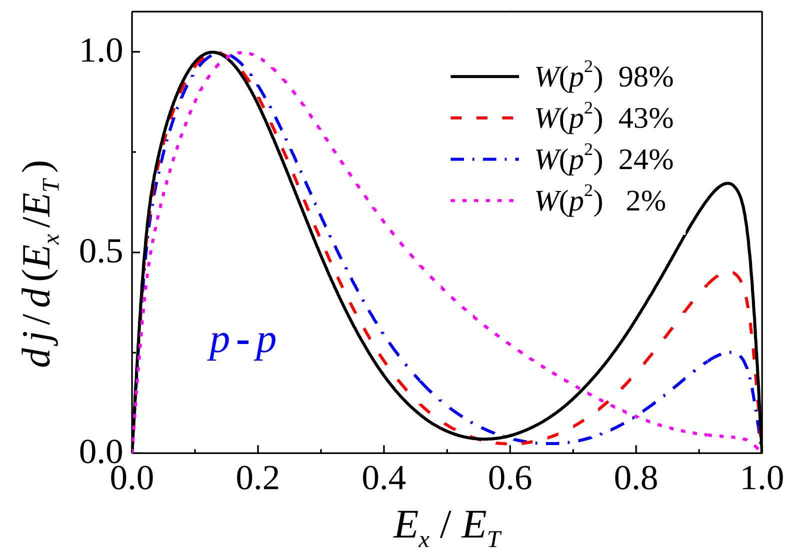
Common properties of correlations

How can we use the correlation information?

- **Energy correlation in the core-p** channel well corresponds to original prediction of Goldansky: energies of the emitted protons tend to be equal.
- **Energy correlation in the p-p channel** in the s-d shell nuclei quantitatively depend on the structure
- **Energy correlation in the p-p channel** in the p-f shell nuclei qualitatively depend on the structure







Between theory and experiment

Monte-Carlo codes

Observables in reactions:
Nuclear structure +
Reaction mechanism +
Final state interaction

M.S.Golovkov et al., PRL **93** (2004) 262501.
M.S.Golovkov et al., PRC **72** (2005) 064612.
L.V. Grigorenko et al., PRC **82** (2010) 014615.
A.S.Fomichev et al., PLB **708** (2012) 6.
I.A. Egorova et al., PRL **109** (2012) 202502.
I. Mukha et al., PRL **115** (2015) 202501.
T.A. Golubkova et al., PLB **762** (2016) 263.

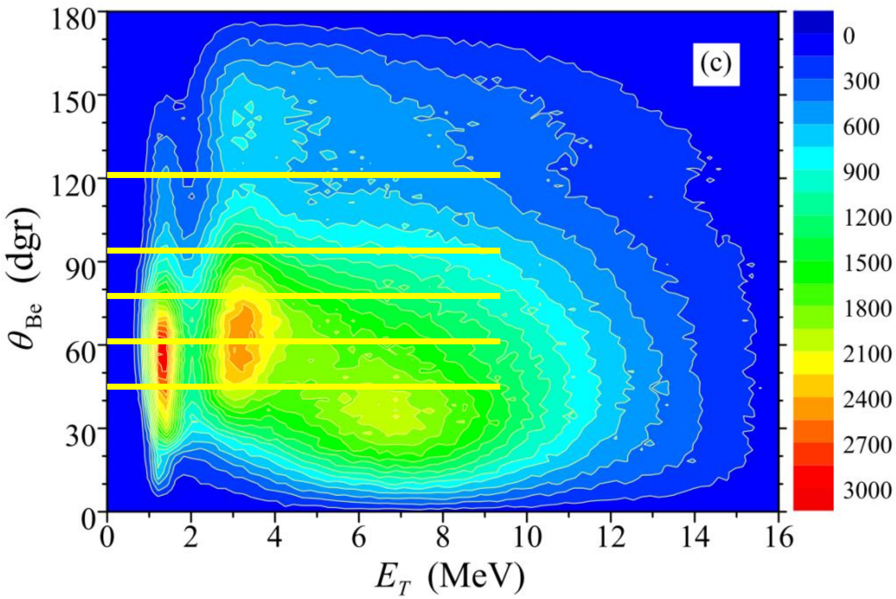
Experimental bias:
Acceptance +
Efficiency +
Resolution +
Physical backgrounds

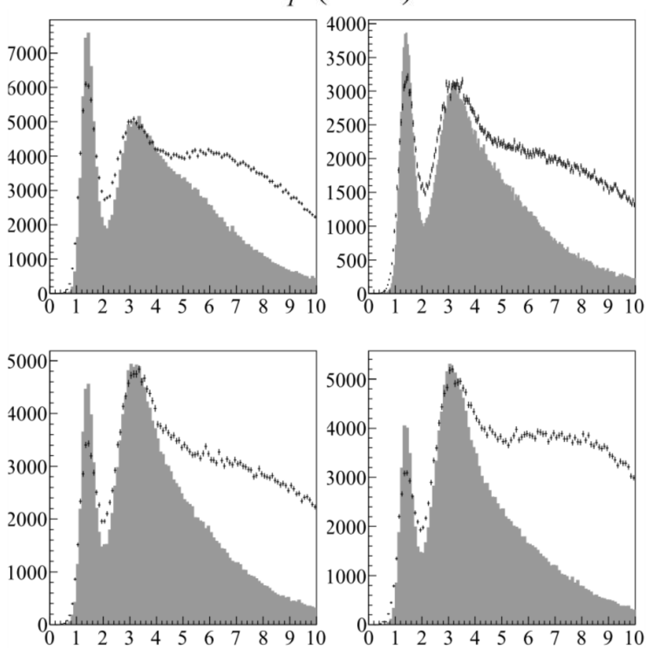
- For studies of correlations full quantum-mechanical Monte Carlo simulations are required
- Decompose experimental particle correlation data over hyperspherical amplitudes in the momentum space. HH amplitudes automatically take into account PP, angular momenta in the subsystems and spin. Calculated or parameterized.
- Density matrix formalism:

$$\frac{dW}{dq\,dE\,d\Omega_5} = \sum_{JM,J'M'} \rho_{JM}^{J'M'}(q,E)\,A^{\dagger}_{J'M'}(E,\Omega_5)\,A_{JM}(E,\Omega_5)$$

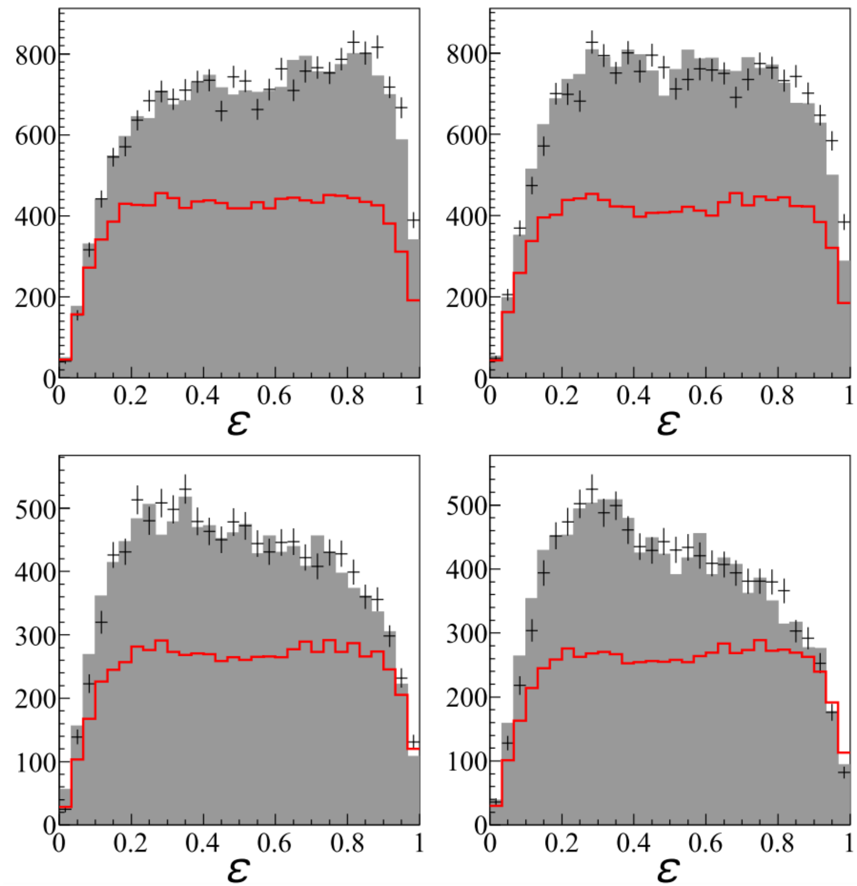
- Density matrix has especially simple form in the system of transferred momentum for direct reactions
- Three-body decay -> eightfold differential cross section

How experiment distort correlations





$^6Be = \alpha + p + p$ populated in (p,n) charge-exchange reaction on ^6Li



Energy correlations for 0^+ state in different angular ranges

Three-body correlations
and
nuclear structure

^{45}Fe: the first found and the best studied

A. Brown, PRC **41** (1991) R1513.

Brown 1991: energy – yes, lifetime – no

Grigorenko 2001: energy – no, lifetime – yes

Pfützner et al., EPJA **14** (2002) 279

Giovinazzo et al., **89** (2002) 102501

Dossat et al., PRC **72** (2005) 054315

$Q_{2p} = 1.154$ MeV

Miernik et al., PRL **99** (2007) 192501

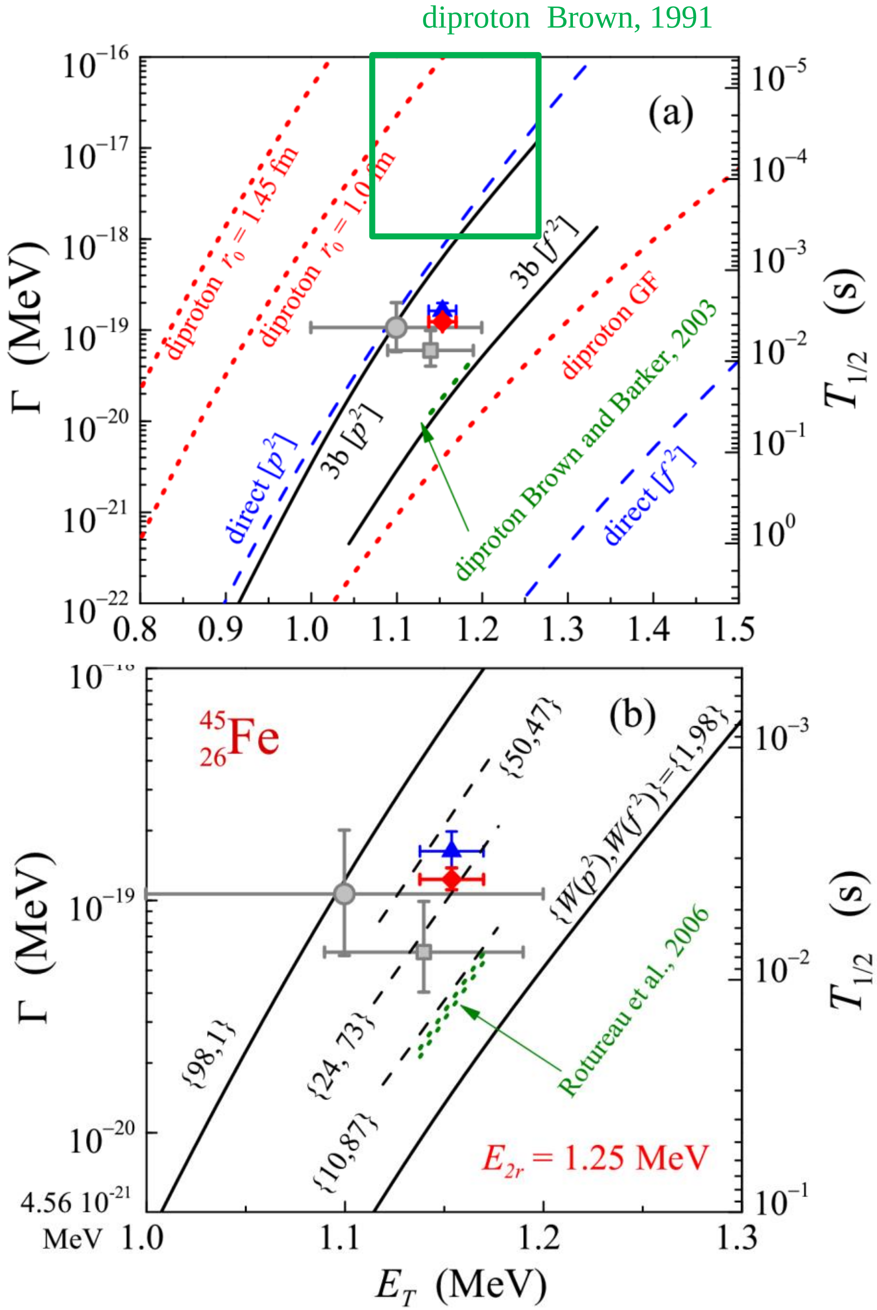
- Special design Optical TPC → nuclear physics "life video"
- Improved lifetime:

$$\Gamma_{2p} = 1.3^{+0.22}_{-0.16} \times 10^{-19} \quad \text{MeV} \quad T_{1/2}(2p) = 3.5(5) \quad \text{ms}$$

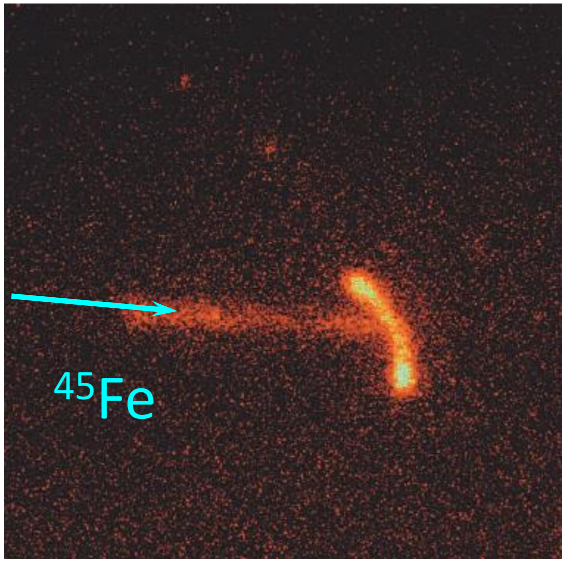
- Complete momentum correlations provided

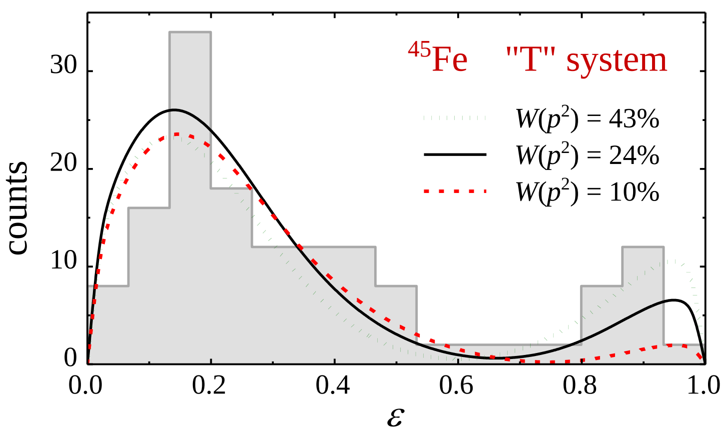
L.Grigorenko et al., PLB **677** (2009) 30

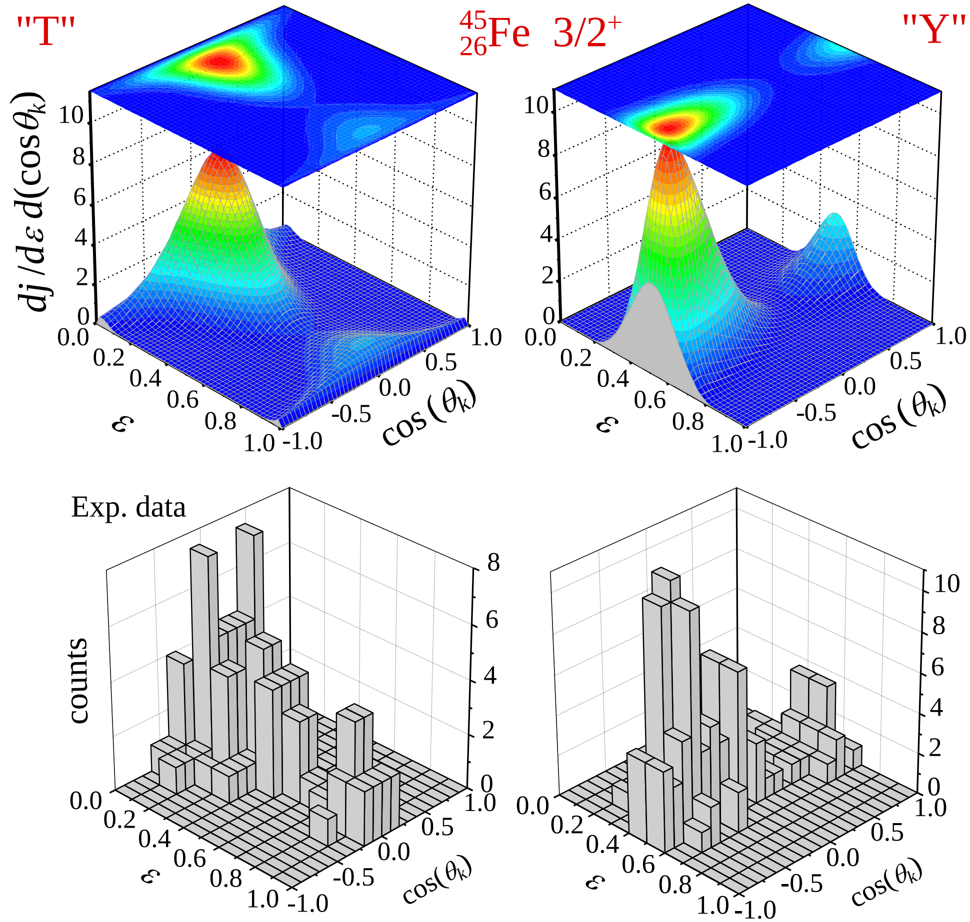
L.Grigorenko et al., PRC **82** (2010) 014615



^{45}Fe: internal correlations







Miernik et al., PRL 99 (2007) 192501

- **Complete kinematics reconstructed**
- **Both lifetime and correlations provide** $W(p^2) \sim 30\%$

High-precision studies of three-body correlations

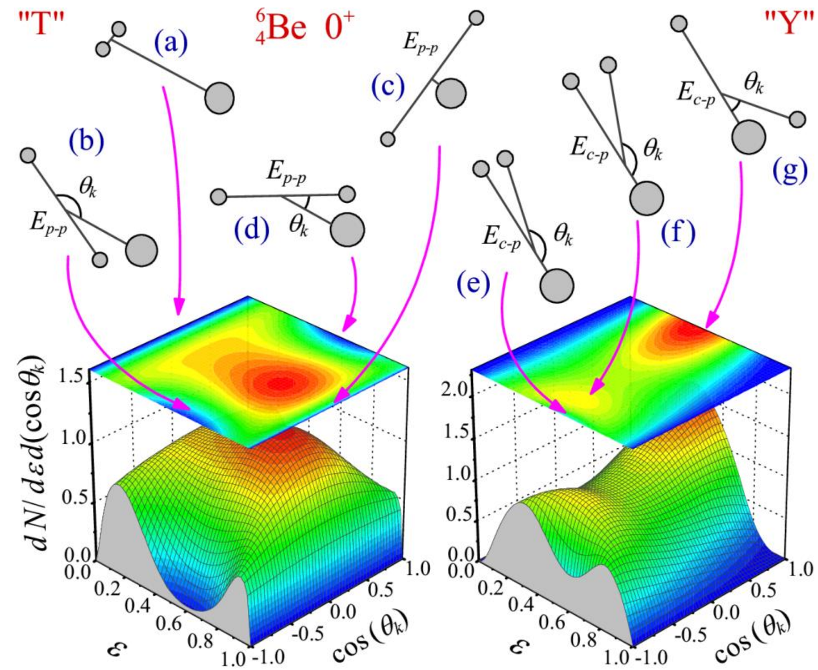
Three-body decay model provides very precise and parameter-free description of correlations in the well-defined three-cluster nuclear systems. This is extensively checked experiementally

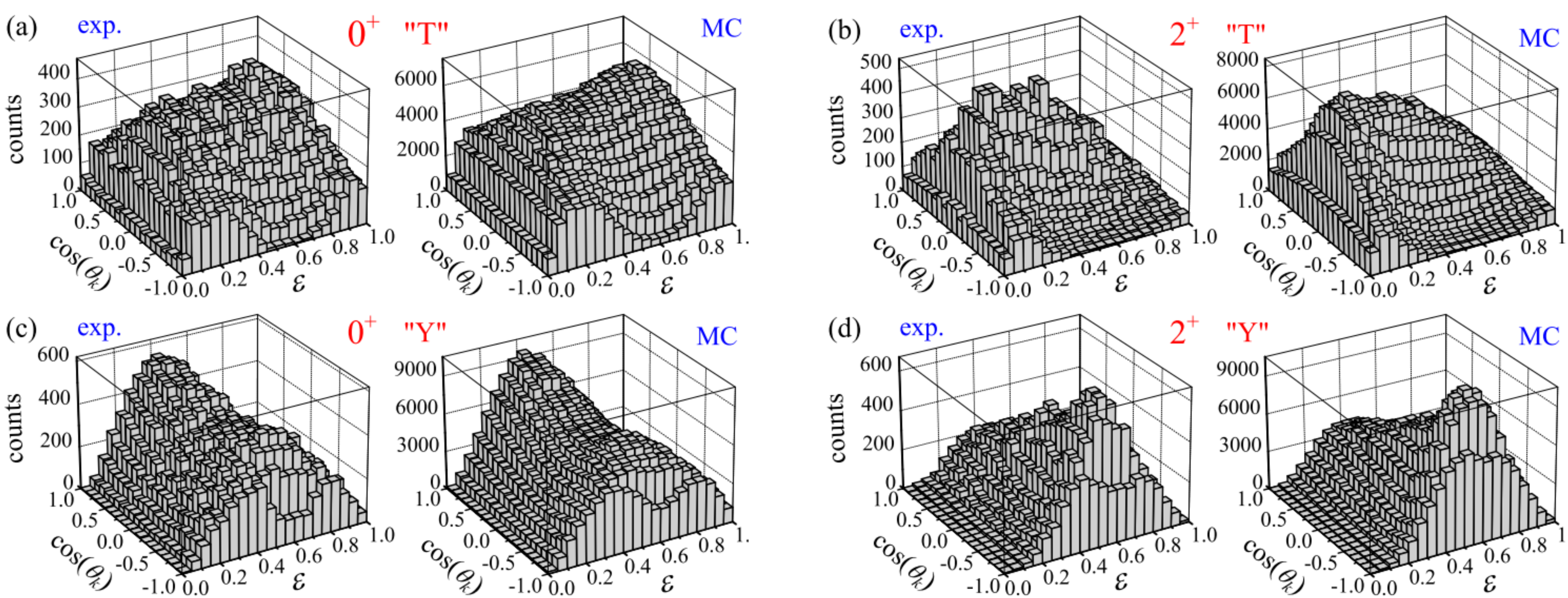
^{6}Be at MSU: correlations on resonance

Experiment:
R. Charity and coworkers, MSU ^{7}Be(^{9}Be,X)^{6}Be

I. Egorova *et al.,* PRL **109** (2012) 202502.

- High statistics (~10^6 events/state)
- High resolution
- Nice agreement with the previous (Texas A&M, Dubna) experimental data





Three-body Coulomb continuum problem

Approximate boundary conditions

Two-body decay WF asymptotic

$$\Psi^{(+)}(r) \sim H^{(+)}(r) \sim \exp[+ikr]$$

In general case the boundary conditions of the three-body Coulomb problem are analytically unknown

Three-body Coulomb problems is one of "eternal problems" of theoretical and mathematical physics

- Boundary conditions are obtained by diagonalization of the three-body Coulomb interaction on the truncated hyperspherical basis.

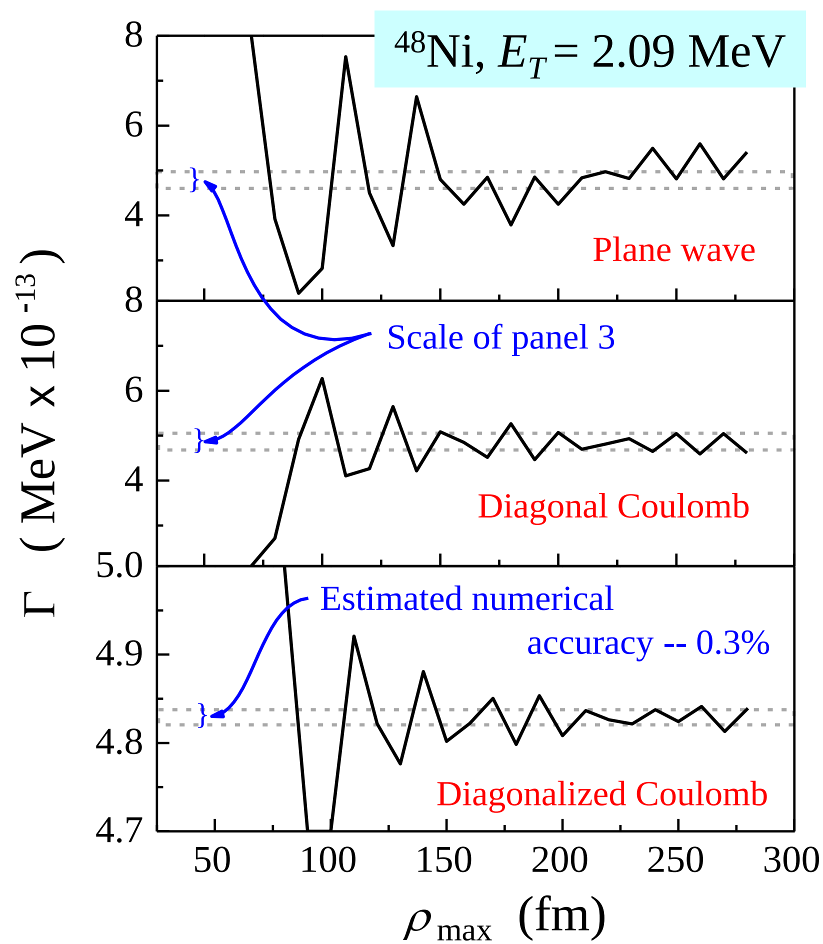
  L. Grigorenko et al., PRC **64** (2001) 054002.

$$V_{K\gamma,K'\gamma'} \sim \frac{\alpha_{K\gamma,K'\gamma'}}{\rho} + \delta_{KK'}\delta_{\gamma\gamma'}\frac{(K+3/2)(K+5/2)}{\rho^2} + \frac{\beta_{K\gamma,K'\gamma'}}{\rho^{N_{K\gamma,K'\gamma'}\geq 3}}$$

$$\tilde{V}_{K\gamma,K'\gamma'} \sim \delta_{KK'}\delta_{\gamma\gamma'}\frac{\tilde{\alpha}_{K\gamma}}{\rho} + \frac{\lambda_{K\gamma,K'\gamma'}}{\rho^2} + \frac{\tilde{\beta}_{K\gamma,K'\gamma'}}{\rho^3}$$

$$\tilde{\Psi}^{(+)}_{K\gamma}(\rho) \sim H^{(+)}_{\tilde{\alpha}_{K\gamma},\lambda_{K\gamma,K'\gamma'}}(\kappa\rho) \quad \rightarrow \quad \Psi^{(+)}_{K\gamma,K'\gamma'}(\rho)$$

- Procedure is exact on the truncated HH basis
- The above procedure was first proposed by Merkuriev. It provide very stable results for the true three-body decays.



Classical extrapolation

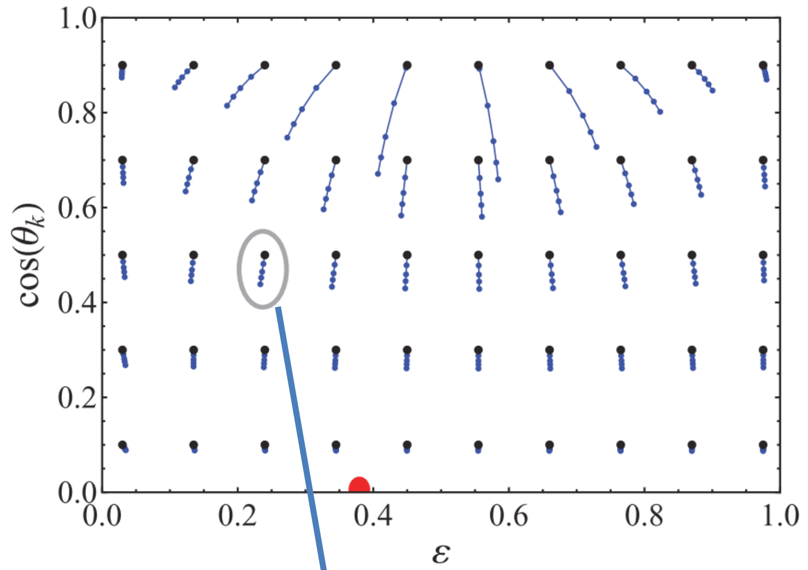
L. Grigorenko *et al.*, PRC **82** (2010) 014615.

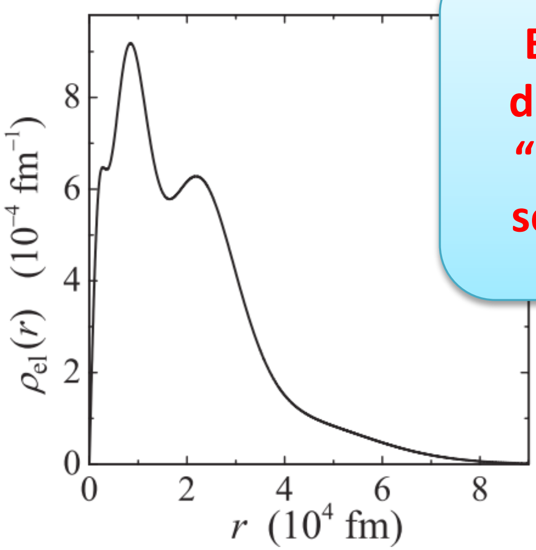
- Approximate boundary conditions do not work good enough for momentum distributions.
- Improvement of the momentum distributions by classical trajectory extrapolation.

$$M_x \ddot{\mathbf{X}} = \frac{\alpha Z_1 Z_2 \mathbf{X}}{X^3} - \frac{\alpha Z_2 Z_3 c_1 \mathbf{r}_{23}}{r_{23}^3} + \frac{\alpha Z_3 Z_1 c_2 \mathbf{r}_{31}}{r_{31}^3},$$

$$M_y \ddot{\mathbf{Y}} = \frac{\alpha Z_2 Z_3 \mathbf{r}_{23}}{r_{23}^3} + \frac{\alpha Z_3 Z_1 \mathbf{r}_{31}}{r_{31}^3}.$$

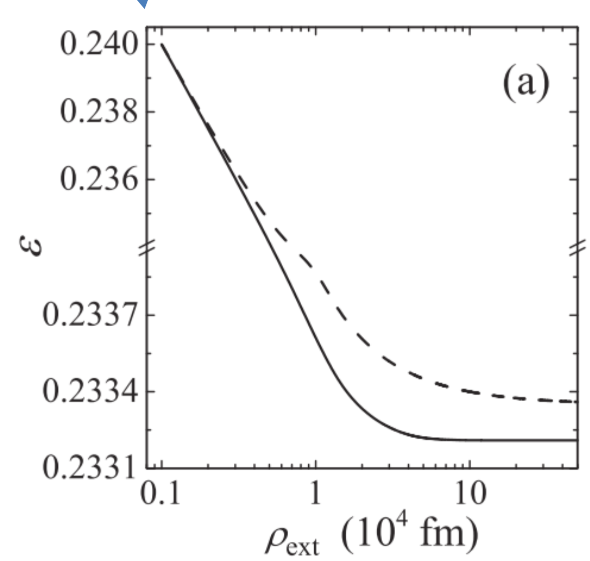
- Trajectories 1000, 1400, 2200, 4000, and 10^5 fm
- Arrive to borderline with atomic phenomena





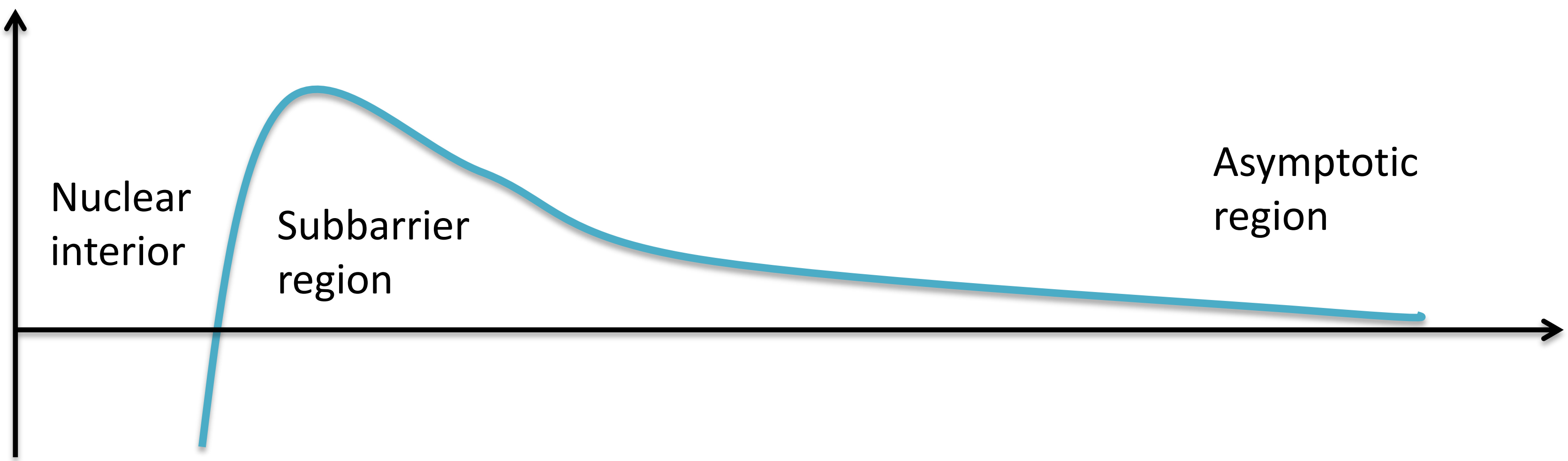
Electron density -> "plasma" screening

^{45}Fe, E_T = 1.154 MeV



How far we need to go?

Three radial scales of the calculations for three-body Coulomb problem



Energy of state: 20-30 fm

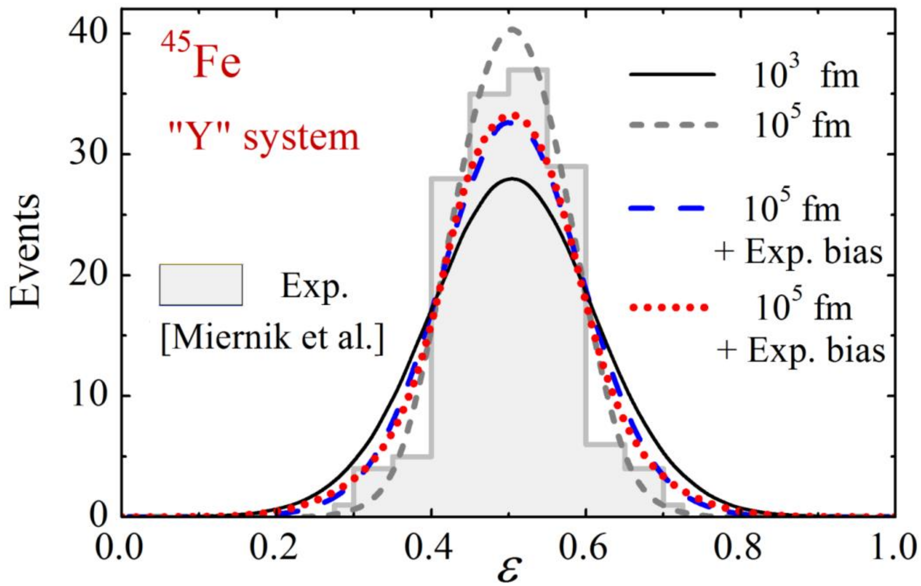
Width of state: 200-300 fm

Three-body correlations: 10^4-10^5 fm

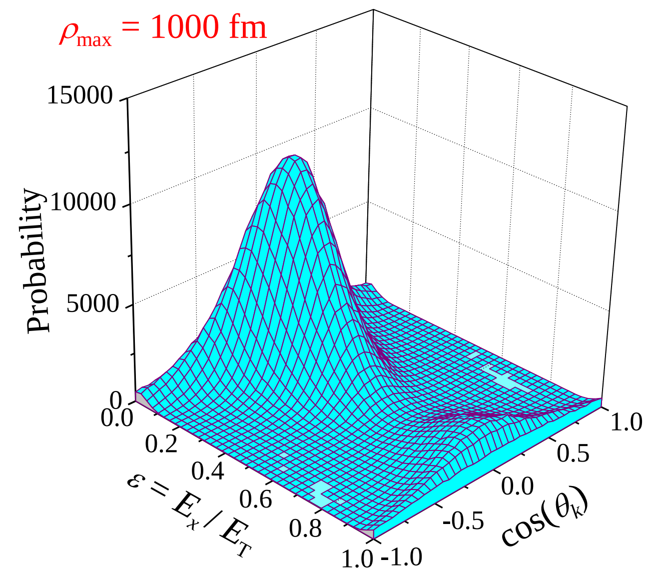
Long-range character of 3-body Coulomb by example of ^{45}Fe

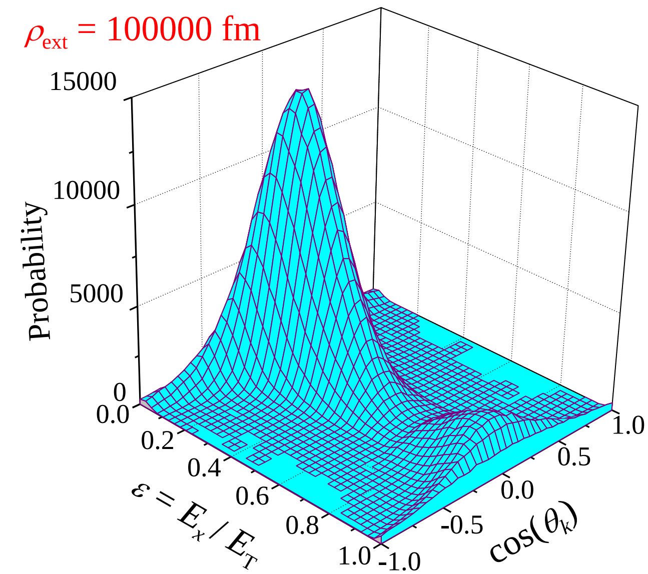
L. Grigorenko *et al.*, PLB **677** (2009) 30.

- Start point for extrapolation: typical range of 1000 fm in ρ value
- End point for extrapolation: typical range of 100000 fm in ρ value
- Complicated treatment of experimental effects



^{45}Fe, $E_T = 1.154$ MeV





Consistence, with data but no solid evidence

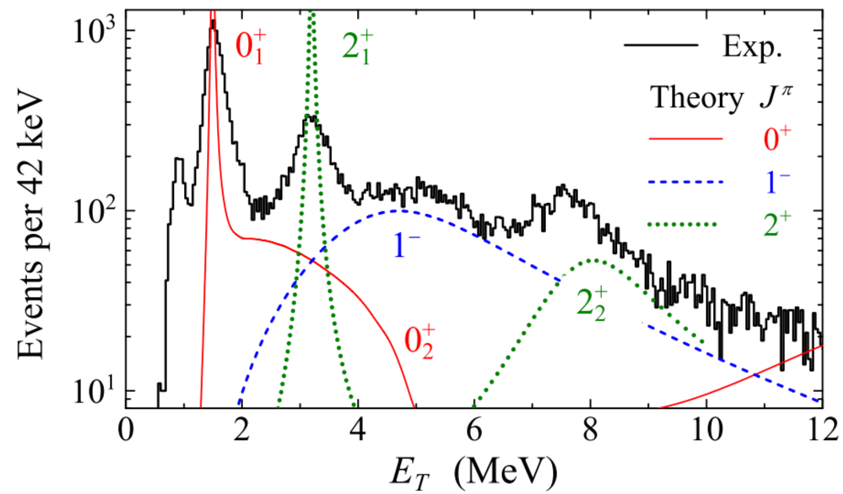
Long-range character of three-body Coulomb by example of ^{16}Ne

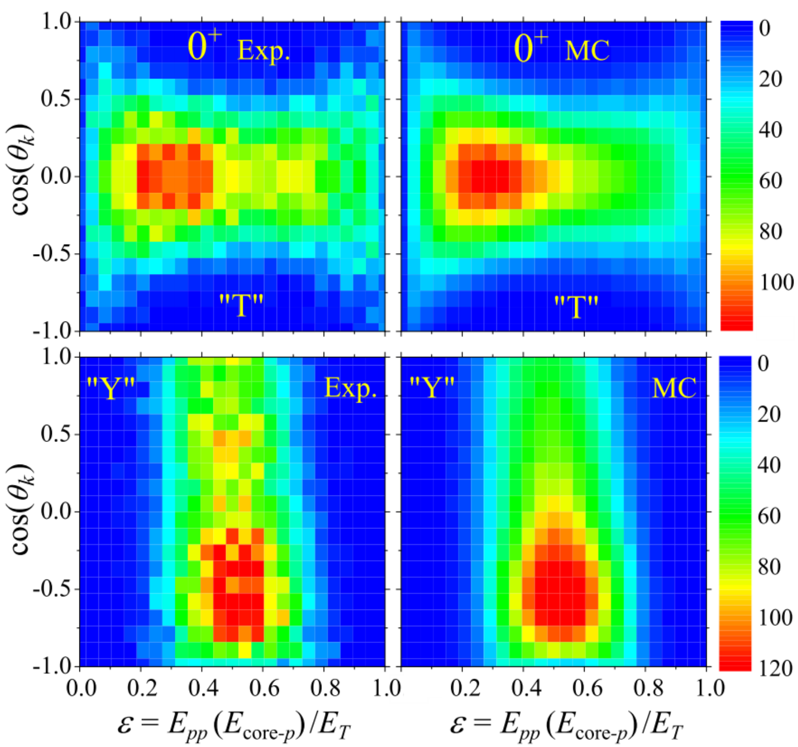
^{16}Ne g.s., $E_T = 1.476$ MeV

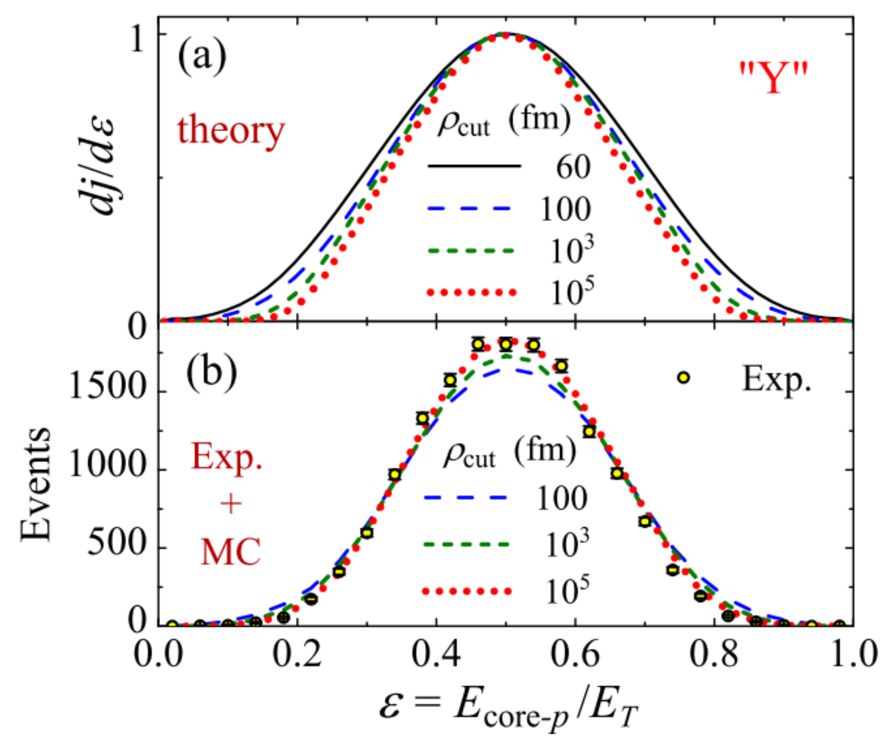
- New level of experimental precision. MSU 2013: ^{16}Ne populated in n knockout from ^{17}Ne

K. Brown *et al.*, PRL **113** (2014) 232501

- The energy distribution in "Y" Jacobi system only reproduced for extreme range of calculation



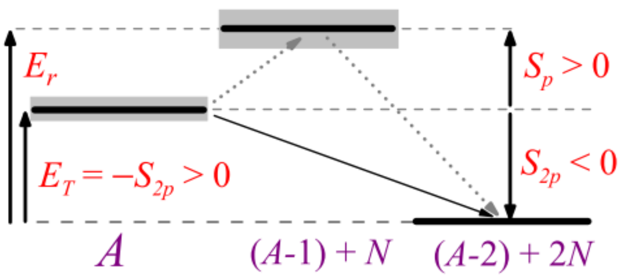




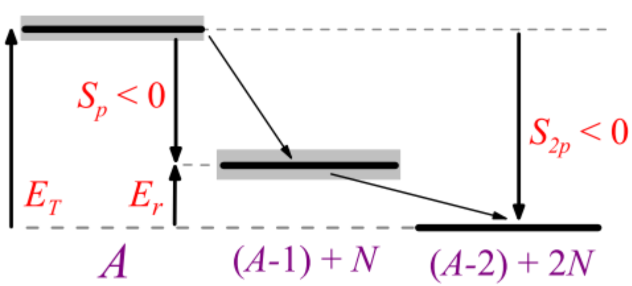
Three-body decay mechanisms

Energy conditions and few-body phenomena

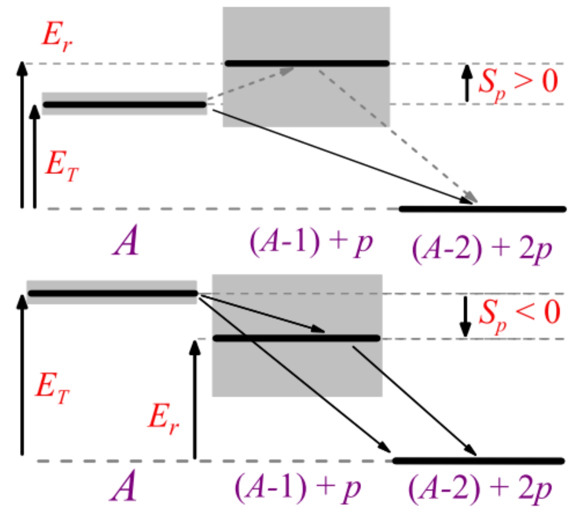
2p radioactivity/true 2p decay



Sequential decay

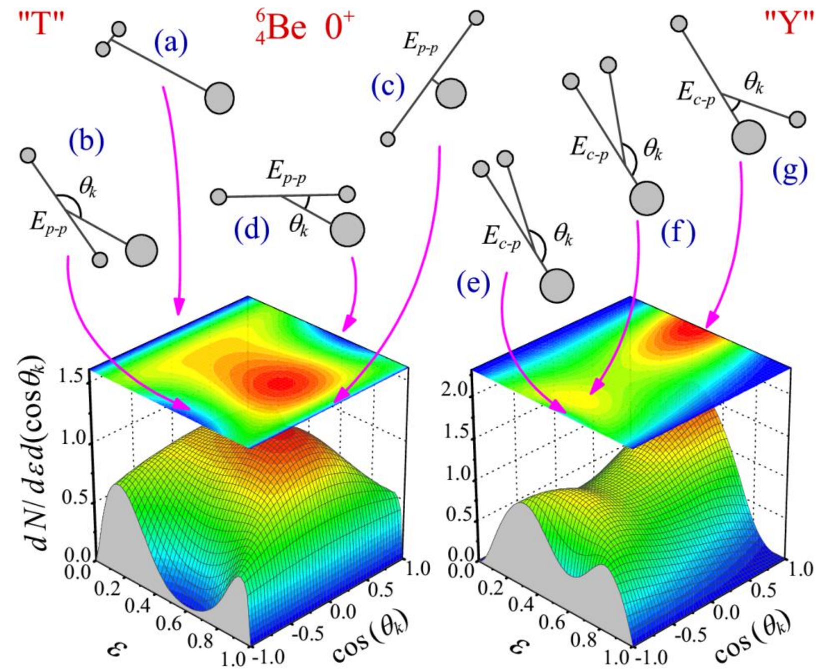


Democratic decays



Light 2p and majority of 2n emitters

^{6}Be at MSU: correlations on resonance

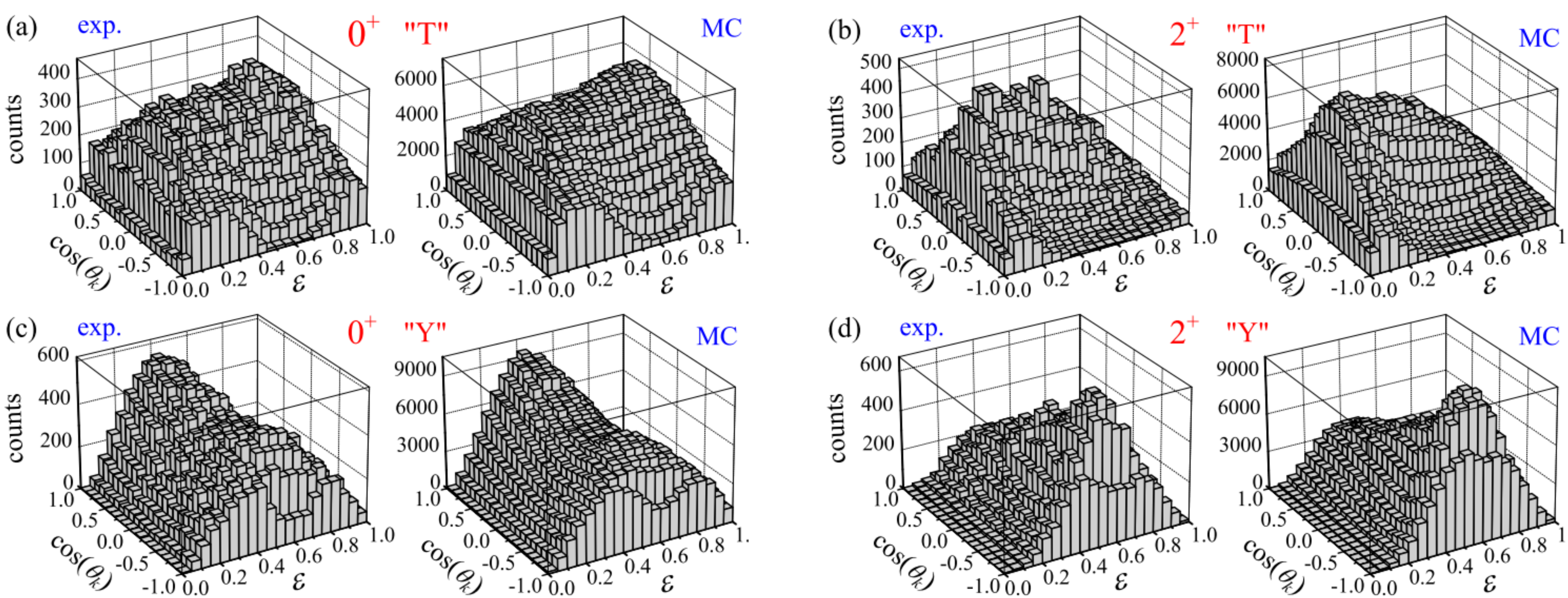


Experiment:

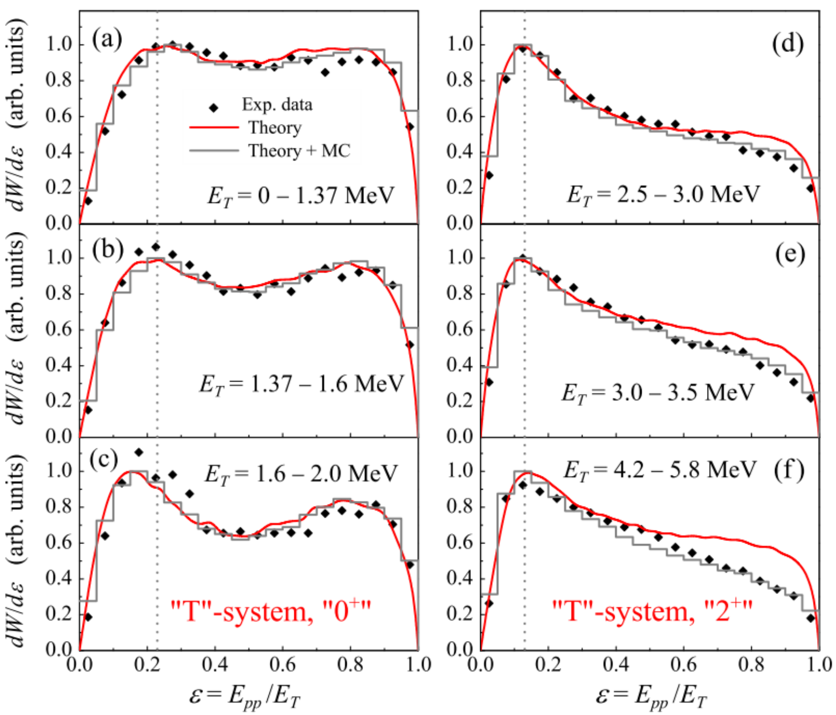
R. Charity and coworkers, MSU ^{7}Be(^{9}Be,X)^{6}Be

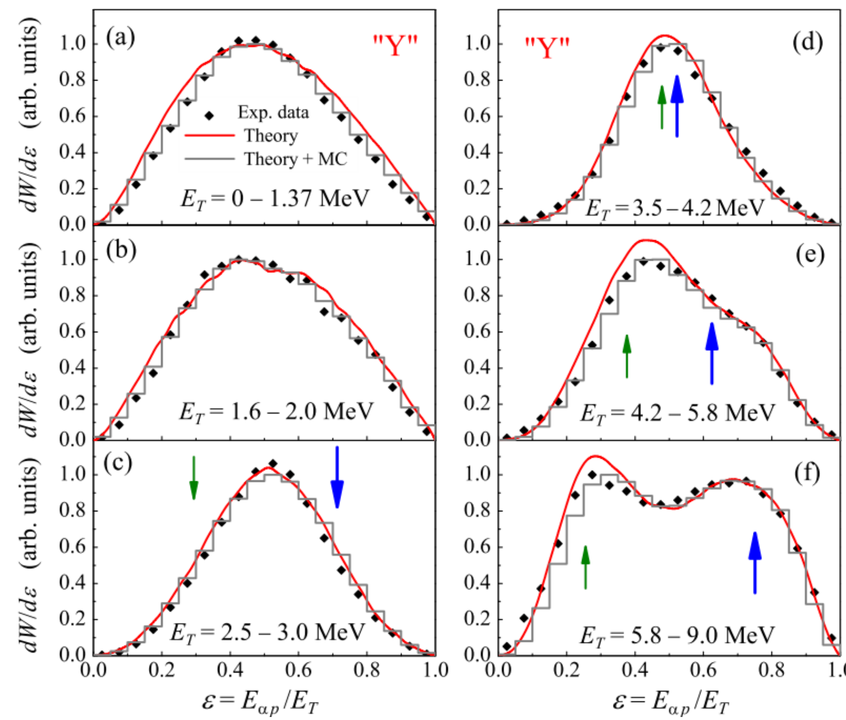
I. Egorova *et al.*, PRL **109** (2012) 202502.

- High statistics (~10^6 events/state)
- High resolution
- Nice agreement with the previous (Texas A&M, Dubna) experimental data



^{6}Be at MSU: energy evolution of correlations





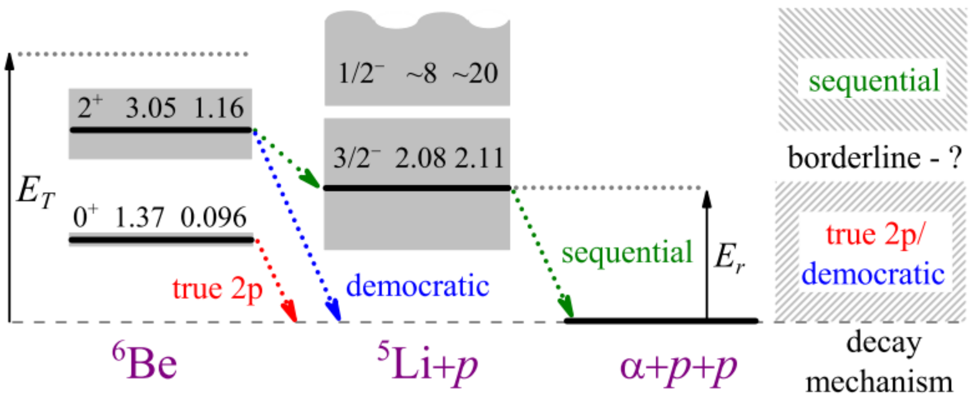
Note: the higher decay energy – the more developed is low-energy p-p correlation ("diproton")

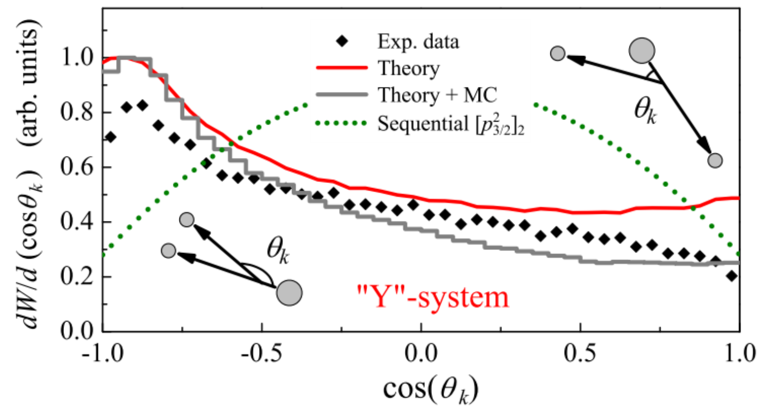
Note: when two-body states enters the decay window the intensity at expected peak position is suppressed

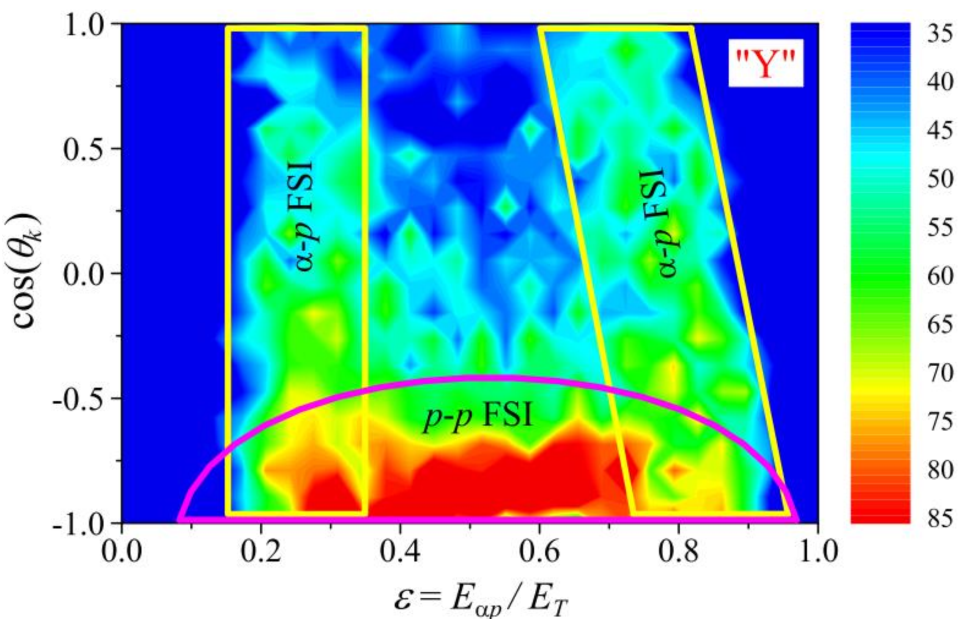
Note: above 2$^+$ the ε distribution is practically insensitive to decay energy

Note: sequential decay patterns appears only for $E_T > 2E_r + \Gamma$

^{6}Be at MSU: where is borderline democratic/sequential?



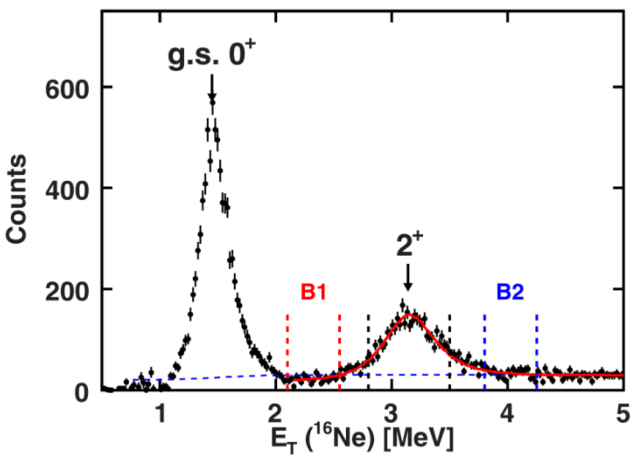




- Sequential decay pattern in energy distribution is formed at 6-9 meV.
- Is the decay sequential? NO.
- Angular correlation shows complex behavior
- Energy angular correlations elucidate the actual situation: mixture of p-p and a-p FSIs

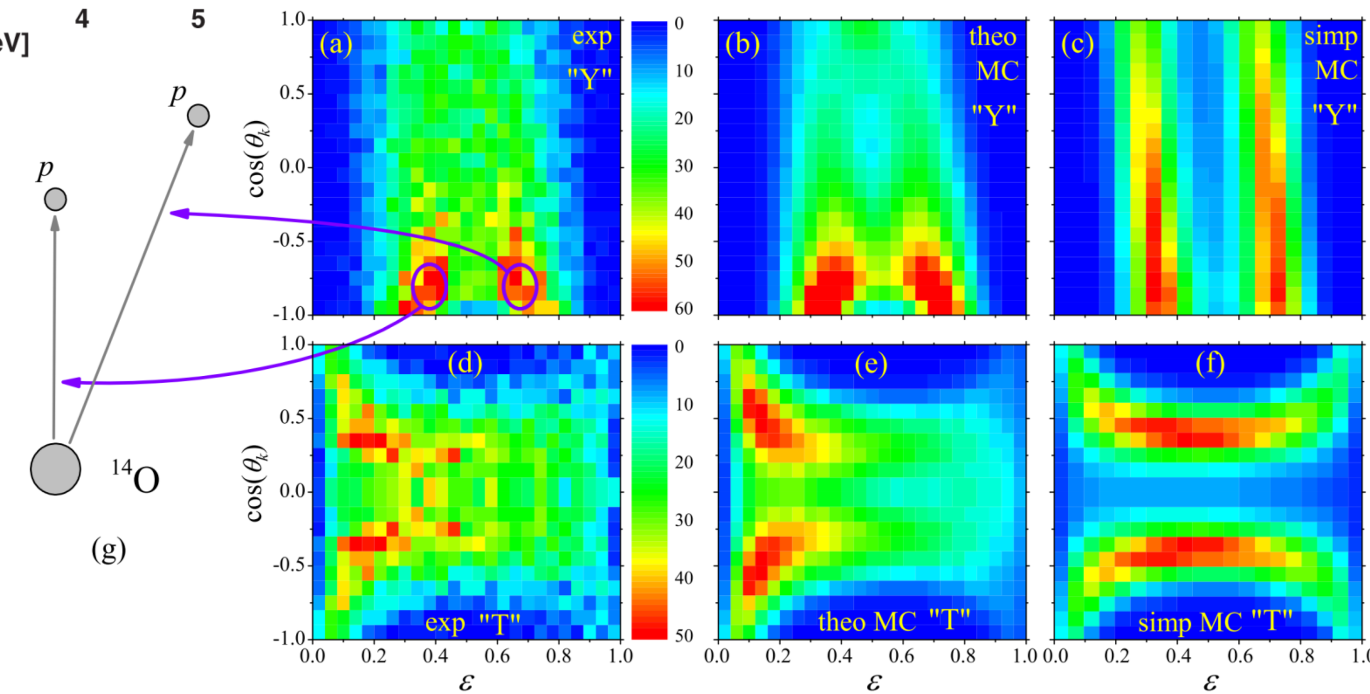
Highly detailed data allows deep insights in the decay dynamics

Interplay between sequential and prompt two-proton decay from the first excited state of ^{16}Ne

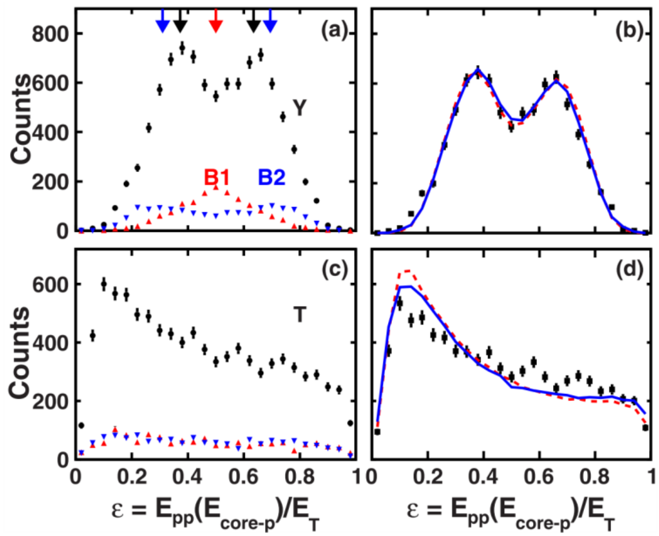


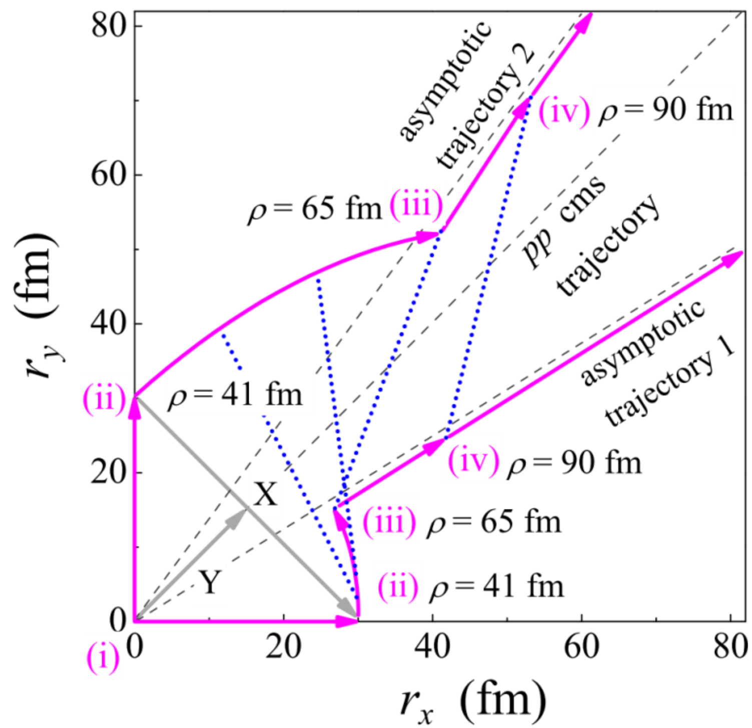
K. Brown et al., PRC **113** (2015) 034329

Data vs full three-body vs sequential



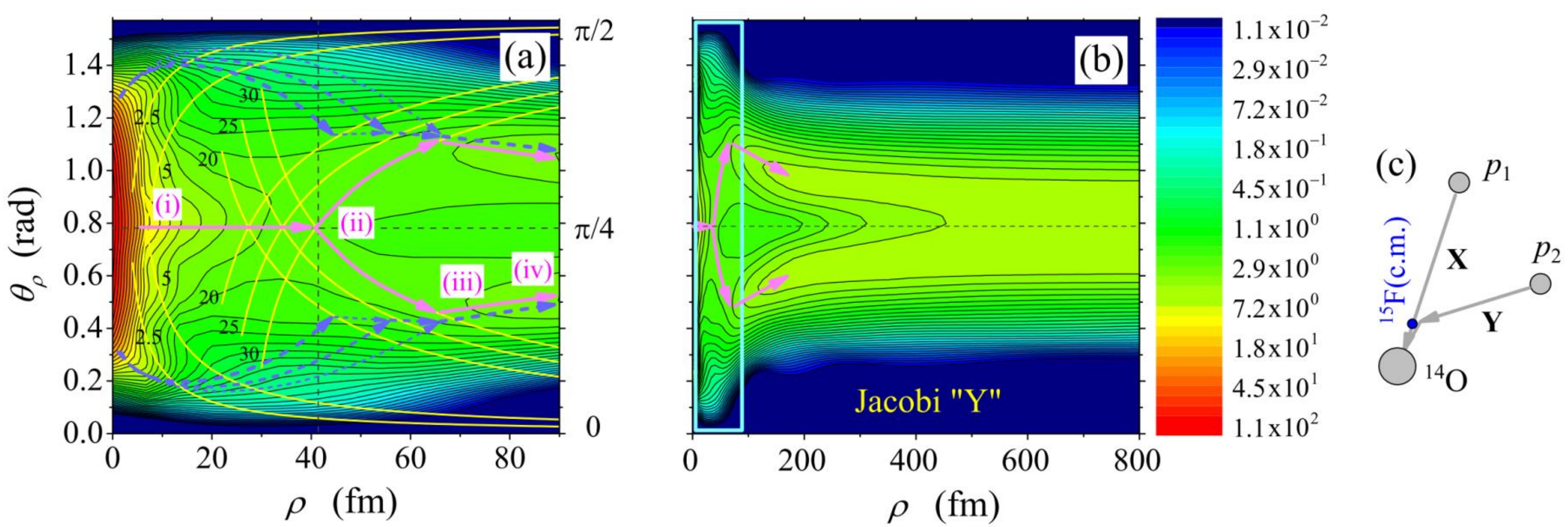
“Tethered decay mechanism” for the ^{16}Ne 2$^+$ state





$$\theta_\rho \sim \pi/4 \quad \rightarrow \quad X \sim Y \sim \rho/\sqrt{2}.$$

$$\theta_\rho \rightarrow \theta_\varkappa, \quad E_x = E_T \sin^2(\theta_\varkappa), \quad E_y = E_T \cos^2(\theta_\varkappa).$$



Simplified approaches to three-body decays

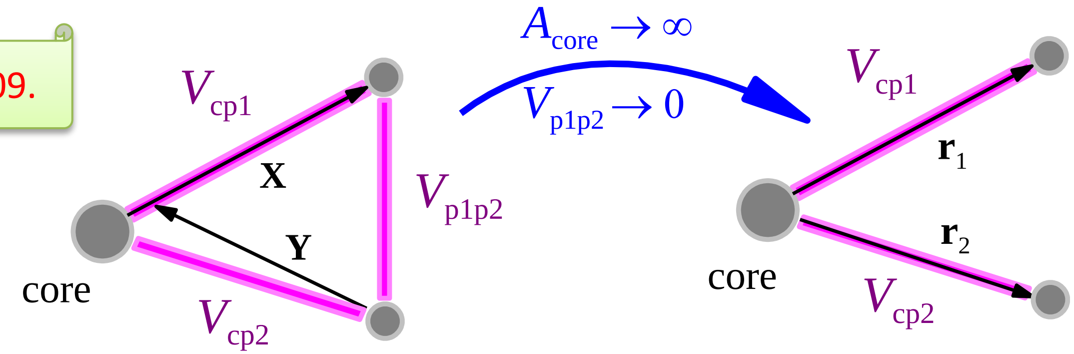
One do not necessarily need extremely complicated computational codes to get decent idea about certain aspects of three-body decays. Important results could be obtained based on the models with simplified three-body Hamiltonians leading to compact analytical expressions

Simplified approach to 2p decay: direct decay model

Grigorenko and Zhukov, PRC **76** (2007) 014009.

- Green's function for the three-body Coulomb problem is unknown in a general analytical form
- Approximations
  - proton-proton interaction is neglected (reasonable in heavy-core systems)
  - One of core-proton potentials assumed to depend on Jacobi Y coordinate (becomes precise as core mass goes to infinity)
- Analytical Green's function
- Resonance is provided by three-body potential depending on ρ.
- "Correction procedure"

$$\Gamma_{corr} = \frac{j_{corr}}{N_{corr}} \equiv \Gamma_{appr} = \frac{j_{appr}}{N_{appr}}$$



$$\bar{G}_E^{(+)}(XX',YY') = \frac{1}{2\pi i}\int_{-\infty}^{\infty} d\varepsilon\, \bar{G}_\varepsilon^{(+)}(XX')\bar{G}_{E-\varepsilon}^{(+)}(YY')$$

$$\Psi_{corr}^{(+)}(X,Y) = \int dX'dY'\, \bar{G}_E^{(+)}(XX',YY')\left[\bar{V}-V\right]\Psi_{appr}^{(+)}(X',Y')$$

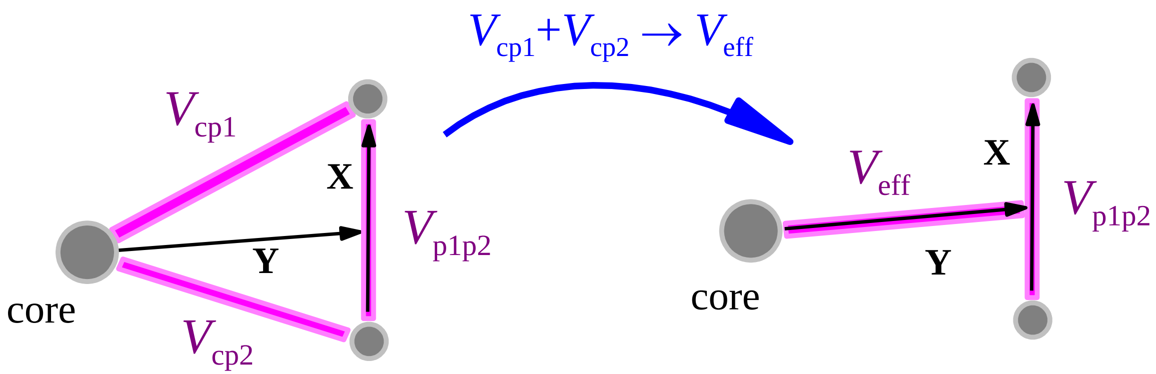
$$j_{corr} = \frac{1}{M_x}\sum_m R^2 \frac{1}{M}\,\mathrm{Im}\left[X^2 d\Omega \int dY\, \Psi_{corr}^{(+)*}\nabla_x \Psi_{corr}^{(+)}\right]$$

$$\Gamma_{corr} = \frac{j_{corr}}{N_{corr}} = \frac{8}{\pi}\int_0^E d\varepsilon \frac{1}{v_x(\varepsilon)v_y(\varepsilon)}\left|A(\varepsilon)\right|^2$$

$$A(\varepsilon) = \int_0^\infty dXdY\;\; \varphi_{l_x}(k_x X)\varphi_{l_y}(k_y Y)\left[\bar{V}-V\right]\tilde{\psi}_l(X,Y)$$

$$\begin{aligned}\Gamma_{\mathrm{dir}}(E_T) &= \frac{E_T\langle V_3\rangle^2}{2\pi}\int_0^1 d\varepsilon \frac{\Gamma_{p_1}(\varepsilon E_T)}{(\varepsilon E_T - E_{p_1})^2 + \Gamma_{p_1}(\varepsilon E_T)^2/4} \\ &\times \frac{\Gamma_{p_2}((1-\varepsilon)E_T)}{((1-\varepsilon)E_T - E_{p_2})^2 + \Gamma_{p_2}((1-\varepsilon)E_T)^2/4}\end{aligned}$$

Diproton model: no, thank you

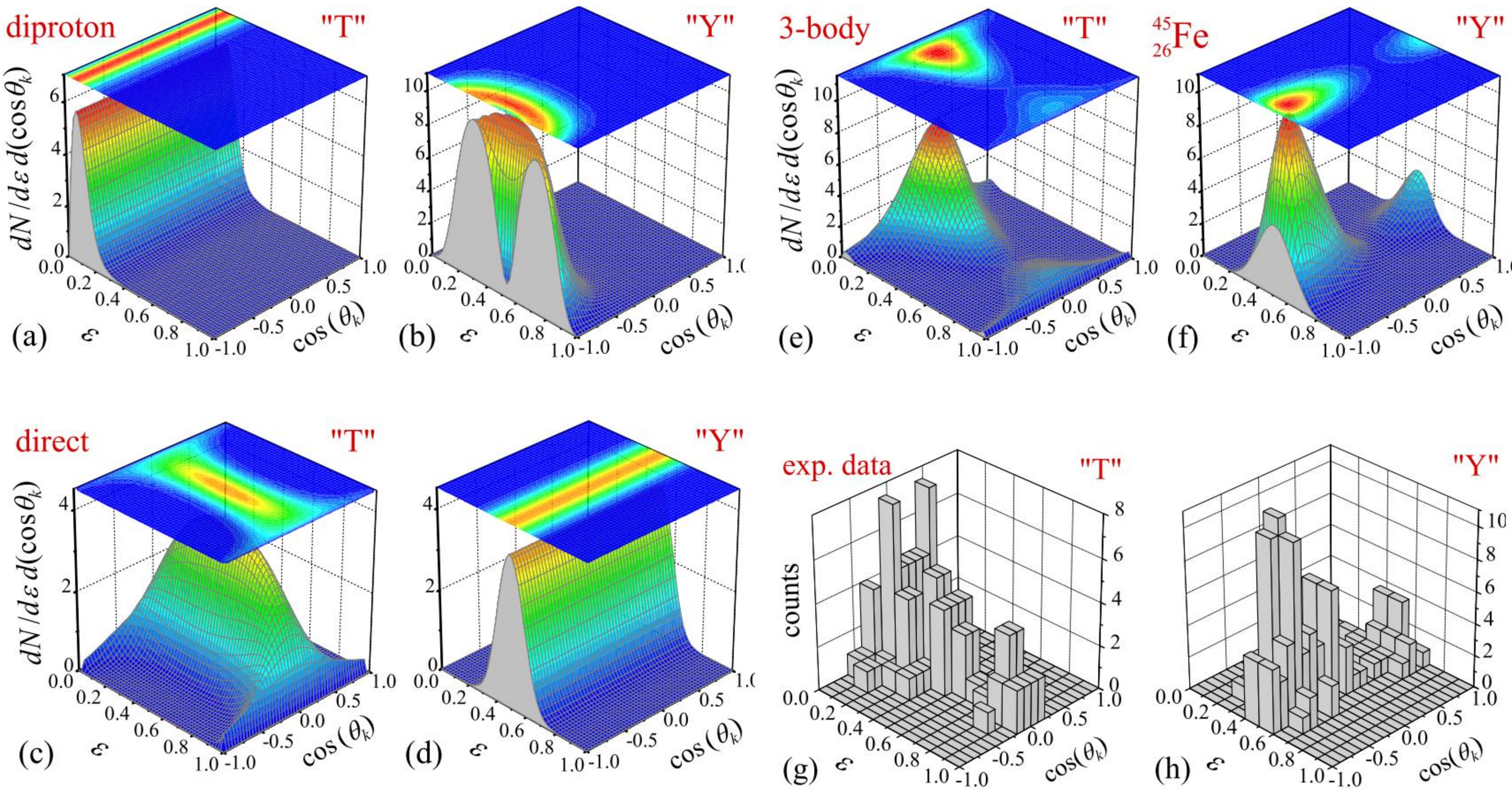


$$\Gamma_{dp}(E_T) = 2\gamma_{pp}^2 \int_0^1 d\varepsilon \rho(\varepsilon E_T)\, P_0(E_T(1-\varepsilon), R_{\mathrm{dp}}, 2Z_{\mathrm{core}})$$

- Proposed as one of a possible approximations in pioneering paper on 2p decay Goldansky, NPA **19** (1960) 482.
- Factorization of the degrees of freedom in “T” Jacobi system. Exact Green’s function exist for the system.
- Derivation of simple expression for preexponent is not possible.
- Used properly this model underestimate width typically 2 orders of the magnitude.
- Application of this formula is pure phenomenology without theoretical basis.

Correlations in the simplified models

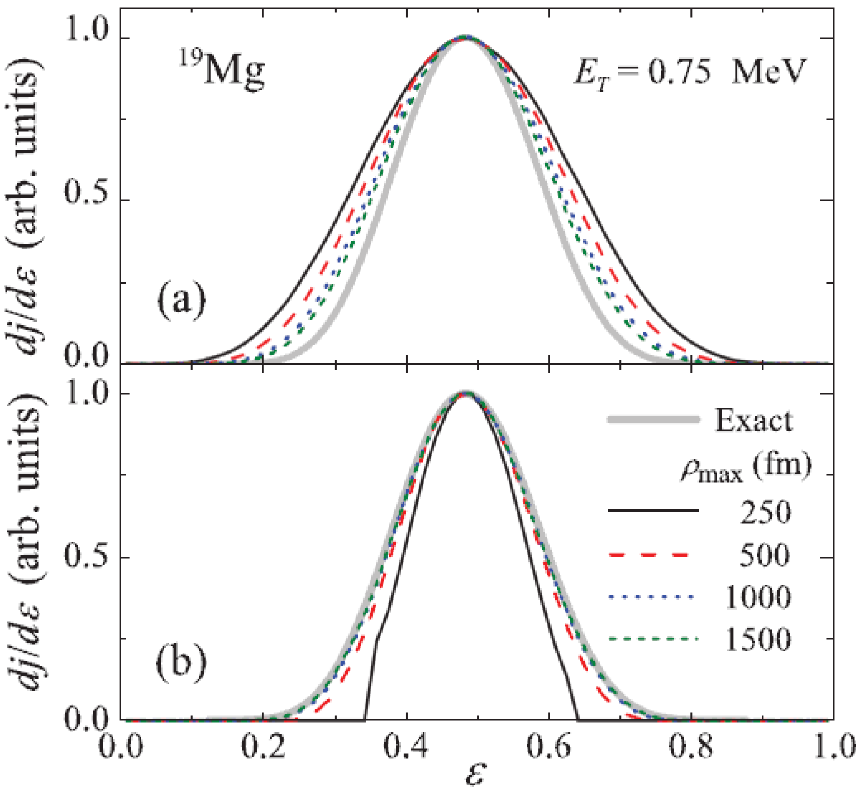
- Three-body model reproduce experimental distributions in details.
- Direct decay model provide some distributions correctly and some not.
- Diproton model does not provide any momentum distributions correctly.



Precision check of the three-body calculations with simplified Hamiltonians

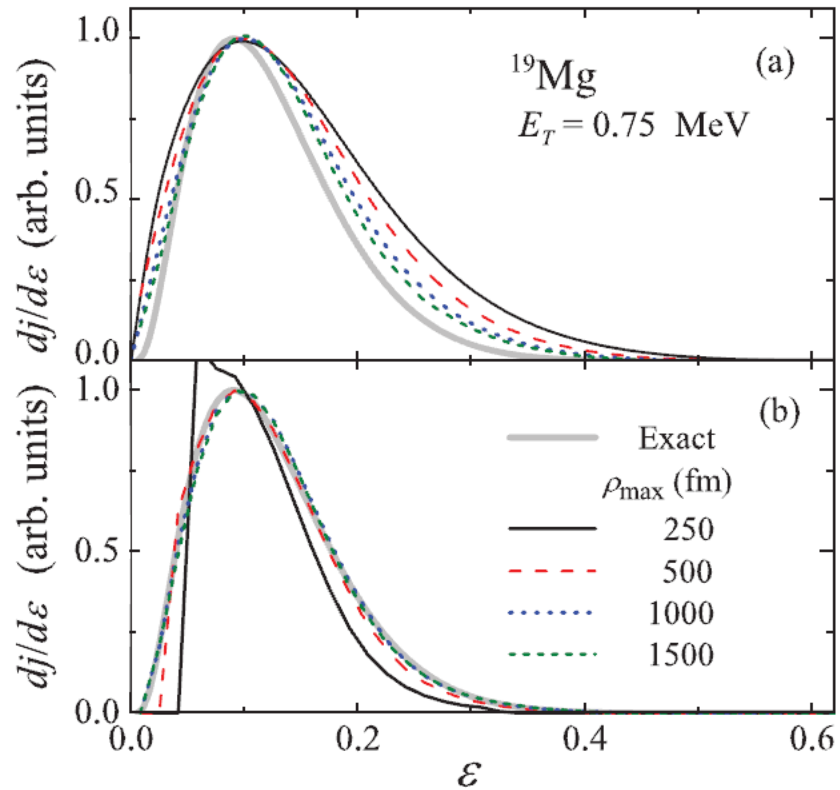
L.V. Grigorenko,et al., PRC **82**, 014615 (2010)

- For direct decay model



$\rho_{ext} = 4\ 10^5$ fm

- For diproton model

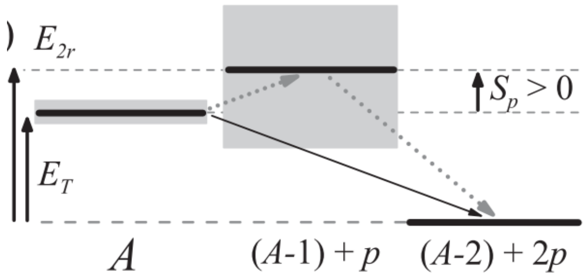


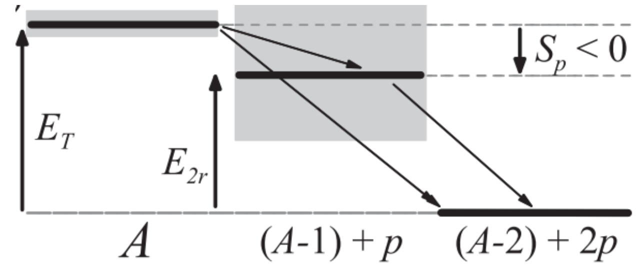
$\rho_{ext} = 10^5$ fm

Transitional dynamics and “phase transition” diagram for the three-body decay

Studies of decays which dynamics lies in between well defined reaction mechanisms may lead to important results as in the case of such transitional dynamics observables have strong sensitivity to parameters typical for phase transition situations

Mechanisms of 2p decay defined by separation energies

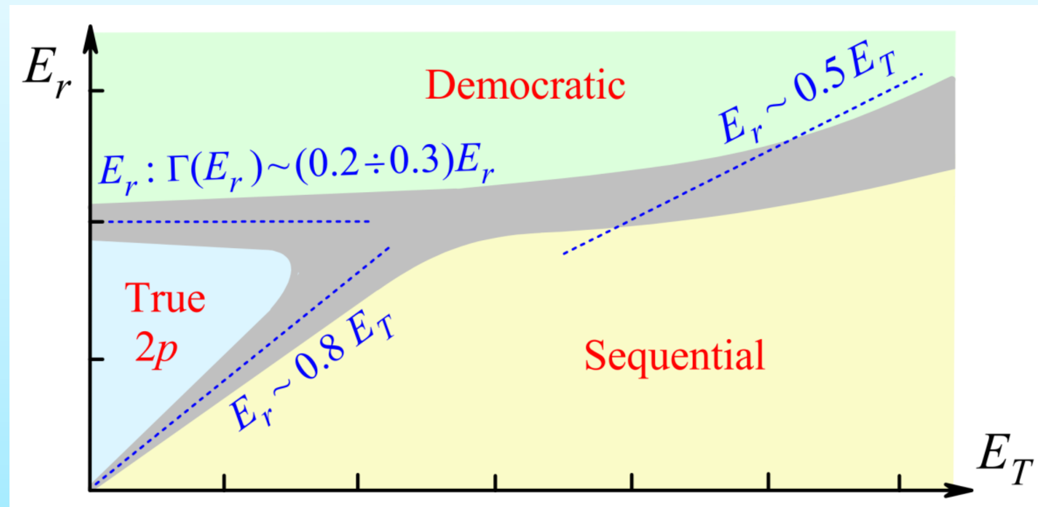




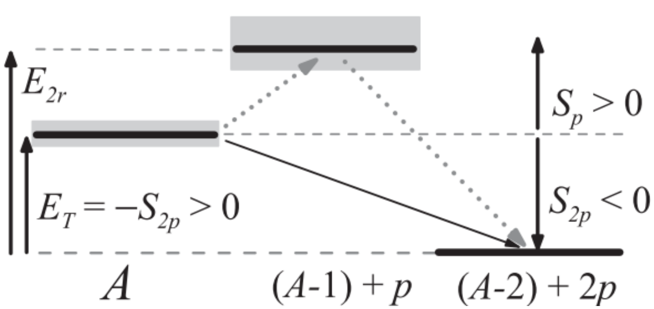
Three major decay mechanisms: True 2p, Democratic 2p Sequential 2p

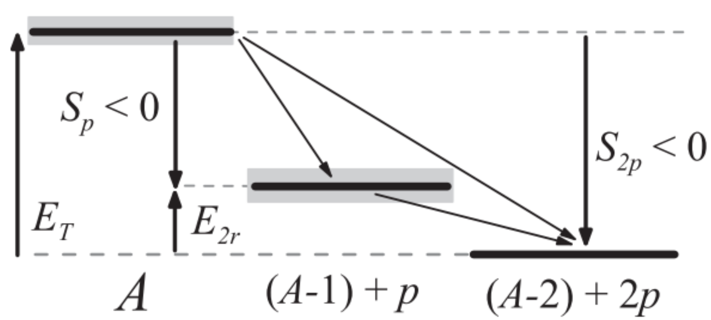
Three principal parameters:

$E_T \;\; E_r \;\; \Gamma_r$

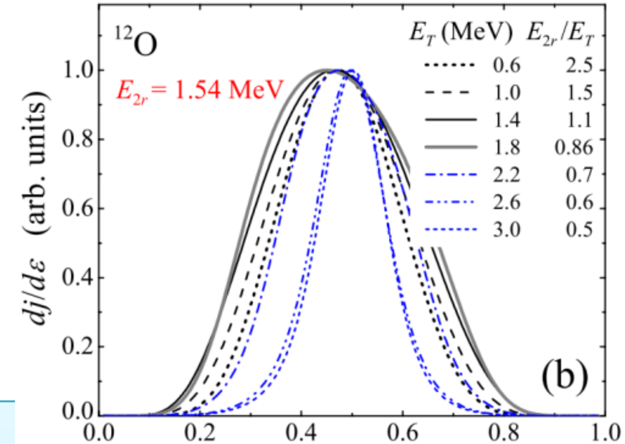


There SHOULD EXIST transition region between them





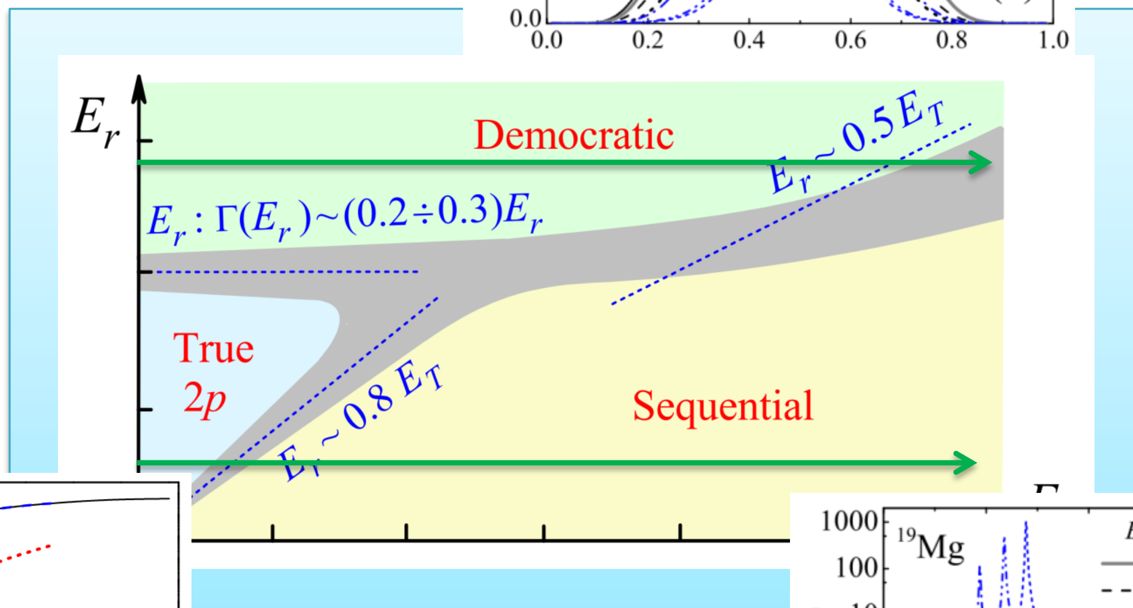
Mechanisms of 2p decay

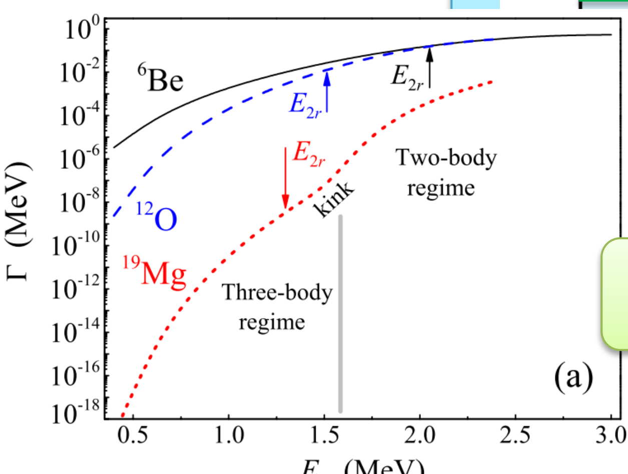


Democratic 2p <-> Sequential 2p

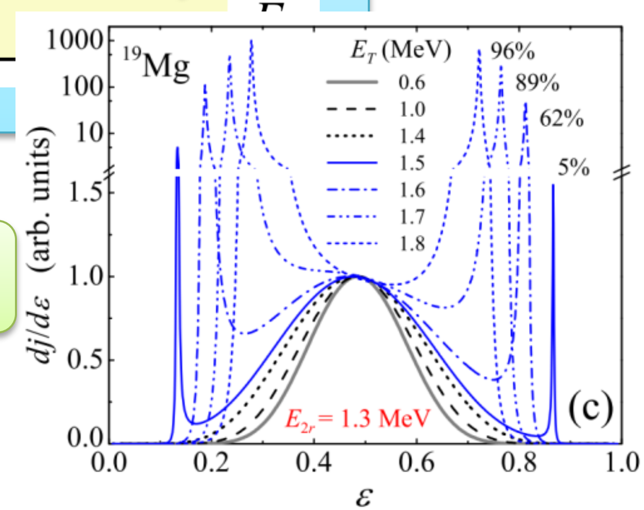
Energy correlations between core and one proton

Lifetime systematics





True 2p <-> Sequential 2p

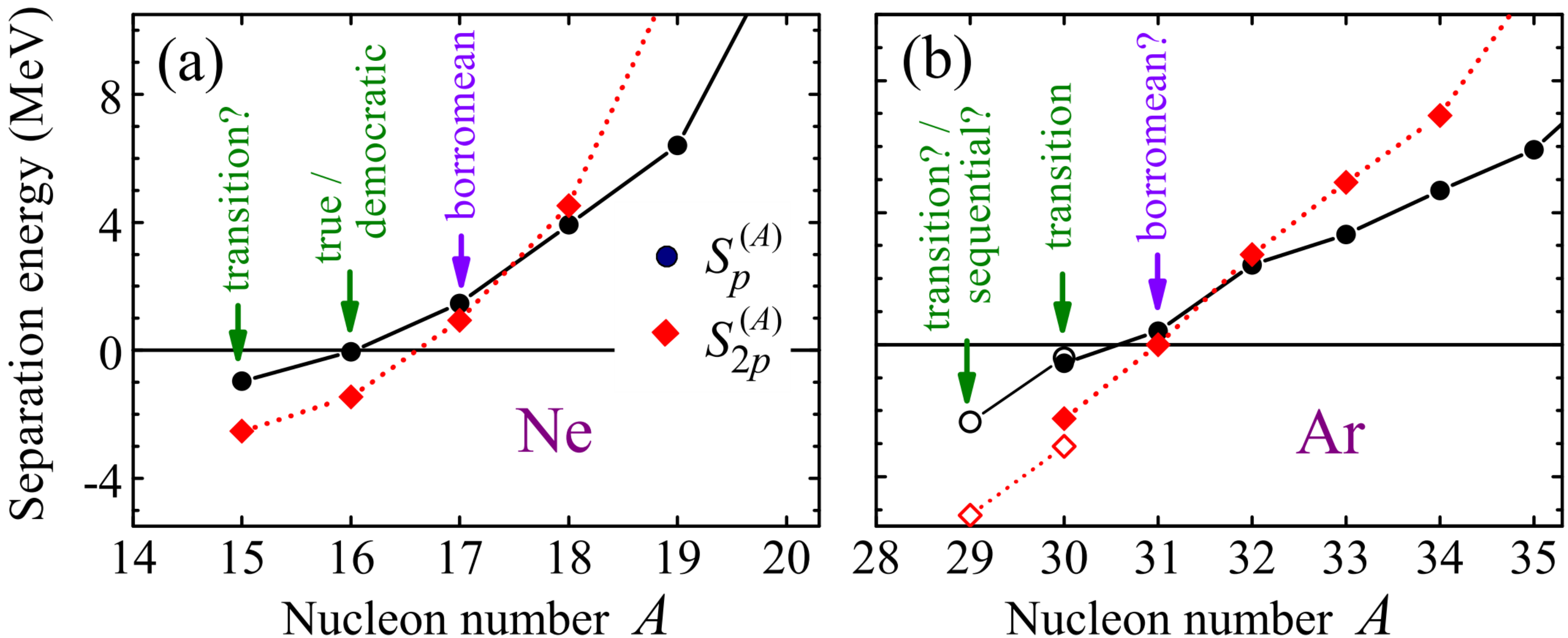


Transition decay mechanism beyond the dripline

Systematics of proton and two-proton separation energies

Even number of protons. Upper and lower s-d shell

Systematic change of dynamics on the way to the dripline and beyond: "normal" -> Borromean -> true 2p -> transitional -> sequential 2p



All three mechanisms of 2p emission as well as transition situation change each other on the move away from the dripline

Transition decay dynamics in simplified semianalytical models

$$\Gamma_{j_1 j_2}(E_T) = \frac{E_T \langle V_3 \rangle^2}{2\pi} \int_0^1 d\varepsilon \, \frac{\Gamma_{j_1}(\varepsilon E_T)}{(\varepsilon E_T - E_{j_1})^2 + \Gamma_{j_1}(\varepsilon E_T)^2/4}$$

$$\times \frac{\Gamma_{j_2}((1-\varepsilon)E_T)}{((1-\varepsilon)E_T - E_{j_2})^2 + \Gamma_{j_2}((1-\varepsilon)E_T)^2/4}$$

Stems from simplified three-body Hamiltonian

Basic feature - strong dependence on two resonances in the subsystems

Recent upgrade to “improved direct decay model”

T.A. Golubkova *et al.*, PLB **762** (2016) 263

$$\frac{d\Gamma(E_{3r})}{d\Omega_\varkappa} = \sum_{LS} \frac{E_{3r}}{2\pi(2L+1)} \sum_{M_L} \left| \sum\nolimits_\gamma A^{LM_L}_{S\gamma}(\Omega_\varkappa) \right|^2 ,$$

$$A^{LM_L}_{S\gamma}(\Omega_\varkappa) = C^{JLS}_\gamma V^J_\gamma \, [l_1 \otimes l_2]_{LM_L} \, A_{j_1 l_1}(E_1) A_{j_2 l_2}(E_2) \, ,$$

$$[l_1 \otimes l_2]_{LM_L} = \sum\nolimits_{m_1 m_2} C^{LM_L}_{l_1 m_1 l_2 m_2} Y_{l_1 m_1}(\hat{r}_1) Y_{l_1 m_1}(\hat{r}_1) \, .$$

$$A^{LM_L}_{S\gamma}(\Omega_\varkappa) \rightarrow \frac{C^{JLS}_\gamma V^J_\gamma \, A^{(pp)}_S(E^T_x)}{E_{r1} + E_{r2} - E_T - i\,[\Gamma_1(E_{r1}) + \Gamma_2(E_{r2})]/2}$$

$$\times \hat{O}_S \left([l^{Y_1}_x \otimes l^{Y_1}_y]_{LM_L} \, A_{j^{Y_1}_x l^{Y_1}_x}(E^{Y_1}_x) \sqrt{\Gamma_1(E^{Y_1}_y)} \right.$$

$$\left. + \, [l^{Y_2}_x \otimes l^{Y_2}_y]_{LM_L} \, A_{j^{Y_2}_y l^{Y_2}_y}(E^{Y_2}_x) \sqrt{\Gamma_2(E^{Y_2}_y)} \right) \, .$$

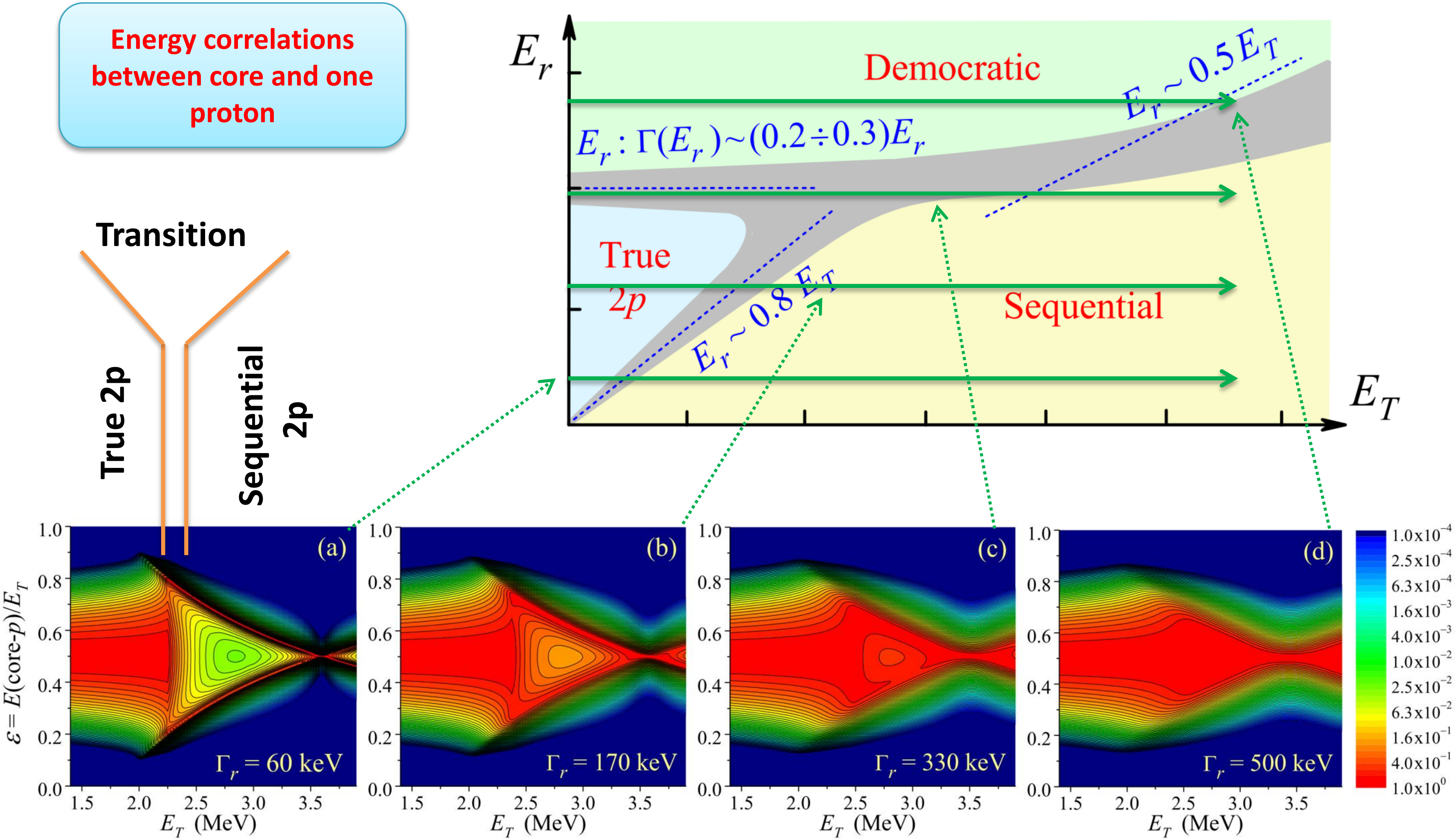
$$A_{jl}(E) = \frac{\sqrt{\Gamma_r(E)}}{E_r - E - i\Gamma_r(E)/2} + A^{(p)}_{jl}(E)$$

Correct angular momentum coupling, amplitude symetries, NN FSI correction, etc.

General view of transitional dynamics

"^{30}Ar"

Energy correlations between core and one proton



EXPERT@SuperFRS@FAIR

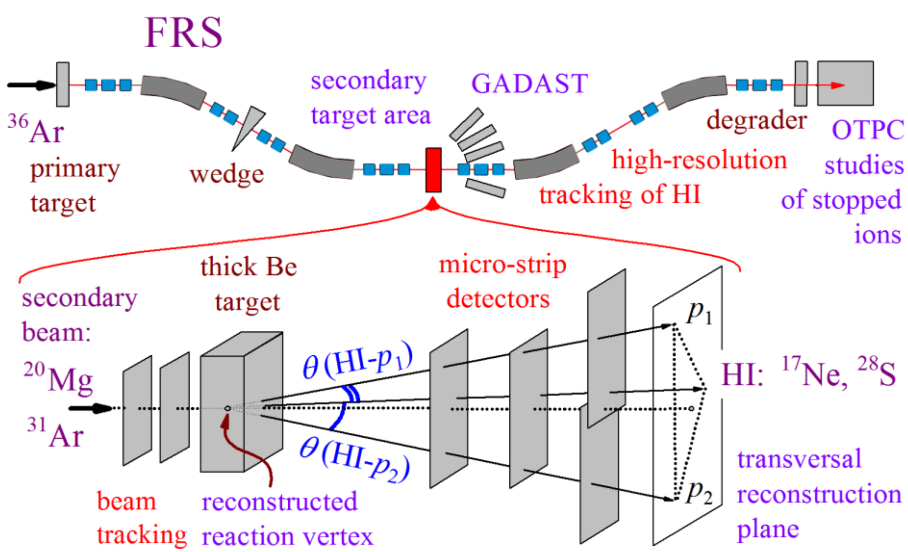
This method features

Simultaneously (i) lifetime in the ps range
(ii) spectroscopy of the short-lived states

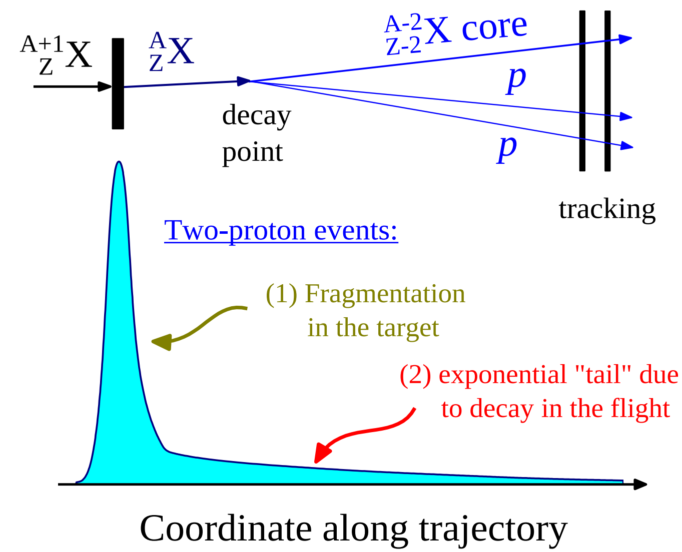
Extremely thick secondary targets up to ~25 mm

Efficient work with poor “coctail” beams

***Basic idea* is old, *technological realization* is modern and sophisticated**



New method of radoactivity studies:
I.G. Mukha *et al.*, PRL **99** (2007) 182501.
Investigation of radioactive decays with particle emission by product tracking

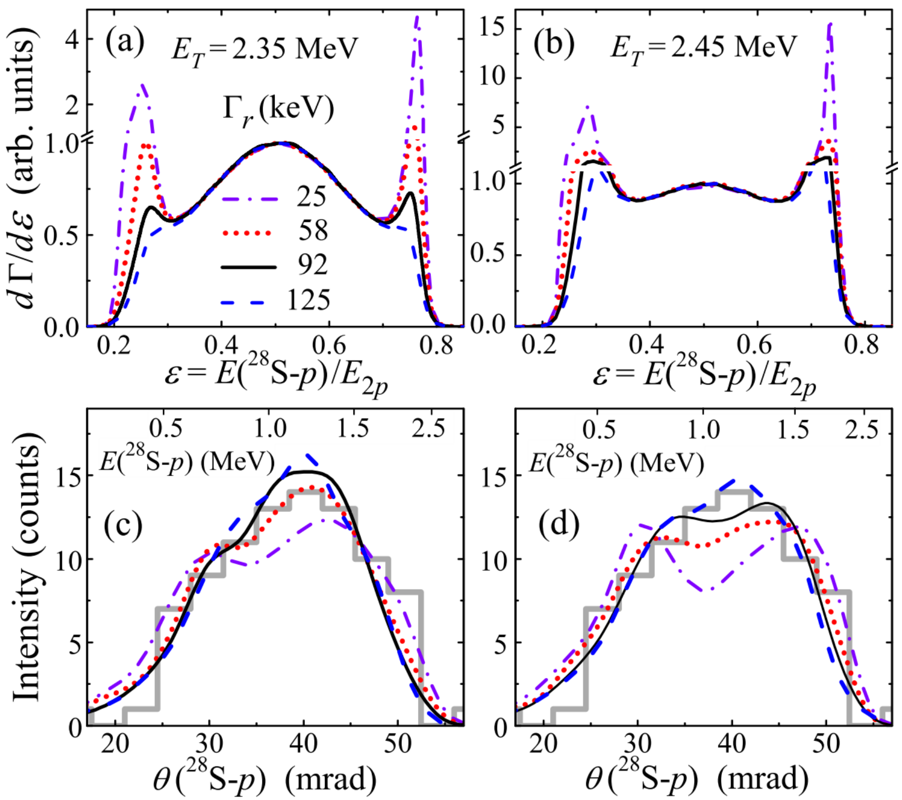


Recent experiment in this technique

I. Mukha et al., PRL **115** (2015) 202501.
A. Lis et al., PRC **91** (2014) 064309.

Simultaneous results from tracking system (decay spectroscopy) and OTPC (β-delayed decays) – two experiments for the price of one

^{29}Cl g.s. width from ^{30}Ar data



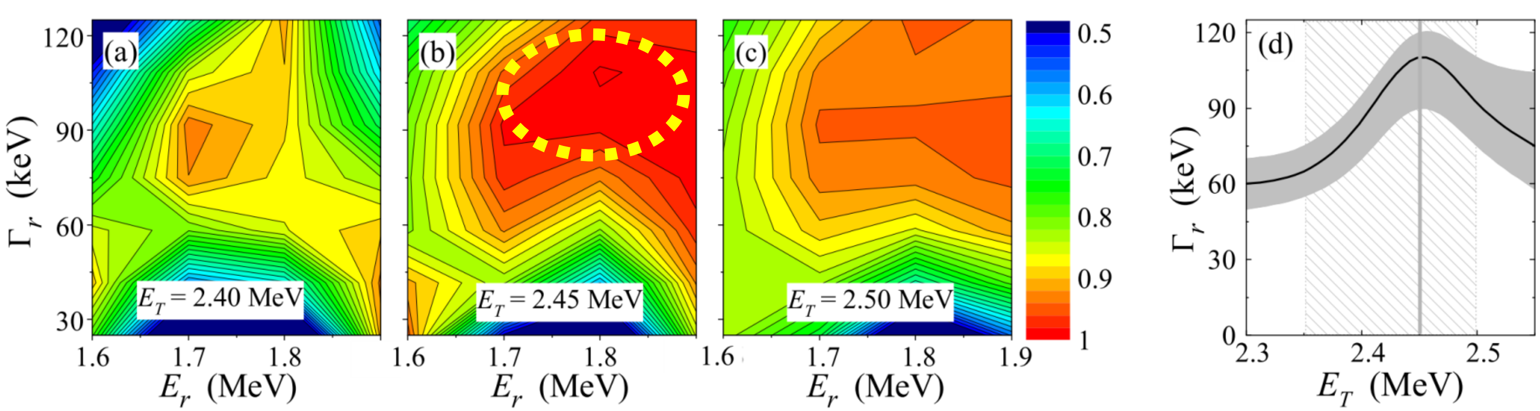
Energy is "easy" to measure, width could be very complicated. From $T_{1/2} \sim 1$ ps to $\Gamma \sim 100$ - 200 keV there is a "blind spot" no accessible for direct measurements

^{30}Ar was found to have transition decay dynamics

Strong dependence of the experimental signal on the g.s. properties of core+p subsystem – ^{29}Cl

Stringent limits for ^{29}Cl g.s. width

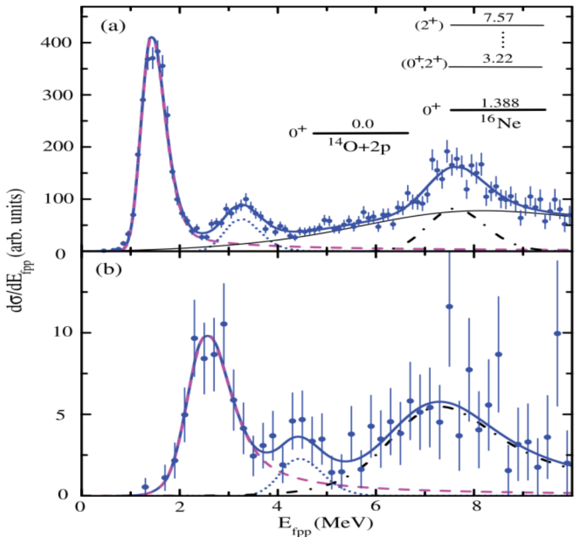
T.A. Golubkova *et al.*, PLB **762** (2016) 263



Prospects to observe transition dynamics in ^{15}Ne

F. Wamers *et al.*, PRL **112** (2014) 132502

^{16}Ne studies and ^{15}Ne discovery, GSI

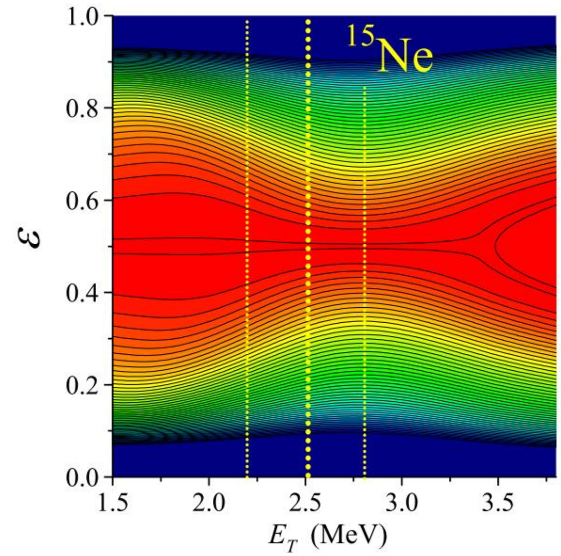


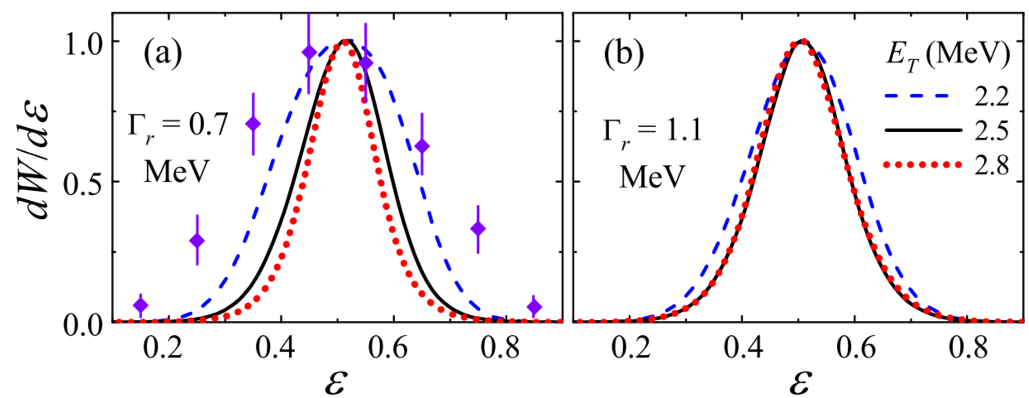
V. Goldberg *et al.*, PLB **692** (2010) 307

^{14}F, TEXAS A&M

Levels in ^{14}F.

E_R (MeV)[a]	E_x[b]	J^π	Γ (keV)	Γ/Γ_{sp}
1.56 ± 0.04	0.00	2^-	910 ± 100	0.85
2.1 ± 0.17	0.54	1^-	~ 1000	0.6
3.05 ± 0.060	1.49	3^-	210 ± 40	0.55
4.35 ± 0.10	2.79	4^-	550 ± 100	0.5





Proposal: to study energy evolution of three-body correlations across the energy of broad (Γ~0.6 MeV) g.s. of ^{15}Ne to extract ^{14}F width

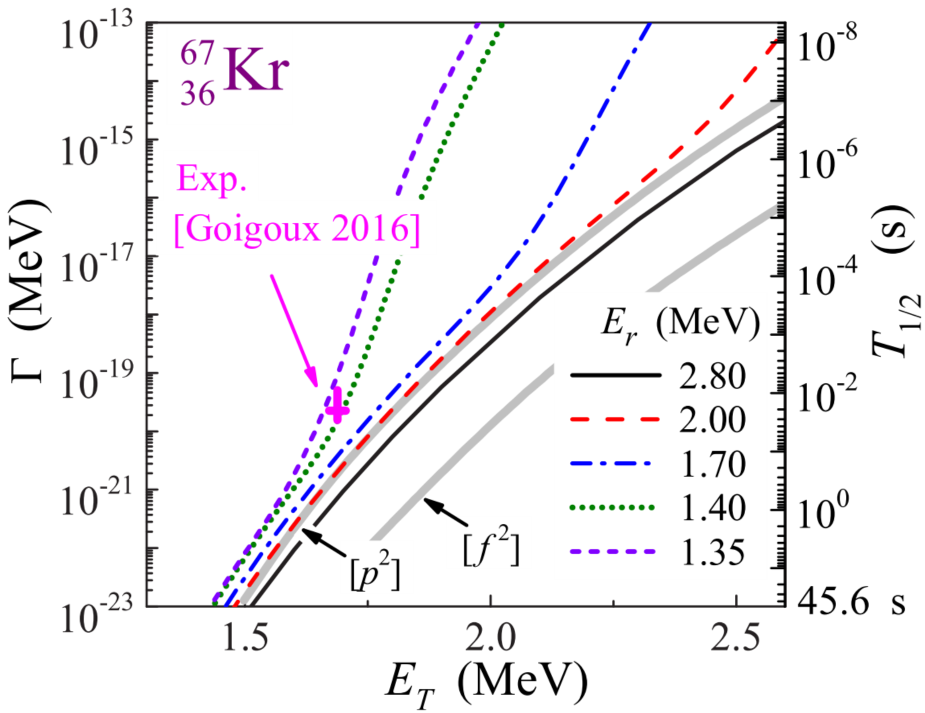
Problem of ^{67}Kr

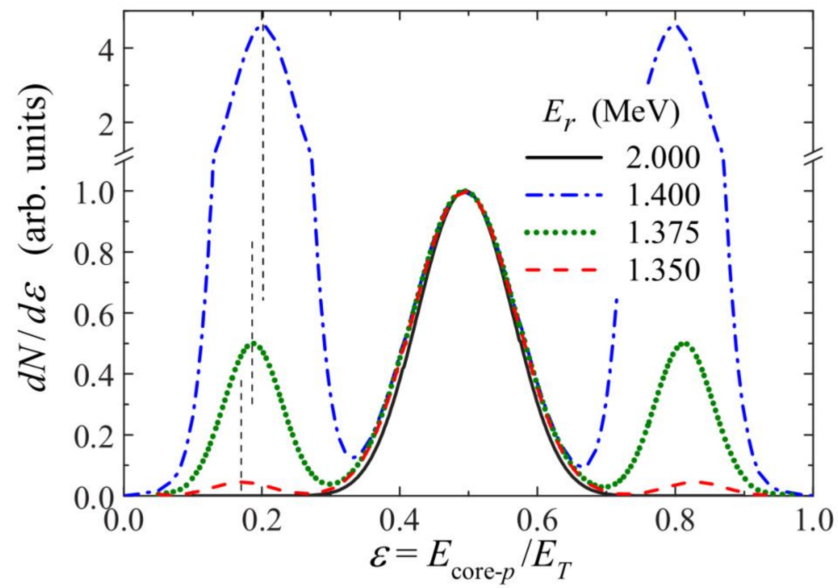
EXP: T. Goigoux et al., Phys. Rev. Lett. **117**, 162501 (2016)

TH: L. Grigorenko et al., Phys. Rev. C **95**, 021601(R) (2017)

^{67}Kr
Decay energy 1690(17)
Decay width 7.3(3.0) ms

^{67}Kr width is larger than any prediction of the three-body model. Possible reason: not true 2p, but transitional dynamics of 2p decay.





Proposal: to study correlations as the indicative signature of decay mechanism

Multi-neutron radioactivity

It could be that it is more probable to find very long-living (radioactivity timescale) four-neutron emitters than two-neutron.

HH equations for N particles

For "true" N-body systems which dynamics is well described by finite set of hyperspherical equations the effective centrifugal barriers

The minimal effective centrifugal barriers grows as the number of particles grows

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}\rho^2} - \frac{\mathcal{L}(\mathcal{L}+1)}{\rho^2} + 2M\left\{E_{\mathrm{T}} - V_{K\gamma,K\gamma}(\rho)\right\}\right]\chi_{K\gamma}(\rho) = \sum_{K'\gamma'} 2MV_{K\gamma,K'\gamma'}(\rho)\chi_{K'\gamma'}(\rho) + f_{K\gamma}(\rho)\,,$$

$$V_{K\gamma,K'\gamma'}(\rho) = \int \mathrm{d}\Omega_\rho\, \mathcal{J}^{\dagger}_{K\gamma}(\Omega_\rho)\left[\sum_{i>j}\hat{V}(\mathbf{r}_{ij})\right]\mathcal{J}_{K'\gamma'}(\Omega_\rho)$$

$$\mathcal{L} = K + (3A-6)/2$$

A = 3 min = 3/2

A = 4 min = 3

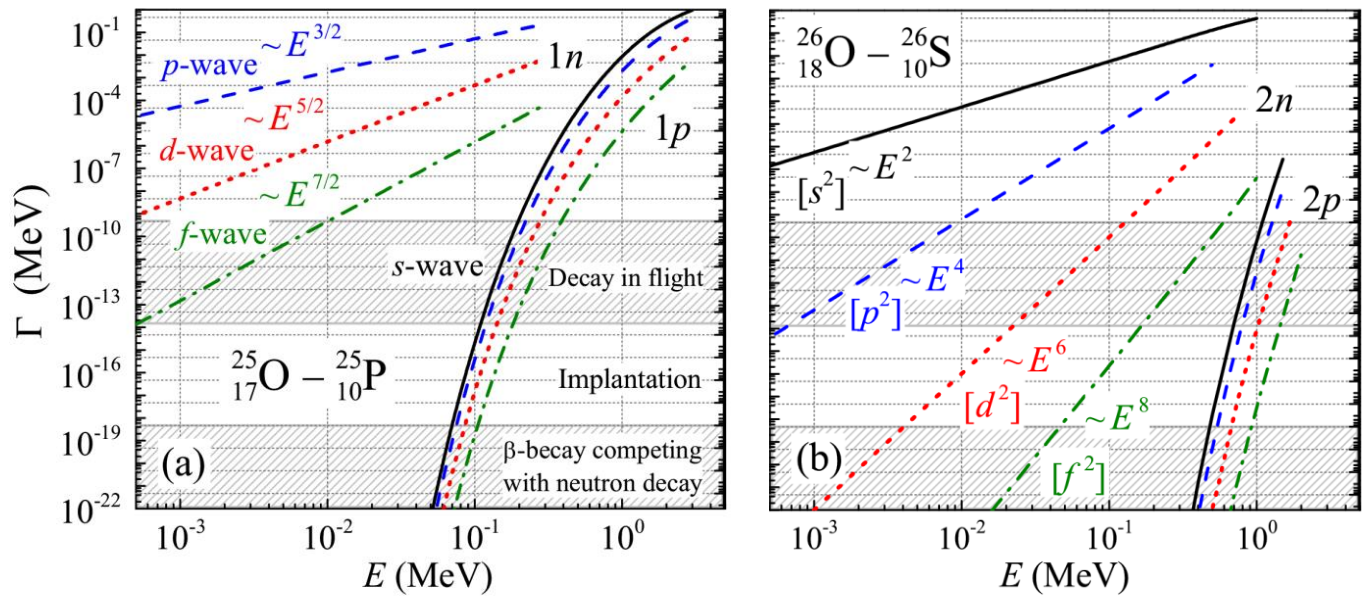
A = 5 min = 9/2

What about neutron radioactivity?

Two-proton radioactivity is the long awaited and the most recently found mode of the radioactive decay.

Natural question: Can neutron radioactivity exist?

L.V. Grigorenko, I.G. Mukha, C. Scheidenberger, and M.V. Zhukov, PRC **84** (2011) 021303(R)



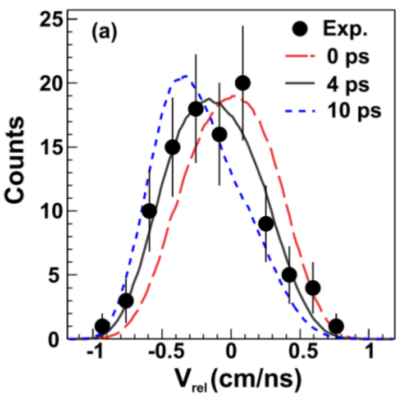
- Estimates: **one-neutron** radioactivity is highly unlikely.
- There are additional effective few-body “centrifugal” barriers making few-body emission relatively slower.
- Long-living **Two-neutron** decay states are reasonably probable.

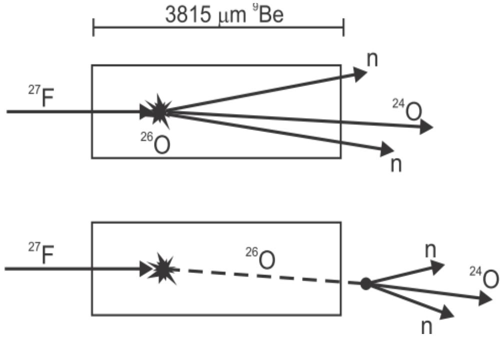
^{26}O Recent studies of 2n decay in :

- Lunderberg *et al.,* PRL **108**, 142503 (2012)
- C. Caesar *et al.,* PRC **88**, 034313 (2013)
- Z. Kohley *et al.,* PRL **110**, 152501 (2013)
- Y. Kondo et al., PRL **116**, 102503 (2016)

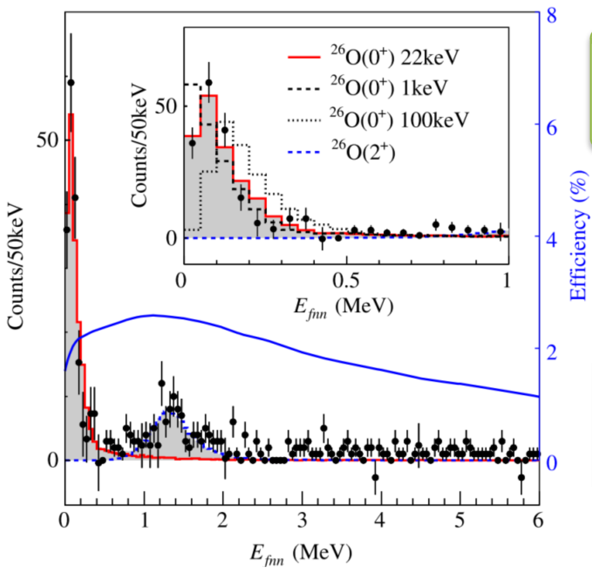
2n radioactivity in ^{26}O ?

Z. Kohley *et al.*, PRL **110**, 152501 (2013)





$T_{1/2}$ = 4.5 ps: 2*n* radioactivity discovered



Y. Kondo *et al.*, PRL **116**, 102503 (2016)

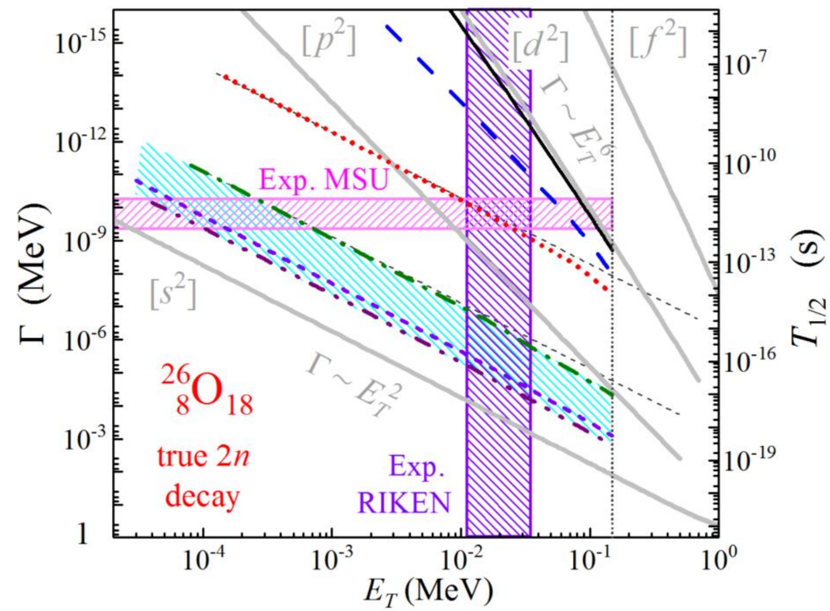
***E_T* = 22$^{+14}_{-11}$ keV: 2*n* radioactivity unlikely**

L.V. Grigorenko, I.G. Mukha, M.V. Zhukov, PRL **111** (2013) 042501

Importance of fine three-body effects

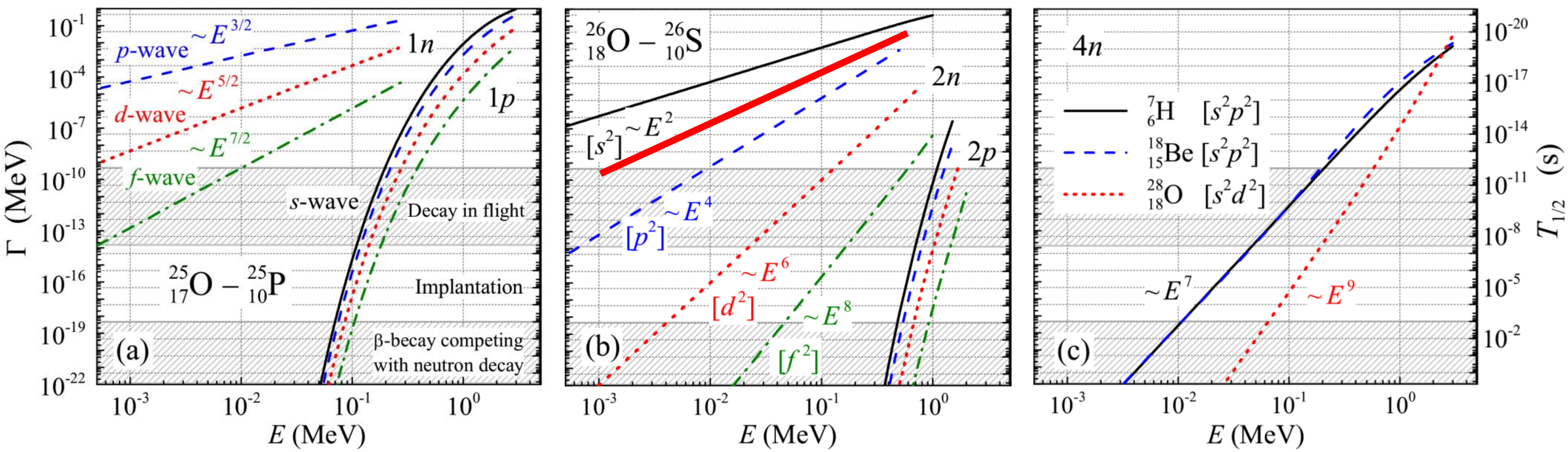
2p radioactivity:
Core recoil – negligible
NN FSI – factor 200-500

2n radioactivity:
Core recoil – factor 5-10
NN FSI – factor 2000-10000



?????????????????

Four-neutron radioactivity search prospects



n **radioactivity realistic only in f- and higher waves. Not achieved regions of the dripline**

2n **radioactivity. Simple estimate spoiled by paring interaction leading to d^2 -> s^2 "diffusion". Realistic energy < 1 keV**

4n **radioactivity. Minimal effetive barrier is high. Also minimal quantum configuration is Pauli-prohibited. Minimal is [s^2p^2] and L = 13/2**

4n **radioactivity. Realistic energy window is 100-200 keV.**